STRUCTURAL ANALYSIS – II

UNIT - 1

ROLLING LOAD AND INFLUENCE LINES:
Rolling load analysis for simply supported beams for several point loads and UDL. Influence line diagram for reaction, SF and BM at a given section for the cases mentioned in above uinit 1   **6 Hours**

UNIT - 2

SLOPE DEFLECTION METHOD:
Introduction, Sign convention, Development of slope-deflection equations and Analysis of Beams and Orthogonal Rigid jointed plane frames (non-sway) with kinematic redundancy less than/equal to three. (Members to be axially rigid)   **8 Hours**

UNIT - 3
MOMENT DISTRIBUTION METHOD:
Introduction, Definition of terms- Distribution factor, Carry over factor, Development of method and Analysis of beams and orthogonal rigid jointed plane frames (nonsway) with kinematic redundancy less than/equal to three. (Members to be axially rigid)   **8 Hours**

UNIT - 4
SWAY ANALYSIS:
Analysis of rigid jointed plane frames (sway, members assumed to be axially rigid and kinematic redundancy £ 3) by slope deflection and moment distribution methods. **4 Hours**

UNIT - 5
FLEXIBILITY MATRIX METHOD OF ANALYSIS:
Introduction, Development of flexibility matrix for plane truss element and axially rigid plane framed structural elements and Analysis of plane truss and axially rigid plane frames by flexibility method with static indeterminacy ≤ 3.   **7 Hours**

UNIT - 6

STIFFNESS MATRIX METHOD OF ANALYSIS:
Introduction, Development of stiffness matrix for plane truss element and axially rigid plane framed structural elements. And Analysis of plane truss and axially rigid plane frames by stiffness method with kinematic indeterminacy £3. **7 Hours**

REFERENCE BOOKS:
1. **Basic Structural Analysis**- Reddy C.S. - Second Edition, Tata McGraw Hill Publication Company Ltd.
2. **Theory of Structures Vol. 2** - S.P. Gupta, G.S. Pandit and R. Gupta, Tata McGraw Hill Publication Company Ltd.
3. Structural Dynamics-by M.Mukhopadhyay,
4. **Structural Analysis-II** -S. S. Bhavikatti – Vikas Publishers, New Delhi.
5. **Basics of Structural Dynamics and Aseismic Design** By Damodhar Swamy and Kavita PHI Learning Private Limited
6. **Structural Analysis**- D.S. Prakash Rao,, A Unified Approach, University Press
7. **Structural Analysis**, 4th SI Edition by Amit Prasanth & Aslam Kassimali, Thomson Learning.

TABLE OF CONTENT

UNIT - 1
ROLLING LOAD AND INFLUENCE LINES

1 Introduction: Variable Loadings

So far in this course we have been dealing with structural systems subjected to a specific set of loads. However, it is not necessary that a structure is subjected to a single set of loads all of the time. For example, the single-lane bridge deck in Figure1 may be subjected to one set of a loading at one point of time (Figure1a) and the same structure may be subjected to another set of loading at a different point of time. It depends on the number of vehicles, position of vehicles and weight of vehicles. The variation of load in a structure results in variation in the response of the structure. For example, the internal forces change causing a variation in stresses that are generated in the structure. This becomes a critical consideration from design perspective, because a structure is designed primarily on the basis of the intensity and location of maximum stresses in the structure. Similarly, the location and magnitude of maximum deflection (which are also critical parameters for design) also become variables in case of variable loading. Thus, multiple sets of loading require multiple sets of analysis in order to obtain the critical response parameters.
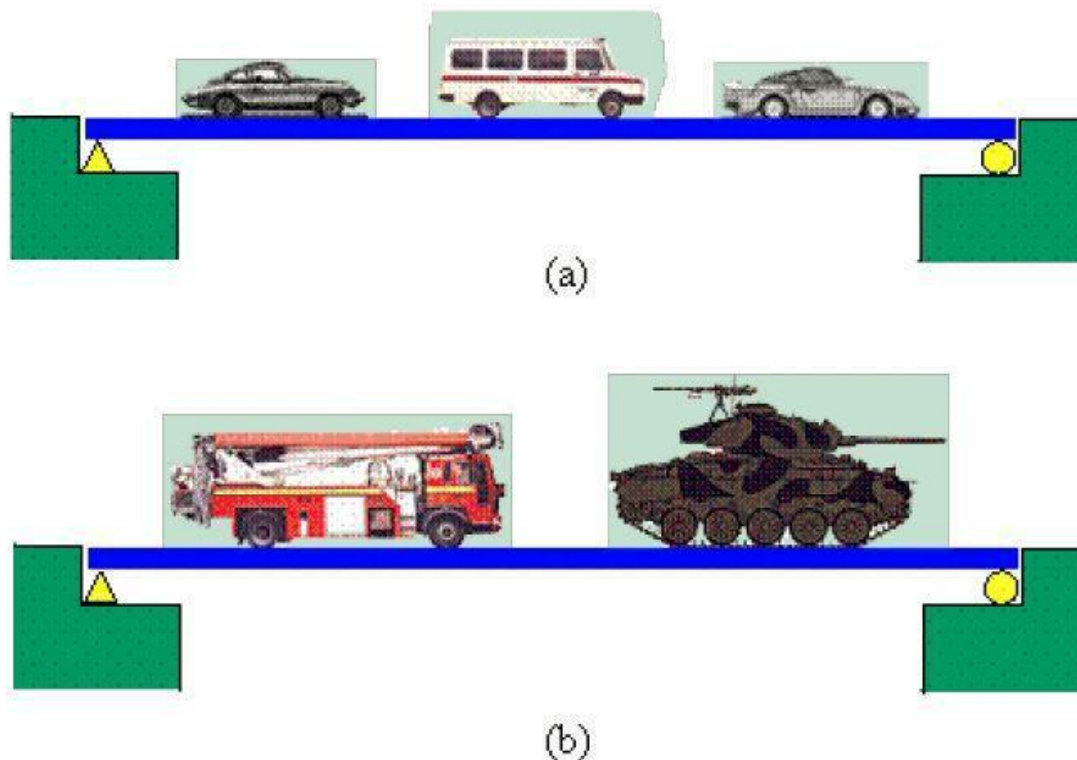

(a)

(b)

Figure 1 Loading condition on a bridge deck at different points of time

Influence lines offer a quick and easy way of performing multiple analyses for a single structure. Response parameters such as *shear force* or *bending moment at a point* or *reaction at a support* for several load sets can be easily computed using influence lines.

For example, we can construct influence lines for (shear force at B) or (bending moment at) or (vertical reaction at support D) and each one will help us calculate the corresponding response parameter for different sets of loading on the beam AD (Figure 2).
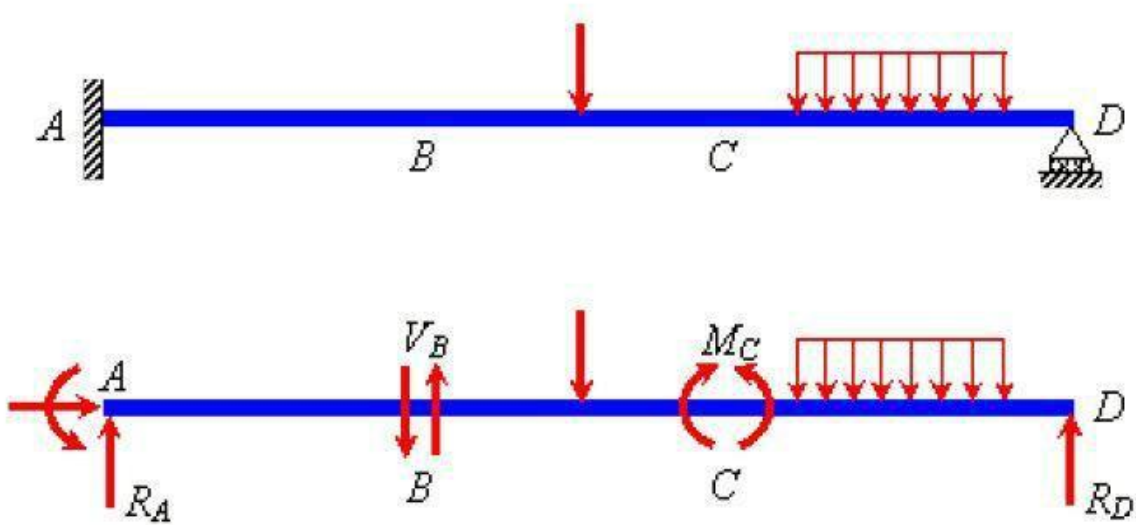
Figure 2 Different response parameters for beam AD

An influence line is a diagram which presents the variation of a certain response parameter due to the variation of the position of a unit concentrated load along the length of the structural member. Let us consider that a unit downward concentrated force is moving from point A to point B of the beam shown in Figure 3a. We can assume it to be a wheel of unit weight moving along the length of the beam. The magnitude of the vertical support reaction at A will change depending on the location of this unit downward force. The influence line for (Figure3b) gives us the value of for different locations of the moving unit load. From the ordinate of the influence line at C, we can say that when the unit load is at point C.
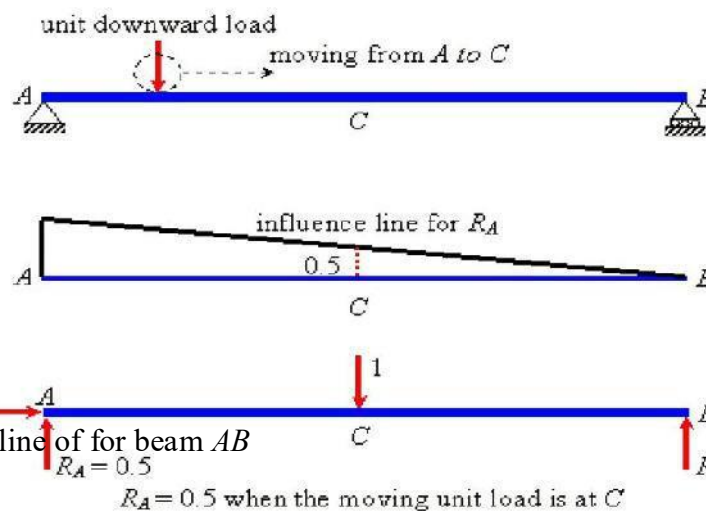
Figure 3b Influence line of for beam AB

Thus, an influence line can be defined as a curve, the ordinate to which at any abscissa gives the value of a particular response function due to a unit downward load acting at the point in the structure corresponding to the abscissa. The next section discusses how to construct influence lines using methods of equilibrium.

2 Construction of Influence Lines using Equilibrium Methods

The most basic method of obtaining influence line for a specific response parameter is to solve the static equilibrium equations for various locations of the unit load. The general procedure for constructing an influence line is described below.

1. Define the positive direction of the response parameter under consideration through a free body diagram of the whole system.

2..For a particular location of the unit load, solve for the equilibrium of the whole system and if required, as in the case of an internal force, also for a part of the member to obtain the response parameter for that location of the unit load.This gives the ordinate of the influence line at that particular location of the load.

3. Repeat this process for as many locations of the unit load as required to determine the shape of the influence line for the whole length of the member. It is often helpful if we can consider a generic location (or several locations) x of the unit load.

4. Joining ordinates for different locations of the unit load throughout the length of the member, we get the influence line for that particular response parameter. The following three examples show how to construct influence lines for a support reaction, a shear force and a bending moment for the simply supported beam AB .

Example 1 Draw the influence line for (vertical reaction at A) of beam AB in Fig.1
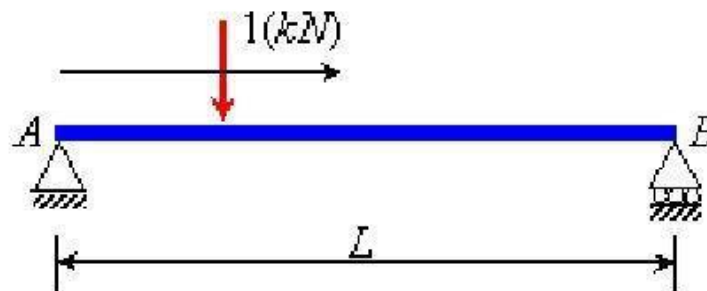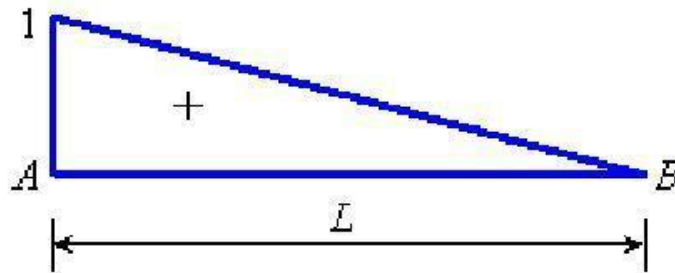

Solution:
Free body diagram of AB :
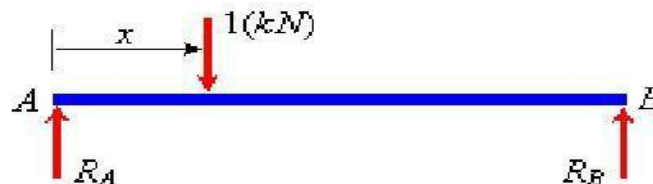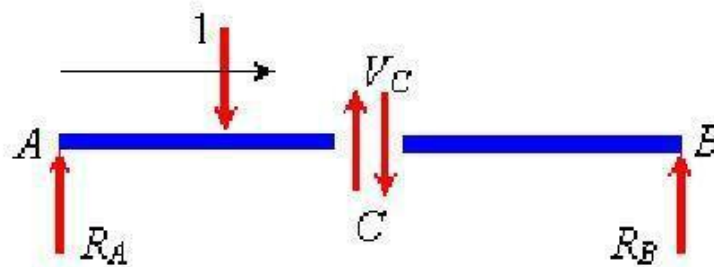

$$\sum F_y = 0 \Rightarrow R_A = 1 - R_B$$

$$\sum M (about B) = 0 \Rightarrow R_A(L) = 1(L - x)$$

$$\Rightarrow R_A = 1 - \frac{x}{L}$$
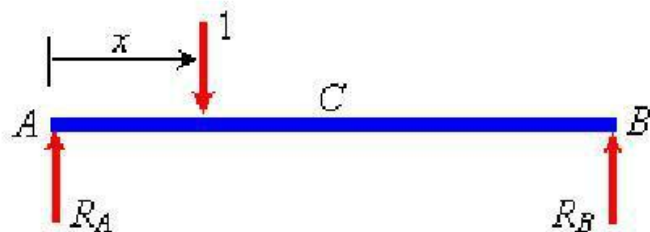
So the influence line of :

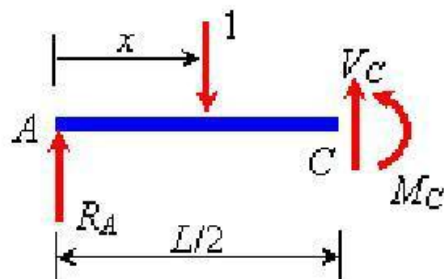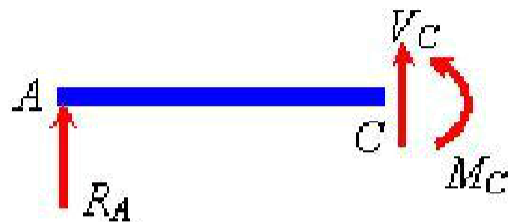**Example 2** Draw the influence line for (shear force at mid point) of beam *AB* in Fig.2.

Solution:

$$\sum M(about\ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$
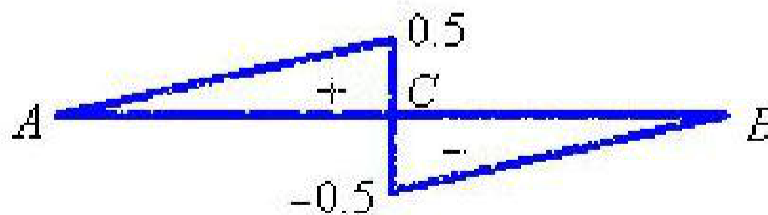
For $x < \dfrac{L}{2}$

$$\sum F_y = 0 \Rightarrow V_C = 1 - R_A = \frac{x}{L}$$
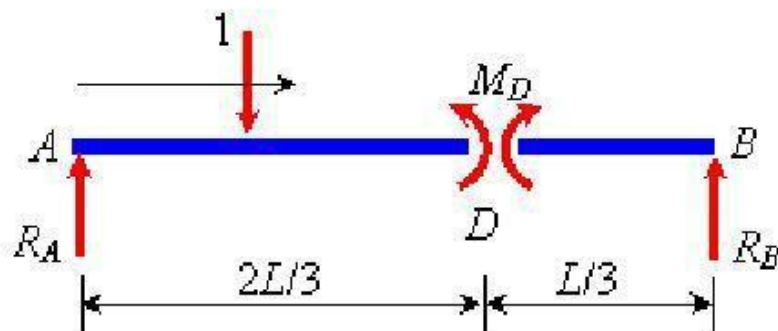
For $x > \dfrac{L}{2}$

$$\sum F_y = 0 \Rightarrow V_C = -R_A = \frac{x}{L} - 1$$

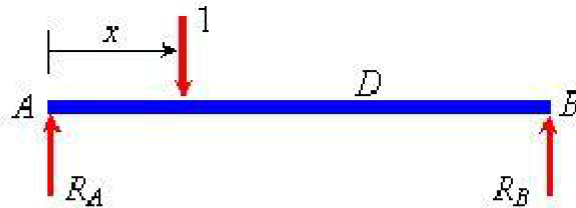So the influence line for V_C:

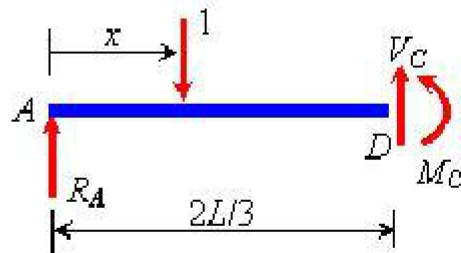**Example 3** Draw the influence line for (bending moment at) for beam *AB* in Fig.3.

Solution:

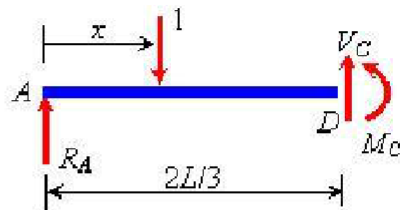$$\sum M(about\ B) = 0 \Rightarrow R_A = 1 - \frac{x}{L}$$

For $x < \frac{2L}{3}$

$$\sum M(about\ D) = 0$$

$$\Rightarrow M_D = R_A\left(\frac{2L}{3}\right) - 1\left(\frac{2L}{3} - x\right) = \left(1 - \frac{x}{L}\right)\left(\frac{2L}{3}\right) - \frac{2L}{3} + x$$

$$= -\frac{2x}{3} + x = \frac{x}{3}$$

For $x < \frac{2L}{3}$

$$\sum M(about\ D) = 0$$

$$\Rightarrow M_D = R_A\left(\frac{2L}{3}\right) - 1\left(\frac{2L}{3} - x\right) = \left(1 - \frac{x}{L}\right)\left(\frac{2L}{3}\right) - \frac{2L}{3} + x$$
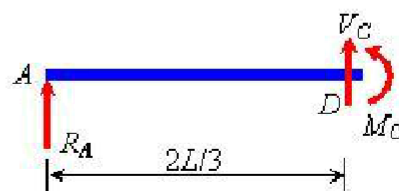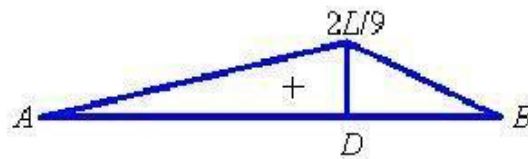
$$= -\frac{2x}{3} + x = \frac{x}{3}$$

For $x > \frac{2L}{3}$

$$\sum M(about\ D) = 0 \Rightarrow M_D = R_A\left(\frac{2L}{3}\right) = \frac{2L}{3} - \frac{2x}{3}$$

M_D

Similarly, influence lines can be constructed for any other support reaction or internal force in the beam. However, one should note that equilibrium equations will not be sufficient to obtain influence lines in indeterminate structures, because we cannot solve for the internal forces/support reactions using only equilibrium conditions for such structures.

3 Use of Influence Lines

In this section, we will illustrate the use of influence lines through the influence lines that we have obtained in Section 2. Let us consider a general case of loading on the simply supported beam (Figure 4a) and use the influence lines to find out the response parameters for their loading. We can consider this loading as the sum of three different loading conditions, (A), (B) and (C) (Figure 4b), each containing only one externally applied force.
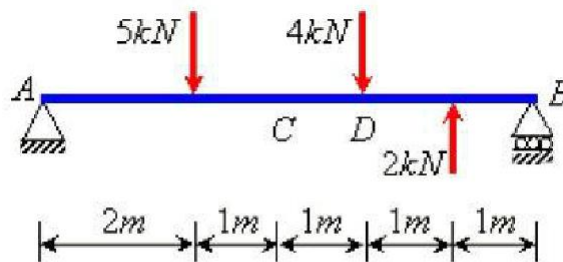
(a)
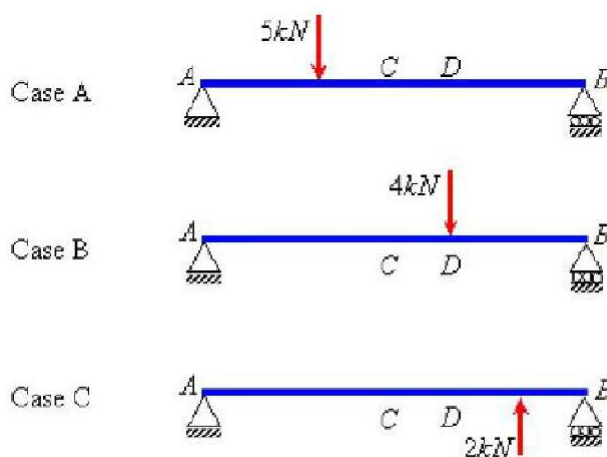
(b)

Figure4: Application of influence lines for a general loading: (a) all the loads, and (b) the general loading is divided into single force systems

For loading case (A), we can find out the response parameters using the three influence lines. Ordinate of an influence line gives the response for a unit load acting at a certain point.

Therefore, we can multiply this ordinate by the magnitude of the force to get the response due to the real force at that point. Thus

$$R_A = 5kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 2m) = 5(1 - 2/6) = 3.33kN$$

$$V_C = 5kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 2m) = 5(2/6) = 1.67kN$$

$$M_D = 5kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 2m) = 5(2/3) = 3.33kNm$$

Similarly, for loading case (B):

$$R_A = 4kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 4m) = 4(1 - 4/6) = 1.33kN$$

$$V_C = 4kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 4m) = 4(4/6 - 1) = -1.33kN$$

$$M_D = 4kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 4m) = 4(2 \times 6/9) = 5.33kNm$$

And for case (C),

$$R_A = -2kN \times (\text{ordinate of the } IL \text{ of } R_A \text{ at } x = 5m) = -2(1 - 5/6) = -0.33kN$$

$$V_C = -2kN \times (\text{ordinate of the } IL \text{ of } V_C \text{ at } x = 5m) = -2(5/6 - 1) = 0.33kN$$

$$M_D = -2kN \times (\text{ordinate of the } IL \text{ of } M_D \text{ at } x = 5m) = -2(2 \times 6/3 - 2 \times 5/3) = -1.33kNm$$

By the theory of superposition, we can add forces for each individual case to find the response parameters for the original loading case (Figure4a). Thus, the response parameters in the beam *AB* are:

$$R_A = (3.33 + 1.33 - 0.33)kN = 4.33kN$$

$$V_C = (1.67 - 1.33 + 0.33)kN = 0.67kN$$

$$M_D = (3.33 + 5.33 - 1.33)kN = 7.33kNm$$

One should remember that the method of superposition is valid only for linear elastic cases with small displacements only. So, prior to using influence lines in this way it is necessary to check that these conditions are satisfied.

It may seem that we can solve for these forces under the specified load case using equilibrium equations directly, and influence lines are not necessary. However, there may be requirement for obtaining these responses for multiple and more complex loading cases. For example, if we need to analyse for ten loading cases, it will be quicker to find only three influence lines and not solve for ten equilibrium cases.

The most important use of influence line is finding out the location of a load for which certain response will have a maximum value. For example, we may need to find the location of a moving load (say a gantry) on a beam (say a gantry girder) for which we get the maximum bending moment at a certain point. We can consider bending moment at point *D* of

Example3, where the beam *AB* becomes our gantry girder. Looking at the influence line of one can say that will reach its maximum value when the load is at point *D* . Influence lines can be used not only for concentrated forces, but for distributed forces as well, which is discussed in the next section.

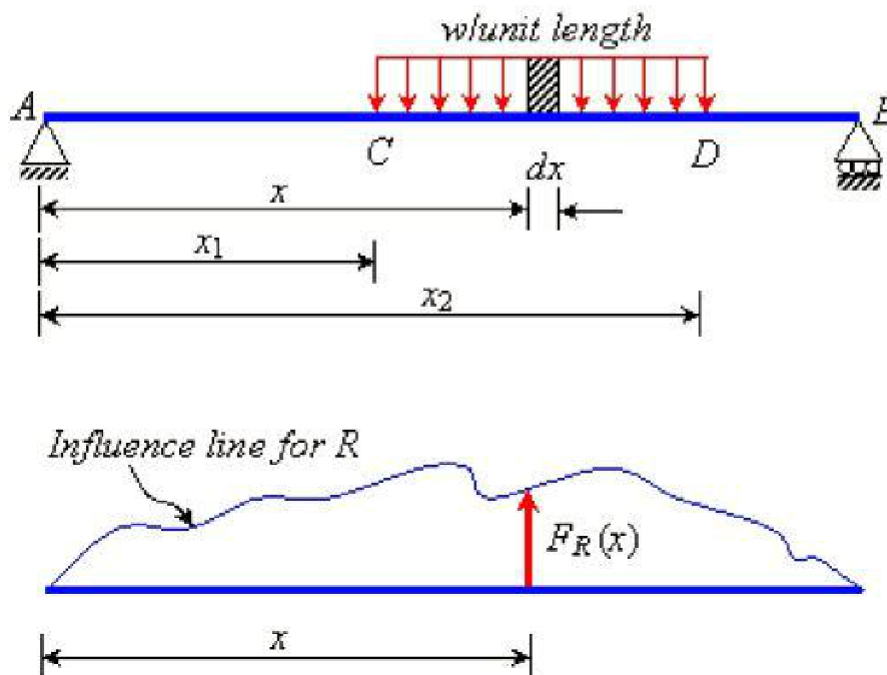**4 Using Influence Lines for Uniformly Distributed Load**

      Consider the simply-supported beam *AB* in Figure 6.5, of which the portion *CD* is acted upon by a uniformly distributed load of intensity *w/unit length* . We want to find the value of a certain response function *R* under this loading and let us assume that we have already constructed the influence line of this response function. Let the ordinate of the influence line at a distance *x* from support *A* be . If we consider an elemental length *dx* of the beam at a distance *x* from *A* , the total force acting on this elemental length is *wdx* . Since *dx* is infinitesimal, we can consider this force to be a concentrated force acting at a distance *x* .The contribution of this concentrated force *wdx* to *R* is:

$$dR = (wdx)F_R(x)$$

Therefore, the total effect of the distributed force from point *C* to *D* is:

$$R = \int_C^D dR = \int_{x_1}^{x_2} w F_R(x)dx$$

$$= w \int_{x_1}^{x_2} F_R(x)dx = w \text{ (area under the influence line from } C \text{ to } D \text{)}$$

ifluence line for a uniformly distributed loading

Thus, we can obtain the response parameter by multiplying the intensity of the uniformly distributed load with the area under the influence line for the distance for which the load is acting. To illustrate, let us consider the uniformly distributed load on a simply supported beam (Figure 6). To find the vertical reaction at the left support, we can use the influence line for that we have obtained in Example 1. So we can calculate the reaction as:

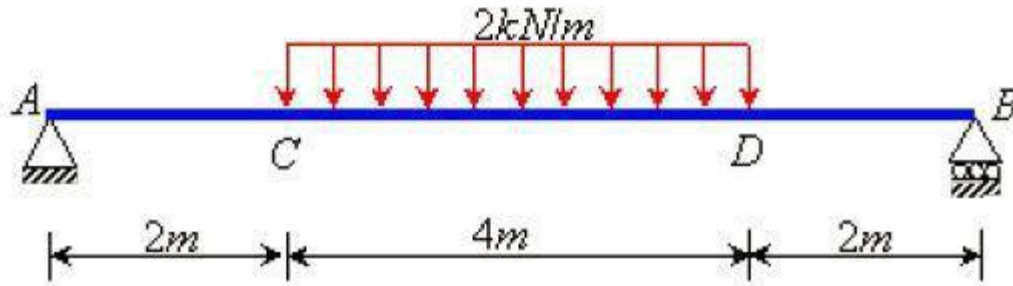$$R_A = 2kN/m \times \{0.5(3/4 + 1/4) \times 4m\} = 4kN$$

Figure 6.6 Uniformly distributed load acting on a beam

Similarly, we can find any other response function for a uniformly distributed loading using their influence lines as well. For non-uniformly distributed loading, the intensity w is not constant through the length of the distributed load. We can still use the integration formulation:

$$R = \int_C^D dR = \int_{x1}^{x2} wF_R(x)dx$$

However, we cannot take the intensity w outside the integral, as it is a function of x.

UNIT – 2

SLOPE DEFLECTION METHOD

In this method the end moments or support moments expressed in terms of slopes, deflections, stiffness and length of the members. The unknown slope values (slopes) are determined from the condition of equilibrium of joints for moments that is

$$M_{BA} + M_{BC} = 0$$
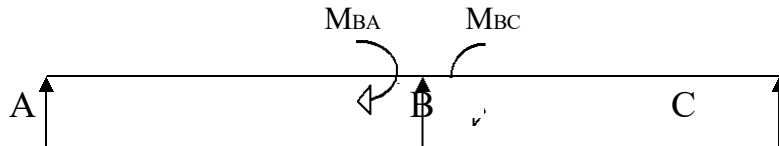
Assumption

1. All the joints of the frame are rigid that is angle between the members do not change at a joint even after deformation.
2. The joints are assumed to rotate as a whole
3. Directions due to axial and shear stress are neglected because they are negligible or small

Sign Conventions

1. Moments:-

   Clockwise moment +ve
   Anticlockwise moment –ve

2. Rotation:-

   Clockwise Rotation +ve
   Anticlockwise Rotation –ve

3. Sinking of support

   If right support sinks down   is +ve
   If left support sinks down   is –ve

4. Bending Moments

   Sagging BM is +ve and Hogging BM is –ve

5. Shear Force

   Left side upward the SF is +ve
   Left side downward the SF is -ve
   Right side upward the SF is -ve
   Right side downwardward the SF is +ve

Structural Analysis – II

1. Draw BMD, Elastic curve and SFD by slope deflection Method
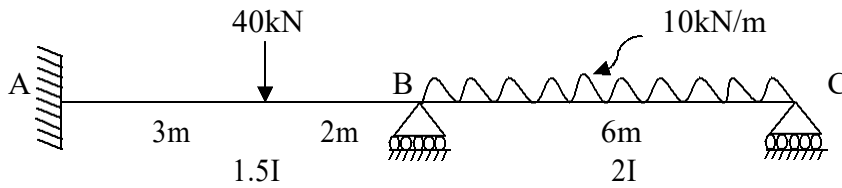
Fixed End Moment:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} \equiv \frac{-40\,X\,3\,X\,2^2}{5^2} \equiv -19.2 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} \equiv \frac{40\,X\,2\,X\,3^2}{5^2} \equiv 28.8 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} \equiv \frac{-10\,X\,6^2}{12} \equiv -30 \text{ kN-m}$$

$$M_{FCB} = \frac{wl^2}{12} \equiv \frac{10\,X\,6^2}{12} \equiv 30 \text{ kN-m}$$

Slope Deflection Equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -19.2 + \frac{2\,X\,1.5EI}{5}(2\theta_A + \theta_B) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

$$\boxed{M_{AB} = -19.2 + 0.6EI\,\theta_B}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= 28.8 + \frac{2\,X\,1.5EI}{5}(2\theta_B + \theta_A)$$

$$\boxed{M_{BA} = 28.8 + 1.2EI\,\theta_B}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -30 + \frac{2\,X\,2EI}{6}(2\theta_B + \theta_C)$$

$$\boxed{M_{BC} = -30 + 1.33EI\,\theta_B + 0.667EI\,\theta_C}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

$$= 30 + \frac{2\,X\,2EI}{6}(2\theta_C + \theta_B)$$

$$\boxed{M_{CB} = 30 + 1.33EI\,\theta_C + 0.667EI\,\theta_B}$$

Apply the condition of Equilibrium

$M_{BA} + M_{BC} = 0$ and $M_{CB} = 0$

At 'B'

$M_{BA} + M_{BC} = 0$

$28.8 + 1.2EI\ \theta_B - 30 + 1.33EI\ \theta_B + 0.667EI\ \theta_C = 0$

$$2.533EI\ \theta_B + 0.667EI\ \theta_C = 1.2 \longrightarrow 1$$

At 'c'

$M_{CB} = 0$

$30 + 0.667EI\ \theta_C + 0.667EI\ \theta_B = 0$

$$0.667EI\ \theta_B + 1.33EI\ \theta_C = -30 \longrightarrow 2$$

Solving Eq 1 and 2

$$\theta_B = \qquad\qquad \theta_C =$$

$$\frac{7\ 372}{EI} \qquad\quad \frac{-26.194}{EI}$$

Substitute the above values in the S-D Equation

$M_{AB} = -19.2 + 0.6EI\ \theta_B$

$\boxed{M_{AB} = -14.76 \text{ kN-m}}$

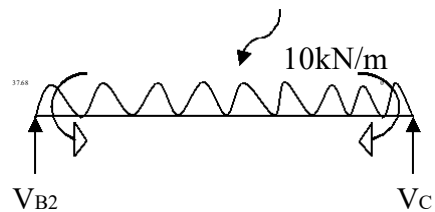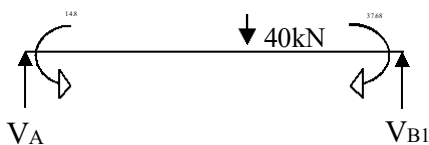$M_{BA} = 28.8 + 1.2EI\ \theta_B$

$\boxed{M_{BA} = 37.68 \text{ kN-m}}$

$M_{BC} = -30 + 1.33EI\ \theta_B + 0.667EI\ \theta_C$

$\boxed{M_{BC} = -37.68 \text{ kN-m}}$

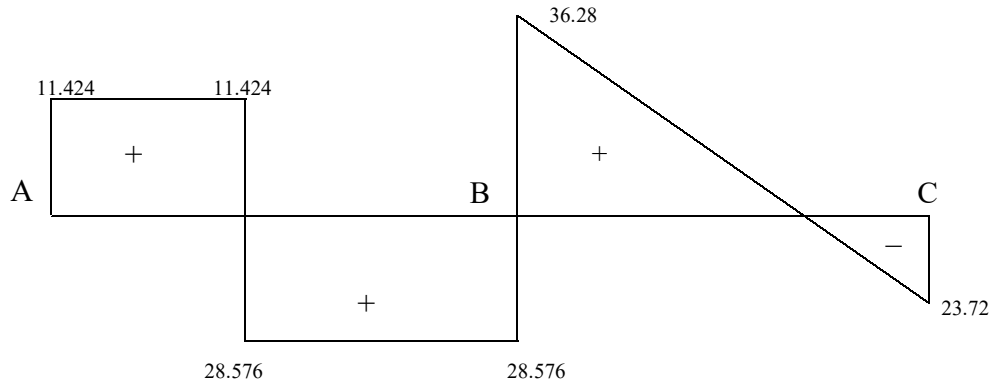$M_{CB} = 30 + 1.33EI\ \theta_C + 0.667EI\ \theta_B$
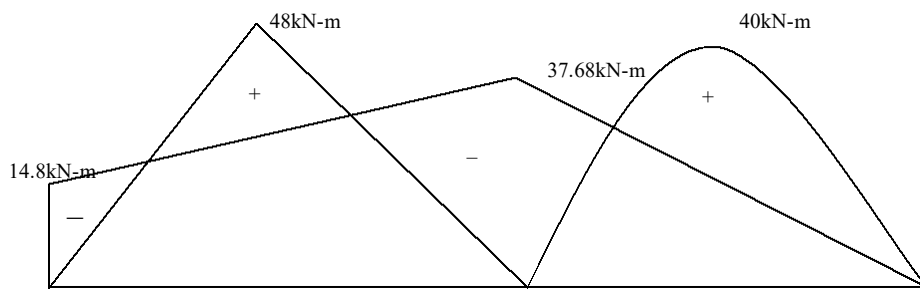
$\boxed{M_{CB} = 0}$

<u>Support Reaction</u>

V_A = 11.424kN, V_{B1} = 28.576kN, V_{B2} = 36.28kN and V_C = 23.72kN
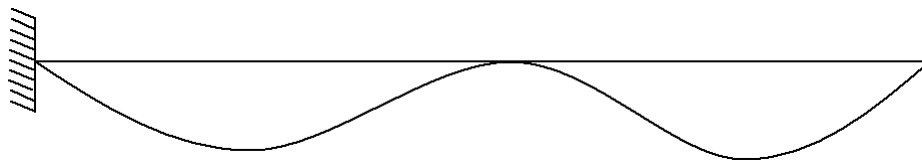
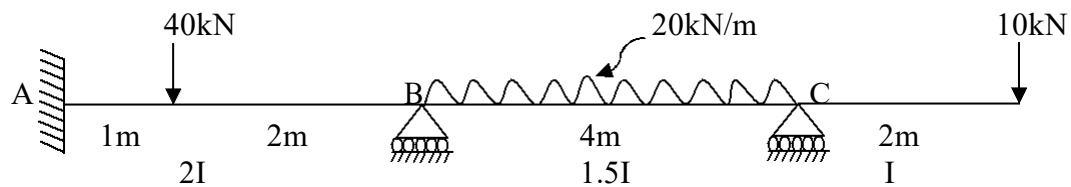V_B = V_{B1}+ V_{B2} = 58.734kN

SFD

BMD

Elastic Curve

2. Draw BMD, Elastic curve and SFD by slope deflection Method

Fixed End Moment:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} = \frac{-40\,X\,1\,X\,2^2}{3^2} = -17.78 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} = \frac{40\,X\,2\,X\,1^2}{3^2} = 8.89 \text{ kN-m}$$

$M_{FBC} = \dfrac{-wl^2}{12} = \dfrac{-20 \times 4^2}{12} = -26.67$ kN-m

$M_{FCB} = \dfrac{wl^2}{12} = \dfrac{20 \times 4^2}{12} = 26.67$ kN-m

$M_{FCD} = M_{CD} = -10 \times 2 = -20$ kN-m

<u>Slope Deflection Equation</u>

$M_{AB} = M_{FAB} + \dfrac{2EI}{l}(2\theta_A + \theta_B)$

$\qquad = -17.78 + \dfrac{2 \times 2EI}{3}(2\theta_A + \theta_B)$ $\qquad\qquad$ ($\theta_A = 0$ due to fixity at support A)

$\boxed{M_{AB} = -17.78 + 1.33 EI \theta_B}$

$M_{BA} = M_{FBA} + \dfrac{2EI}{l}(2\theta_A + \theta_B)$

$\qquad = 8.89 + \dfrac{2 \times 2EI}{3}(2\theta_B + \theta_A)$

$\boxed{M_{BA} = 8.89 + 2.67 EI \theta_B}$

$M_{BC} = M_{FBC} + \dfrac{2EI}{l}(2\theta_B + \theta_C)$

$\qquad = -26.67 + \dfrac{2 \times 1.5EI}{4}(2\theta_B + \theta_C)$

$\boxed{M_{BC} = -26.67 + 1.5EI \theta_B + 0.75EI \theta_C}$

$M_{CB} = M_{FCB} + \dfrac{2EI}{l}(2\theta_C + \theta_B)$

$\qquad = 26.67 + \dfrac{2 \times 1.5EI}{4}(2\theta_C + \theta_B)$

$\boxed{M_{CB} = 26.67 + 1.5EI \theta_C + 0.75EI \theta_B}$

Apply the condition of Equilibrium

$M_{BA} + M_{BC} = 0$ and $M_{CB} + M_{CD}$

$= 0$ At 'B'

$\qquad M_{BA} + M_{BC} = 0$

$8.89 + 2.67EI \theta_B - 26.67 + 1.5EI \theta_B + 0.75EI \theta_C = 0$

$\qquad\qquad 4.17EI \theta_B + 0.75EI \theta_C = 17.78$ $\qquad\longrightarrow$ 1

At 'c'

$M_{CB} + M_{CD} = 0$

$26.67 + 1.5EI\ \theta_C + 0.75EI\ \theta_B - 20 = 0$

$0.75EI\ \theta_B + 1.5EI\ \theta_C = -6.67$ ⟶ 2

Solving Eq 1 and 2

$\theta_B = \dfrac{5.592}{EI} \circlearrowleft$ $\theta_C = \left(\dfrac{-7.241}{EI} \right) \circlearrowleft$

Substitute the above values in the S-D Equation

$M_{AB} = -17.78 + 1.33EI\ \theta_B$

$\boxed{M_{AB} = -10.3426 \text{ kN-m}}$

$M_{BA} = 8.89 + 2.67EI\ \theta_B$

$\boxed{M_{BA} = 23.703 \text{ kN-m}}$

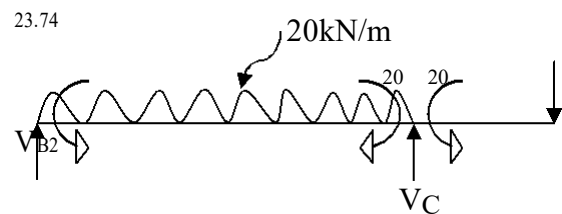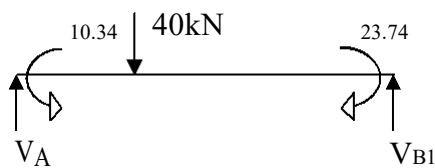$M_{BC} = -26.67 + 1.5EI\ \theta_B + 0.75EI\ \theta_C$

$\boxed{M_{BC} = -23.703 \text{ kN-m}}$

$M_{CB} = 26.67 + 1.5EI\ \theta_C + 0.75EI\ \theta_B$

$\boxed{M_{CB} = 20 \text{kN-m}}$

$\boxed{M_{CD} = -20 \text{kN-m}}$
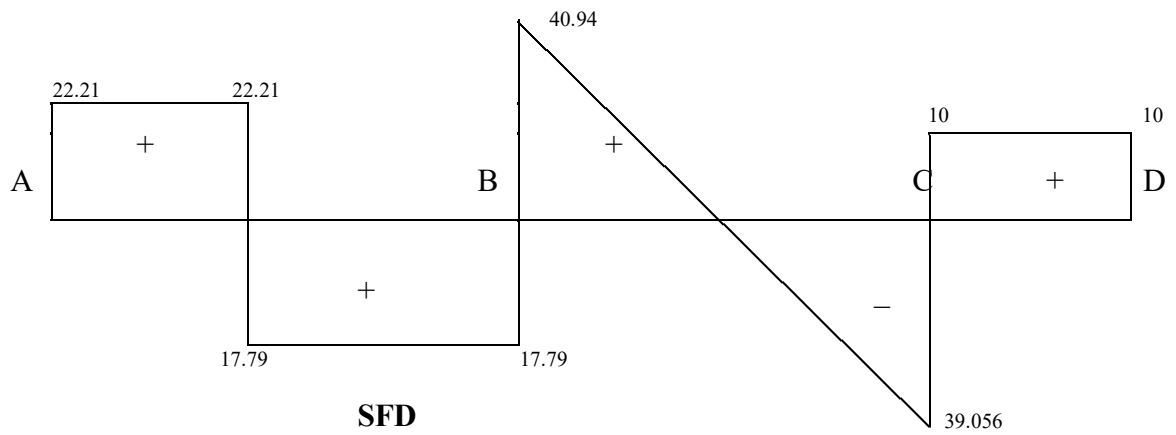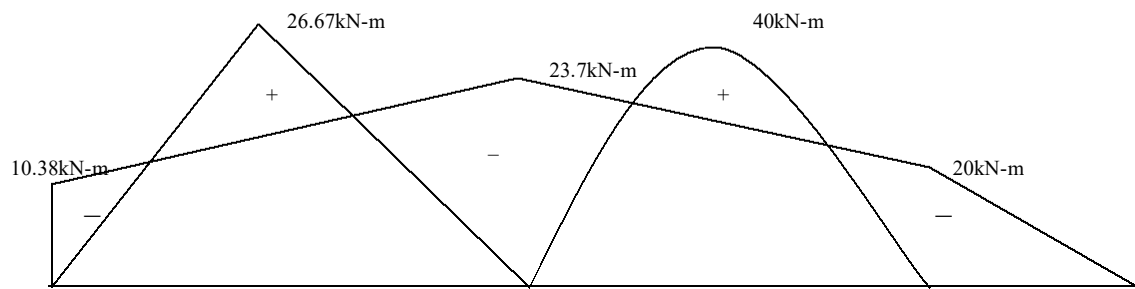
Support Reaction

$V_A = 22.21 \text{kN}, \quad V_{B1} = 17.79 \text{kN}, \quad V_{B2} = 40.935 \text{kN} \text{ and } \quad V_C = 49.065 \text{kN}$
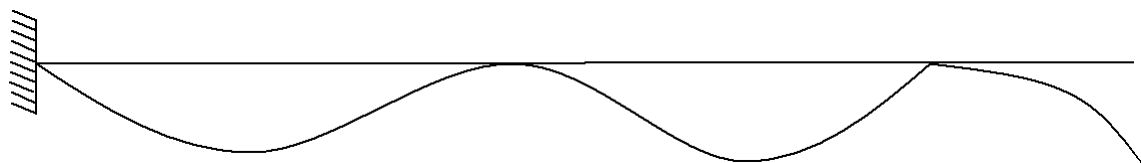
$V_B = V_{B1} + V_{B2} = 58.734 \text{kN}$

40.94

22.21   22.21

A   B   C   +   D

10   10

17.79   17.79

SFD

39.056

26.67kN-m   40kN-m

23.7kN-m

10.38kN-m   20kN-m

BMD

Elastic Curve

SINKING OF SUPPORT

$$M_{AB} = \frac{-6EI\delta}{l^2}$$

$$M_{BA} = \frac{-6EI\delta}{l^2}$$

$$M_{BA} = \frac{6EI\delta}{l^2}$$

$$M_{AB} = \frac{6EI\delta}{l^2}$$
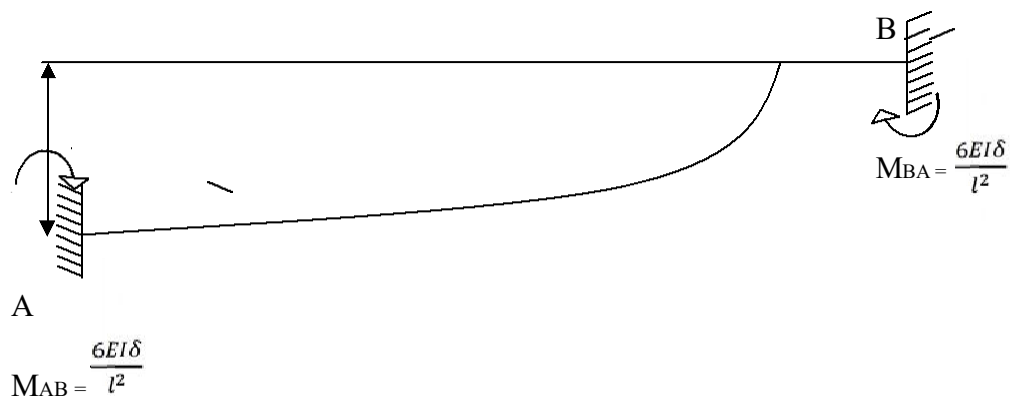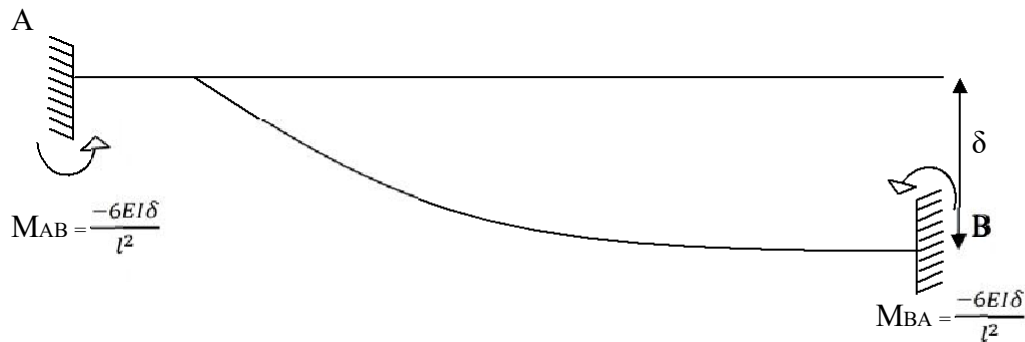
Slope Deflection equations

$$M_{AB} = M_{FAB} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\delta}{l}\right)$$

δ is $+^{ve}$ when right side support sinks
δ is $-^{ve}$ when left side support sinks

1. Analyse the continuous beam by slope deflection method the support B sinks by 5mm Draw BMD, EC and SFD. Take EI = 2 X 10^4 kN-mm^2
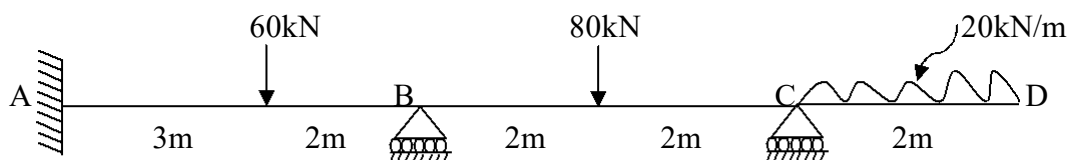
Fixed End Moment:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} = \frac{-60\,X\,3\,X\,2^2}{5^2} = \text{-28.8 kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} = \frac{60\,X\,2\,X\,3^2}{5^2} = \text{43.2 kN-m}$$

$M_{FBC} = \dfrac{-wa\,b^2}{l^2} \equiv \dfrac{-80 \times 2 \times 2^2}{4^2} \equiv -40$ kN-m

$M_{FCB} = \dfrac{wb\,a^2}{l^2} \equiv \dfrac{80 \times 2 \times 2^2}{4^2} \equiv 40$ kN-m

$M_{FCD} = M_{CD} = -20 \times 2 \times 1 = -40$ kN-m

Slope Deflection Equation

$M_{AB} = M_{FAB} + \dfrac{2EI}{l}\left(2\theta_A + \theta_B - \dfrac{3(+\delta)}{l}\right)$

$= -28.8 + \dfrac{2 \times 2 \times 10^4}{4}\left(2\theta_A + \theta_B - \dfrac{3(0.005)}{4}\right)$

(θ_A = 0 due to fixity at support A)

$\boxed{M_{AB} = -52.8 + 0.8 \times 10^4\,\theta_B}$

$M_{BA} = M_{FBA} + \dfrac{2EI}{l}\left(2\theta_A + \theta_B - \dfrac{3\delta}{l}\right)$

$= 43.2 + (2\theta_B + \theta_A - (0.005))$

$\boxed{M_{BA} = 19.2 + 1.6 \times 10^4\,\theta_B}$

$M_{BC} = M_{FBC} + \dfrac{2EI}{l}\left(2\theta_B + \theta_C - \dfrac{3\delta}{l}\right)$

$= -40 + (2\theta_B + \theta_C - (0.005))$

$\boxed{M_{BC} = -2.5 + 2 \times 10^4\,\theta_B + 10^4\,\theta_C}$

$M_{CB} = M_{FCB} + \dfrac{2EI}{l}\left(2\theta_C + \theta_B - \dfrac{3\delta}{l}\right)$

$= 40 + \dfrac{2 \times 1.5EI}{4}\left(2\theta_C + \theta_B - \dfrac{3(-0.005)}{5}\right)$

$\boxed{M_{CB} = 77.5 + 2 \times 10^4\,\theta_C + 10^4\,\theta_B}$

Apply the condition of Equilibrium

$M_{BA} + M_{BC} = 0$ and $M_{CB} + M_{CD}$

= 0 At 'B'

$M_{BA} + M_{BC} = 0$

$19.2 + 1.6 \times 10^4\,\theta_B - 2.5 + 2 \times 10^4\,\theta_B + 10^4\,\theta_C = 0$

$3.6 \times 10^4\,\theta_B + 10^4\,\theta_C = -16.7$

1

At 'c'

$$M_{CB} + M_{CD} = 0$$

$$77.5 + 2 \times 10^4 \theta_C + 10^4 \theta_B - 40 = 0$$

$$10^4 \theta_B + 2 \times 10^4 \theta_C = -37.5 \qquad\qquad \longrightarrow 2$$

Solving Eq 1 and 2

$$\theta_B = 0.66 \times 10^{-4} \qquad\qquad\qquad \theta_C = -19.1 \times 10^{-4}$$

Substitute the above values in the S-D Equation

$$M_{AB} = -52.8 + 0.8 \times 10^4 \theta_B$$
$$\boxed{M_{AB} = -52.275 \text{ kN-m}}$$

$$M_{BA} = 19.2 + 1.6 \times 10^4 \theta_B$$
$$\boxed{M_{BA} = 20.256 \text{ kN-m}}$$

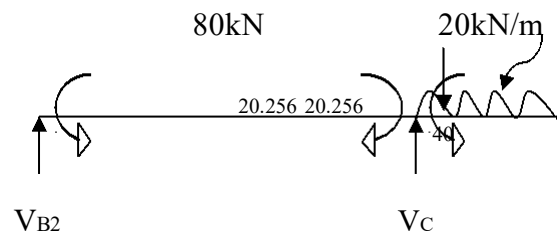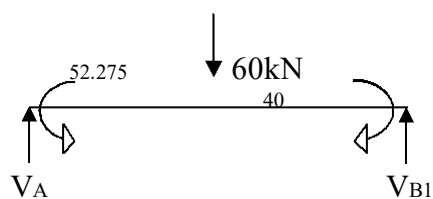$$M_{BC} = -2.5 + 2 \times 10^4 \theta_B + 10^4 \theta_C$$
$$\boxed{M_{BC} = -20.256 \text{ kN-m}}$$

$$M_{CB} = 77.5 + 2 \times 10^4 \theta_C + 10^4 \theta_B$$
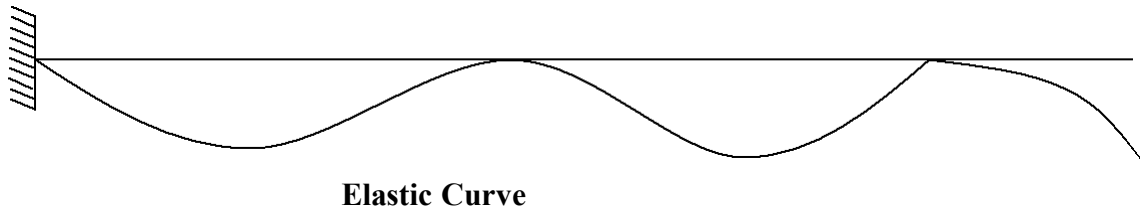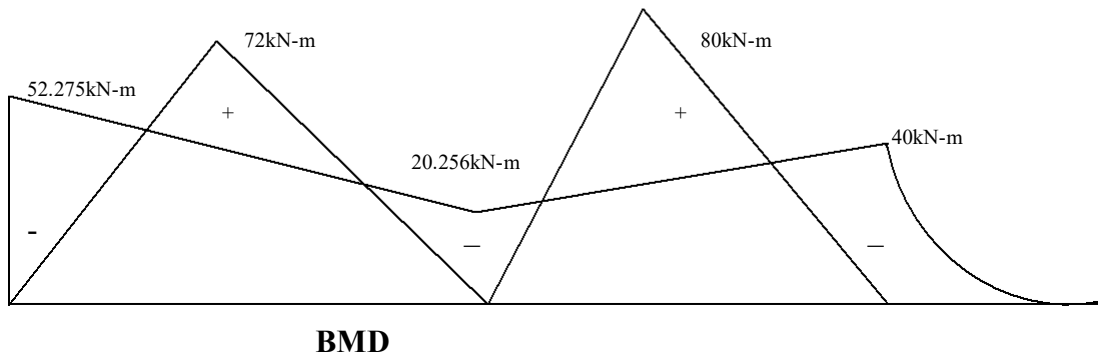$$\boxed{M_{CB} = 40 \text{kN-m}}$$
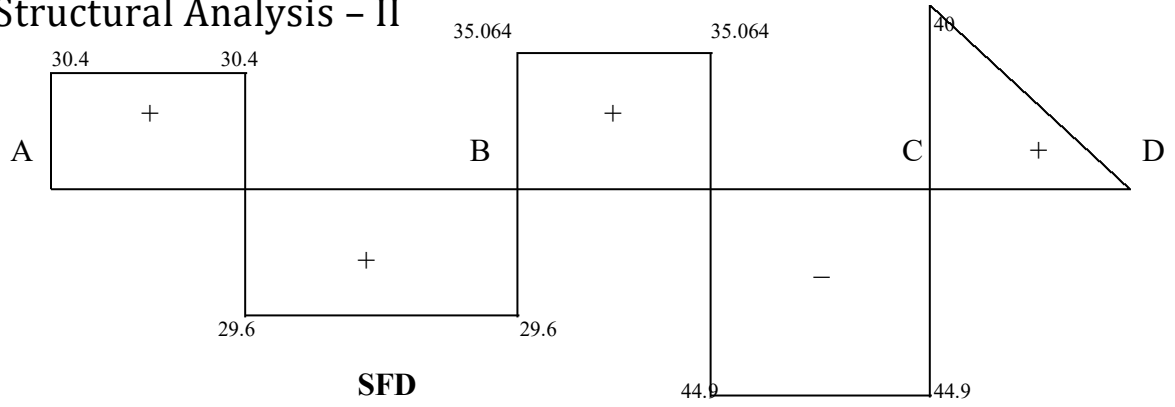
$$\boxed{M_{CD} = -40 \text{kN-m}}$$

Support Reaction

$$V_A = 30.4\text{kN}, \quad V_{B1} = 29.6\text{kN}, \quad V_{B2} = 35.064\text{kN} \quad \text{and} \quad V_C = 84.94\text{kN}$$

$$V_B = V_{B1} + V_{B2} = 64.664\text{kN}$$

SFD

30.4   30.4
+

A   B   C   +   D

35.064   35.064
+

+

29.6   29.6

44.9   44.9

40
−

BMD

72kN-m
52.275kN-m

+   80kN-m

-                   +

20.256kN-m

−   −   40kN-m

Elastic Curve

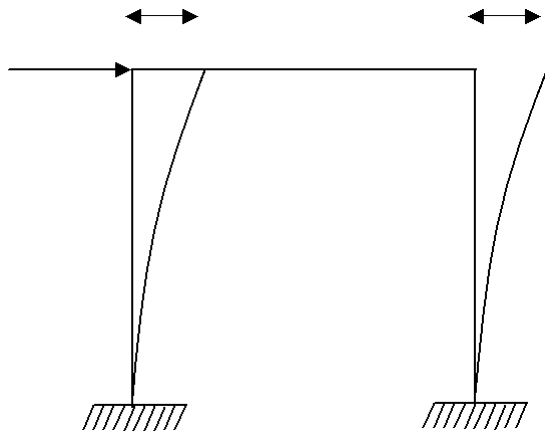## Analysis of NON-SWAY Portal Frames

A frame is a structure having both horizontal and vertical members, such as beams and columns. The joint between any two members is assumed to rotate has a whole when loads are applied (rigid) hence they are called rigid jointed frames.

The frames is which the beams and columns are perpendicular to each other are called orthogonal frames. The moment of the joints in frames in the lateral direction is called Lateral Sway or Sway.

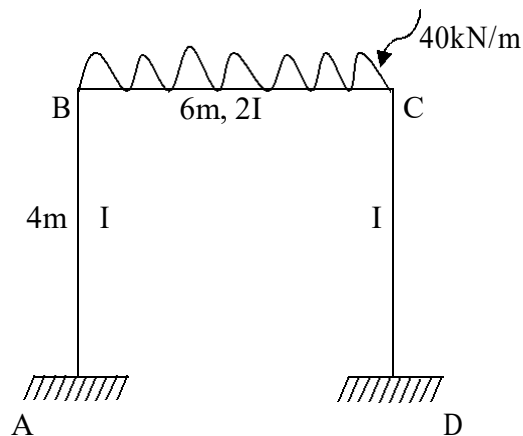The frames which do not sway in lateral direction are called non sway portal frames.

Examples:
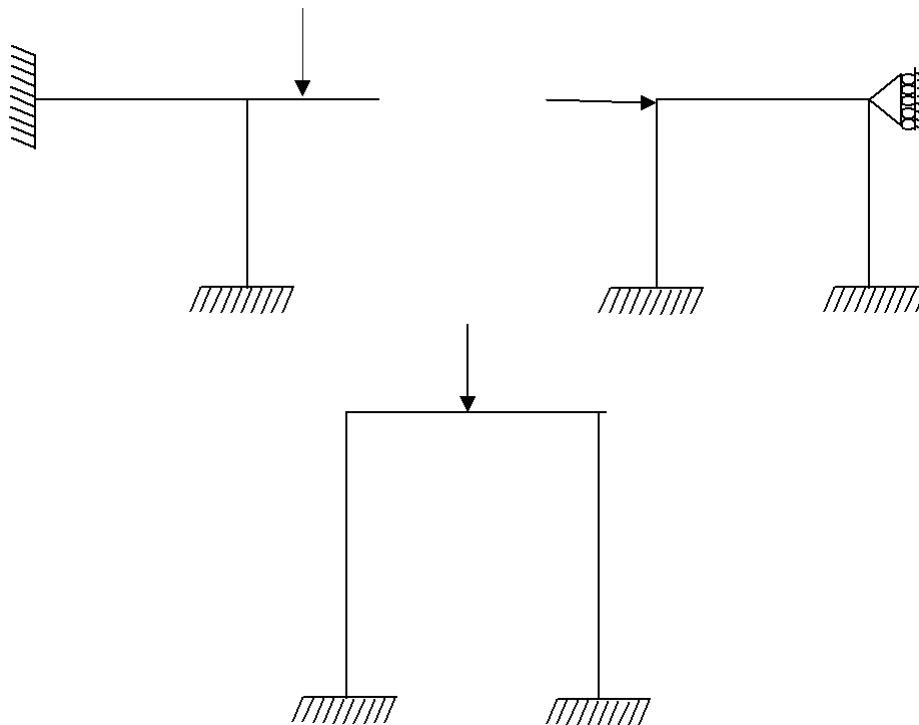
Structural Analysis – II   figure and draw BMD

1. Analyse the frame shown in the

Fixed End Moment:

$M_{FAB} = 0$

$M_{FBA} = 0$

$M_{FBC} = \dfrac{-wl^2}{12} \equiv \dfrac{-40 \times 6^2}{12} \equiv -120 \text{ kN-m}$

$M_{FCB} = \dfrac{wl^2}{12} = \dfrac{40 \times 6^2}{12} \equiv 120 \text{ kN-m}$

$M_{FCD} = M_{FDC} = 0$

Slope Deflection Equation

$M_{AB} = M_{FAB} + \dfrac{2EI}{l}(2\theta_A + \theta_B)$

$= 0 + \dfrac{2EI}{4}(2\theta_A + \theta_B)$ ($\theta_A = 0$ due to fixity at support A)

$\boxed{M_{AB} = 0.5EI\,\theta_B}$

$M_{BA} = M_{FBA} + \dfrac{2EI}{l}(2\theta_A + \theta_B)$

$= 0 + \dfrac{2EI}{4}(2\theta_B + \theta_A)$

$\boxed{M_{BA} = EI\,\theta_B}$

$M_{BC} = M_{FBC} + \dfrac{2EI}{l}(2\theta_B + \theta_C)$

$= -120 + \dfrac{2 \times 2EI}{6}(2\theta_B + \theta_C)$

$\boxed{M_{BC} = -120 + 1.33EI\,\theta_B + 0.667EI\,\theta_C}$

Structural Analysis – II

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

$$= 120 + \frac{2 \times 2EI}{6}(2\theta_C + \theta_B)$$

$$\boxed{M_{CB} = 120 + 1.33EI\,\theta_C + 0.667EI\,\theta_B}$$

$$M_{CD} = M_{FCD} + \frac{2EI}{l}(2\theta_C + \theta_D)$$

$$= 0 + \frac{2EI}{4}(2\theta_C + \overset{0}{\theta_D})$$

$$\boxed{M_{CD} = EI\,\theta_C}$$

$$M_{DC} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_C)$$

$$= 0 + \frac{2EI}{4}(2\overset{0}{\theta_D} + \theta_C)$$

$$\boxed{M_{DC} = 0.5EI\,\theta_C}$$

Apply the condition of Equilibrium

$M_{BA} + M_{BC} = 0$ and $M_{CB} + M_{CD} = 0$

At 'B'

$M_{BA} + M_{BC} = 0$

$EI\,\theta_B - 120 + 1.33EI\,\theta_B + 0.667EI\,\theta_C = 0$

$2.33EI\,\theta_B + 0.667EI\,\theta_C = 120$ $\longrightarrow$ 1

At 'c'

$M_{CB} + M_{CD} = 0$

$120 + 1.33EI\,\theta_C + 0.667EI\,\theta_B + EI\,\theta_C = 0$

$0.667EI\,\theta_B + 2.33EI\,\theta_C = -120$ $\longrightarrow$ 2

Solving Eq 1 and 2

$$\theta_B = \frac{5.592}{EI} \qquad \theta_C = \frac{-7.241}{EI}$$

Substitute the above values in the S-D Equation

$M_{AB} = 0.5EI\ \theta_B$

$\boxed{M_{AB} = 36.14 \text{ kN-m}}$

$M_{BA} = EI\ \theta_B$

$\boxed{M_{BA} = 72.28\text{kN-m}}$

$M_{BC} = -120 + 1.33EI\ \theta_B + 0.667EI\ \theta_C$

$\boxed{M_{BC} = -71.6 \text{ kN-m}}$
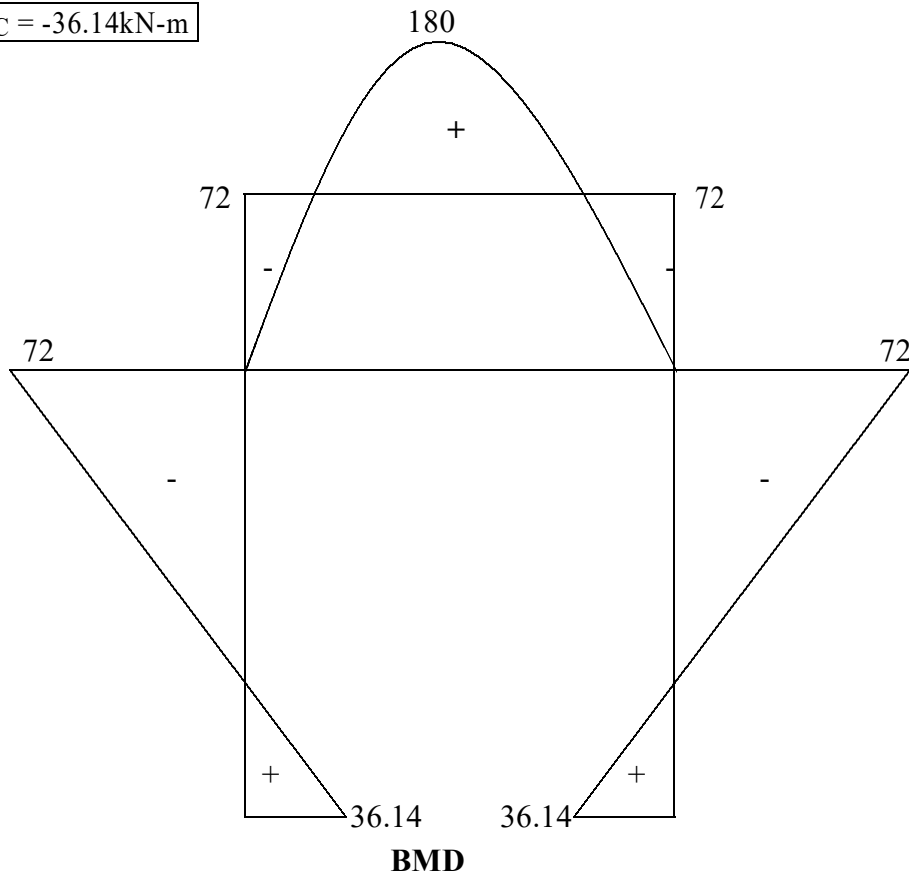
$M_{CB} = 120 + 1.33EI\ \theta_C + 0.667EI\ \theta_B$

$\boxed{M_{CB} = 72.3\text{kN-m}}$

$M_{CD} = EI\ \theta_C$
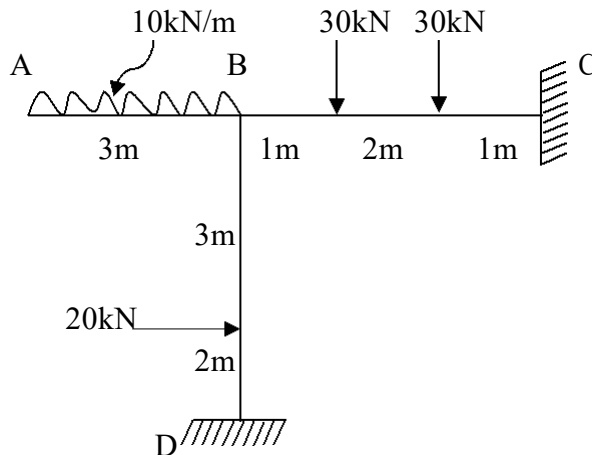
$\boxed{M_{CD} = -72.26\text{kN-m}}$

$M_{DC} = 0.5EI\ \theta_C$

$\boxed{M_{DC} = -36.14\text{kN-m}}$

BMD

2. Draw BMD and EC by slope deflection for the frame shown in figure.

Fixed End Moment:

$M_{FAB} = 0$

$M_{FBA} = 10 \times 3 \times 1.5 = 45 kN\text{-}m$

$M_{FBC} = \dfrac{-wab^2}{l^2} - \dfrac{-wab^2}{l^2} = \dfrac{-30 \times 1 \times 3^2}{4^2} - \dfrac{30 \times 3 \times 1^2}{4^2} = -22.5 \text{ kN-m}$

$M_{FCB} = \dfrac{wba^2}{l^2} + \dfrac{wba^2}{l^2} = \dfrac{30 \times 1 \times 3^2}{4^2} + \dfrac{30 \times 3 \times 1^2}{4^2} = 22.5 \text{ kN-m}$

$M_{FBD} = \dfrac{-wab^2}{l^2} = \dfrac{20 \times 3 \times 2^2}{5^2} = 9.6 kN\text{-}m$

$M_{FDB} = \dfrac{-wba^2}{l^2} = \dfrac{-20 \times 2 \times 3^2}{5^2} = -14.4kN\text{-}m$

Slope Deflection **Equation**

$M_{BC} = M_{FBC} + \dfrac{2EI}{l}(2\theta_B + \theta_C)$

$\quad = -22.5 + \dfrac{2EI}{4}(2\theta_B + 0)$

$\boxed{M_{BC} = -22.5 + EI\,\theta_B}$

$M_{CB} = M_{FCB} + \dfrac{2EI}{l}(2\theta_C + \theta_B)$

$\quad = 22.5 + \dfrac{2EI}{4}(2\theta_C + \theta_B)$

$\boxed{M_{CB} = 22.5 + 0.5EI\,\theta_B}$

$M_{BD} = M_{FCD} + \frac{2EI}{l}(2\theta_B + \theta_D)$

$= 9.6 + \frac{2EI}{5}(2\theta_B + \overset{0}{\theta_D})$

$\boxed{M_{BD} = 9.6 + 0.8EI\,\theta_B}$

$M_{DB} = M_{FDC} + \frac{2EI}{l}(2\theta_D + \theta_B)$

$= 0 + \frac{2EI}{5}(2\overset{00}{\theta_D} + \theta_B)$

$\boxed{M_{DB} = -14.4 + 0.4EI\,\theta_B}$

Apply the condition of Equilibrium

At 'B'

$M_{BA} + M_{BC} + M_{BD} = 0$

$45 - 22.5 + EI\,\theta_B + 9.6 + 0.8EI\,\theta_B = 0$

$1.8EI\,\theta_B = -32.1$ $\longrightarrow$ 1

$\theta_B = $.

Substitute the above value in the S-D Equation

$M_{BA} = 45\text{kN-m} \quad \frac{-17.83}{EI}$

$M_{BC} = -22.5 + EI\,\theta_B$
$\boxed{M_{BC} = -40.33 \text{ kN-m}}$

$M_{CB} = 22.5 + 0.5EI\,\theta_B$
$\boxed{M_{CB} = 13.585\text{kN-m}}$

$M_{BD} = 9.6 + 0.8EI\,\theta_B$
$\boxed{M_{BD} = -4.67\text{kN-m}}$
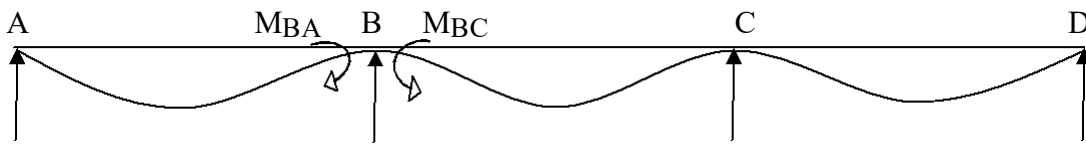
$M_{DB} = -14.4 + 0.4EI\,\theta_B$
$\boxed{M_{DB} = -21.52\text{kN-m}}$

UNIT – 3
MOMENT DISTRIBUTION METHOD

Consider a continuous beam ABC as shown in the figure. The final moment developed at the intermediate support B should follow the condition $M_{BA} + M_{BC} = 0$ but the fixed end moments (FEM) at that support will be $M_{FBA} + M_{FBC} = 0$.

The algebraic sum of the fixed end moments (FEM) thus obtained is the unbalanced moment at B. This unbalanced moment is balanced and redistributed to both M_{FBA} and M_{FBC} depending upon the stiffness of the member. Half of the distributed moment will be carried over to the far end. Hence we will again have unbalanced moments, so that we have to balance and distribute.

This iteration process will continue till we get balanced moments that is $M_{BA} + M_{BC} = 0$, similarly at the last simple support $M_{CB} = 0$.

This iterative method of balancing and redistributing of unbalanced moments to obtain the final balanced moments is called moment distribution method.

In order to distribute the moments we should calculate stiffness factors and distribution factors.

Stiffness Factor (K) :-

Stiffness factor depends upon the support condition at the far end.
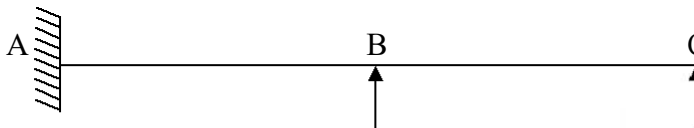
If the far end is continuous or fixed $K = \dfrac{4EI}{l}$

If the far end is simply support or discontinuous $K = \dfrac{3EI}{l}$

Ex: $K_{AB} = \dfrac{4EI}{l}$, $K_{BA} = \dfrac{4EI}{l}$, $K_{BC} = \dfrac{4EI}{l}$, $K_{CB} = \dfrac{4EI}{l}$
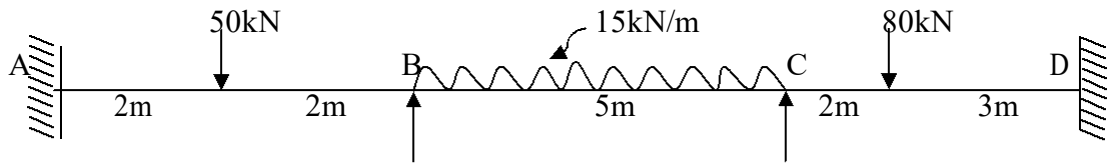
Distribution Factor(Δ):-

The unbalanced moments are distributed only at the intermediate supports, hence distribution factors are calculated only at the intermediate supports.

$$BA = \frac{K_{BA}}{K_{BA} + K_{BC}} \text{ or } \frac{K_{BA}}{\Sigma K_B}$$

$$BC = \frac{K_{BC}}{K_{BA} + K_{BC}} \text{ or } \frac{K_{BC}}{\Sigma K_B}$$

1. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.

Fixed End Moments:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} \equiv \frac{-50 \times 2 \times 2^2}{4^2} \equiv -25 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} \equiv \frac{50 \times 2 \times 2^2}{4^2} \equiv 25 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} = \frac{-15 \times 5^2}{12} \equiv -31.25 \text{ kN-m}$$

$$M_{FCB} = \frac{wl^2}{12} = \frac{15 \times 5^2}{12} \equiv 31.25 \text{ kN-m}$$

$$M_{FCD} = \frac{-wa\,b^2}{l^2} \equiv \frac{-80 \times 2 \times 3^2}{5^2} = -57.6 \text{ kN-m}$$

$$M_{FDC} = \frac{wb\,a^2}{l^2} \equiv \frac{80 \times 3 \times 2^2}{5^2} = 38.4 \text{kN-m}$$

Stiffness Factor(K)

$$K_{BA} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$K_{BC} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI$$

$$K_{CB} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI$$

$$K_{CD} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI$$

Distribution Factor(Δ)

$$BA = \frac{K_{BA}}{K_{BA}+K_{BC}} \equiv \frac{EI}{EI+0.8EI} \equiv 0.56 \qquad CB = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{0.8EI}{0.8EI+0.8EI} \equiv 0.5$$

$$\frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{0.8EI}{0.8EI+EI} = 0.44 \qquad \frac{K_{CD}}{K_{CD}+K_{CB}} = \frac{0.8EI}{0.8EI+0.8EI} \equiv 0.5$$

$$BC = \qquad = \qquad = 0.44 \qquad CD = \qquad = \quad . \quad . \quad .$$

Moment Distribution Table

	A		B		C		D
			0.56	0.44	0.5	0.5	
FEM	-25		25	-31.25	31.25	-57.6	38.4
Balance		+3.5	+2.75	+13.175	+13.175		
Carry over	1.75			6.587	1.375		6.587
Balance		-3.688	-2.89	-0.687	-0.687		
Carry over	-1.84			-0.343	-1.445		-0.343
Balance		+0.192	+0.150	+0.722	+0.722		
Carry over	0.096			0.361	0.075		0.361
Balance		-0.202	-0.158	-0.0375	-0.0375		
Carry over	-0.10			-0.018	-0.079		-0.018
Balance		+0.01	+0.008	+0.04	+0.04		
Carry over	0.005			0.02	0.004		0.02
Balance			-0.01	-0.008	-0.002	-0.002	
Final Moment	-25		24.8	-24.8	44.39	-44.38	45

Support Reaction
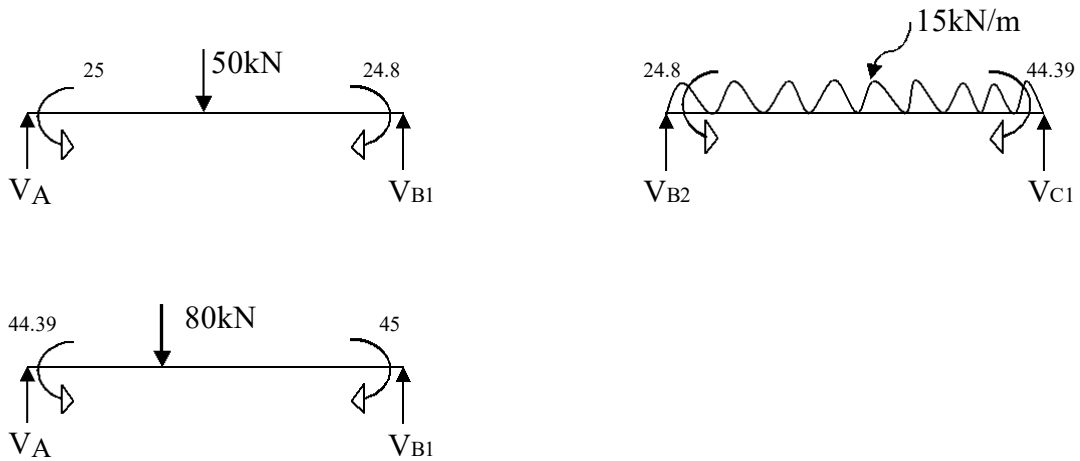
$V_A = 25.05 kN$, $V_{B1} = 24.95 kN$, $V_{B2} = 33.6 kN$, $V_{C1} = 41.42 kN$, $V_{C2} = 47.878 kN$

$V_B = V_{B1} + V_{B2} = 58.55 kN$
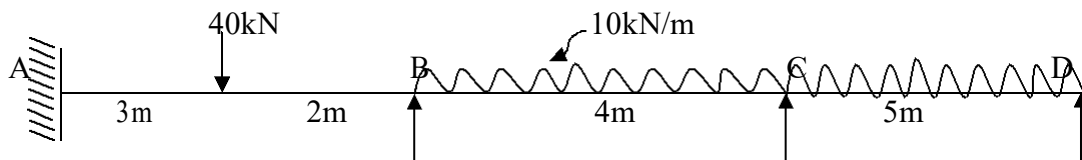
$V_C = V_{C1} + V_{C2} = 89.20 kN$
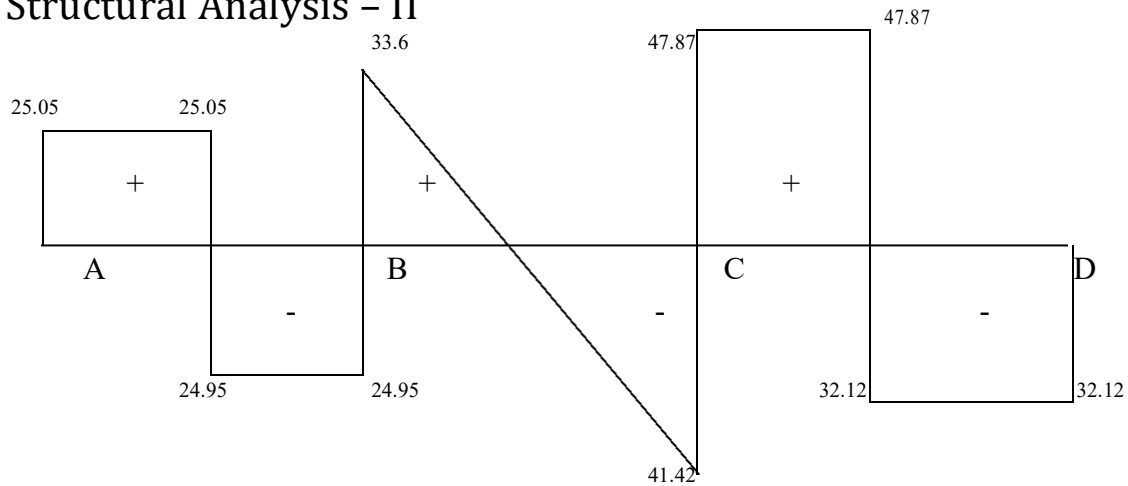
SFD
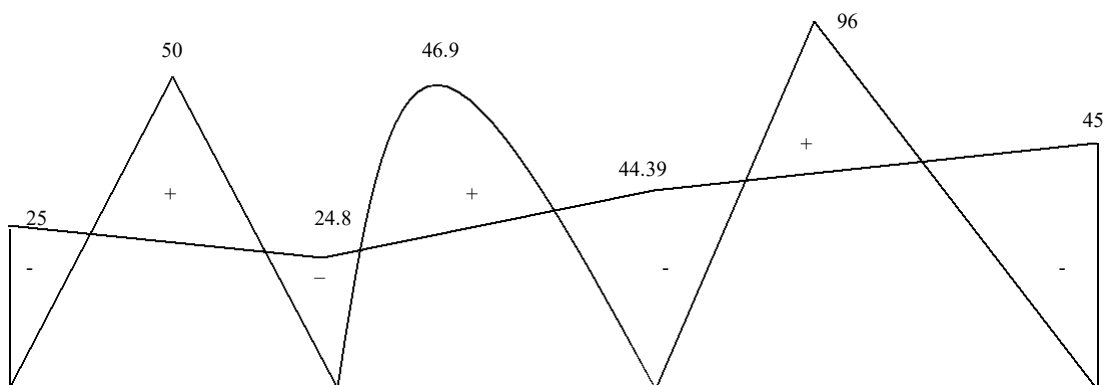
BMD

2. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.

Fixed End Moments:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} \equiv \frac{-40 \times 3 \times 2^2}{5^2} = -19.2 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} \equiv \frac{40 \times 2 \times 3^2}{5^2} = 28.8 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} \equiv \frac{-10 \times 4^2}{12} \equiv -13.33 \text{ kN-m}$$

$$M_{FCB} = \frac{wl^2}{12} \equiv \frac{10 \times 4^2}{12} \equiv 13.33 \text{ kN-m}$$

$$M_{FCD} = \frac{-wl^2}{12} \equiv \frac{-10 \times 5^2}{12} \equiv -20.83 \text{ kN-m}$$

$$M_{FDC} = \frac{wl^2}{12} = \frac{10 \times 5^2}{12} = 20.83 \text{ kN-m}$$

Stiffness Factor(K)

$$K_{BA} = \frac{4EI}{l} = \frac{4EI}{5} = 0.8EI$$

$$K_{BC} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$K_{CB} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$K_{CD} = \frac{3EI}{l} = \frac{3EI}{5} = 0.6EI$$

Distribution Factor(Δ)

$$_{BA} = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{0.8EI}{0.8EI+EI} = 0.44 \qquad\qquad _{CB} = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{EI}{EI+0.6EI} = 0.625$$

$$_{BC} = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{EI}{0.8EI+EI} = 0.56 \qquad\qquad _{CD} = \frac{K_{CD}}{K_{CD}+K_{CB}} = \frac{0.6EI}{0.6EI+EI} = 0.375$$

Moment Distribution Table

	A	B		C	D
		0.44	0.56	0.625	0.375
FEM	-19.2	28.8	-13.33	13.33	-20.83	20.83
Release D						-20.83
Carry over					-10.41	
Initial Moment	-19.2	28.8	-13.33	13.33	-31.24	0
Balance		-6.8	-8.66	11.19	6.72	
Carry over	-3.4		-5.59	-4.33		0
Balance		-2.56	-3.14	2.78	1.62	
Carry over	-1.28		1.39	-1.57		0
Balance		-0.616	-0.765	0.98	0.59	
Carry over	-0.31		0.49	-0.383		0
Balance		-0.23	-0.27	+0.24	+0.144	
Carry over	-0.005		0.02	-0.004		0

Balance		-0.053	-0.067	+0.084	+0.051	
Carry over	-0.027		0.042	-0.034		0
Balance		-0.018	-0.023	-0.002	-0.013	
Final Moment	-24.34	18.54	-18.61	22.15	-22.15	0

Support Reaction
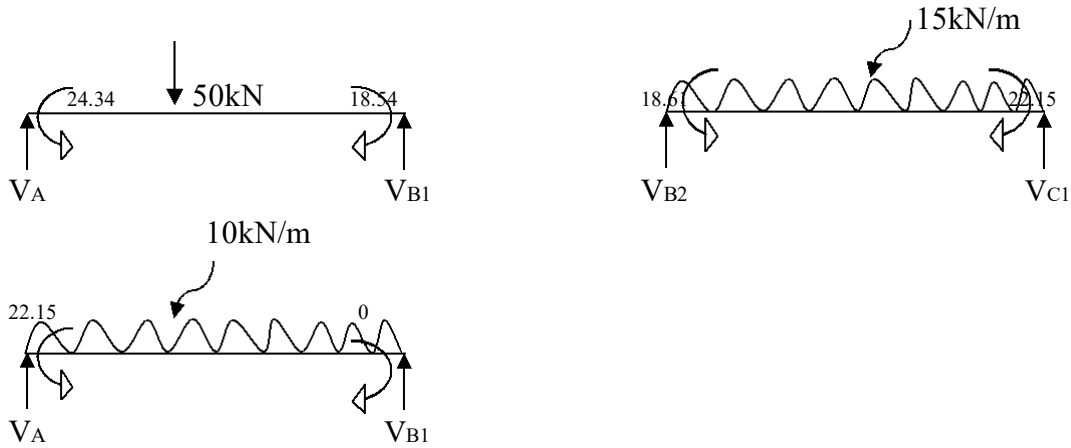
$V_A = 17.15$ kN, $V_{B1} = 22.85$ kN, $V_{B2} = 19.1$ kN, $V_{C1} = 20.9$ kN, $V_{C2} = 29.5$ kN

$V_B = V_{B1} + V_{B2} = 41.95$ kN

$V_C = V_{C1} + V_{C2} = 50.40$ kN

SFD

BMD
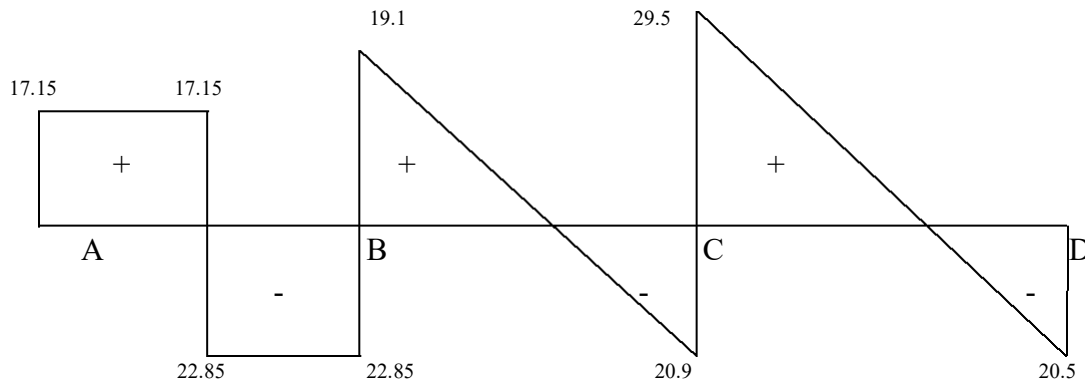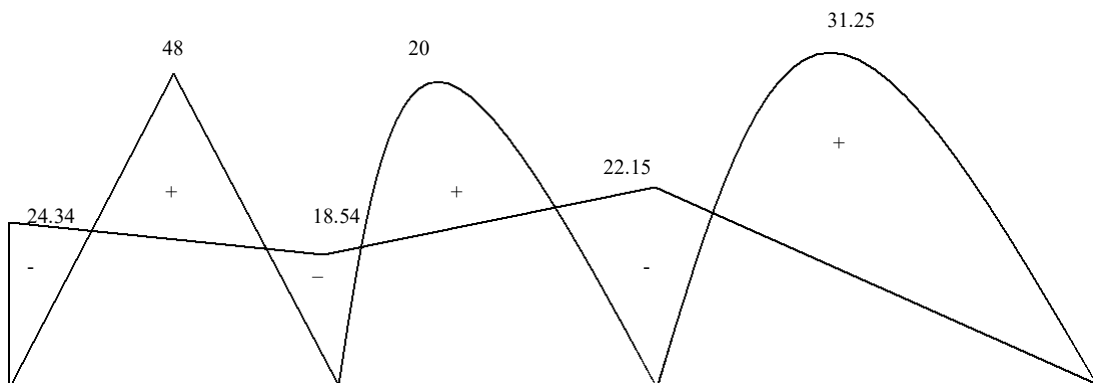
3. Analyse continuous beam shown in the figure by moment distribution method. Draw BMD, EC and SFD.
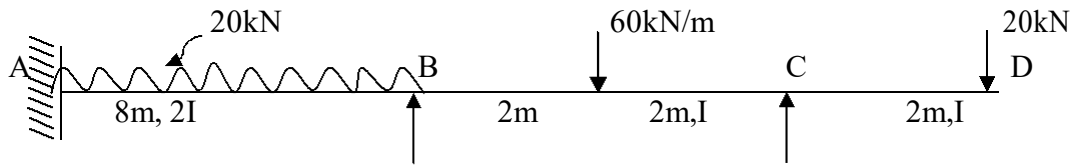
Fixed End Moments:

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20 \times 8^2}{12} = -106.67 \text{ kN-m}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{20 \times 8^2}{12} = 106.67 \text{ kN-m}$$

$$M_{FBC} = \frac{-wa\,b^2}{l^2} = \frac{-60 \times 2 \times 2^2}{4^2} = -30 \text{ kN-m}$$

$$M_{FCB} = \frac{wb\,a^2}{l^2} = \frac{60 \times 2 \times 2^2}{4^2} = 30 \text{ kN-m}$$

$$M_{FCD} = M_{CD} = -20 \times 2 = -40 \text{ kN-m}$$

Stiffness Factor(K)

$$K_{BA} = \frac{4EI}{l} = \frac{4 \times 2EI}{8} = EI$$

$$K_{BC} = \frac{3EI}{l} = \frac{3EI}{4} = 0.75EI$$

$$K_{CB} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$K_{CD} = 0$$

Distribution Factor(Δ)

$$BA = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{EI}{EI+0.75EI} = 0.57$$

$$CB = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{EI}{EI+0} = 1$$

$$\frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{0.75EI}{EI+0.75EI} = 0.43$$

$$\frac{K_{CD}}{K_{CD}+K_{CB}} = ($$

$$BC = \qquad = \;\cdot\; = 0.43 \qquad\qquad CD = \qquad = 0$$

Moment Distribution Table

	A		B		C	D
		0.57	0.43		1	0
FEM	-106.67	106.67	-30		30	-40
BalanceOH					+10	
Carry over			+5			
Initial Moment	-106.67	106.67	-25		40	-40
Balance		-46.55	-35.12			
Carry over	-23.28				0	
Final Moment	-129.95	60.12	-60.12		40	-40

Support Reaction
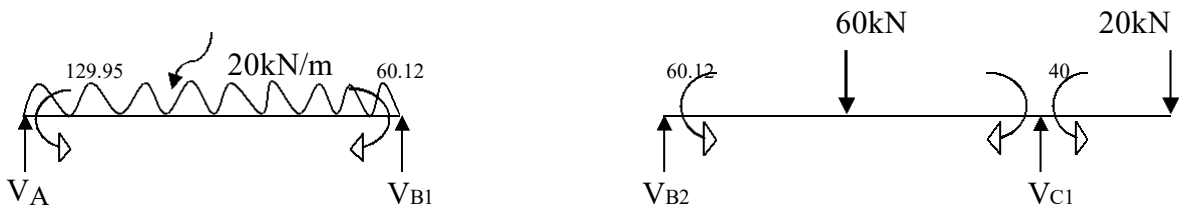
V_A = 88.73kN, V_{B1} = 71.27kN, V_{B2} = 35.03kN, V_C = 106.3kN

$V_B = V_{B1} + V_{B2}$ = 41.95kN

SFD

BMD
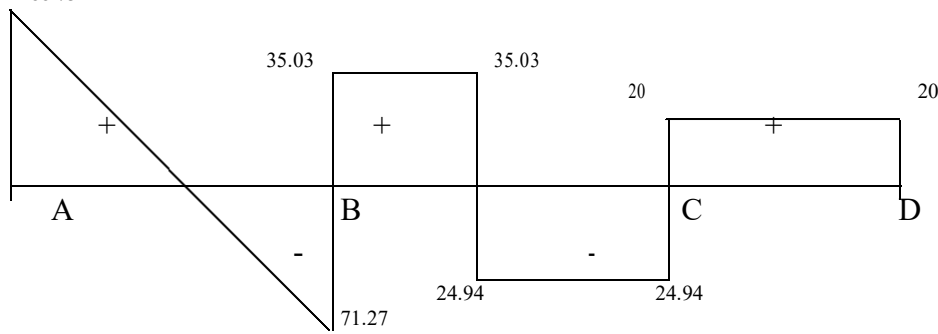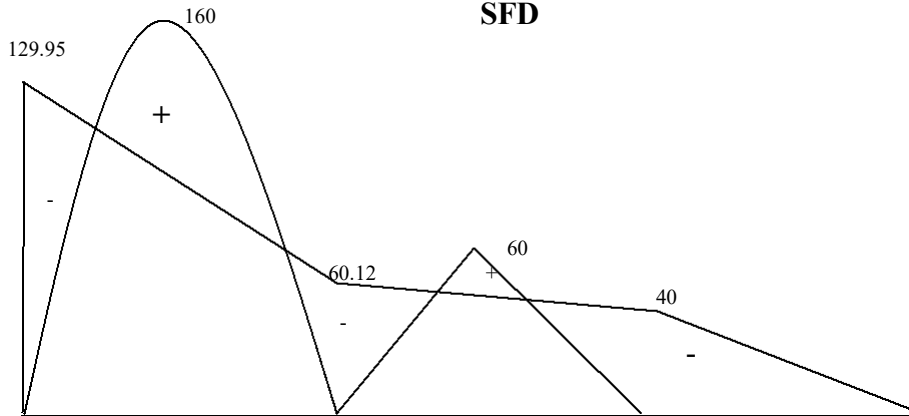
SINKING OF SUPPORT

1. Moments due to sinking

$$M = \frac{-6EI\delta}{l^2} \quad \text{If right support sinks down } \delta \text{ is +ve}$$

If left support sinks down δ is –ve

2. Moments due to Rotation
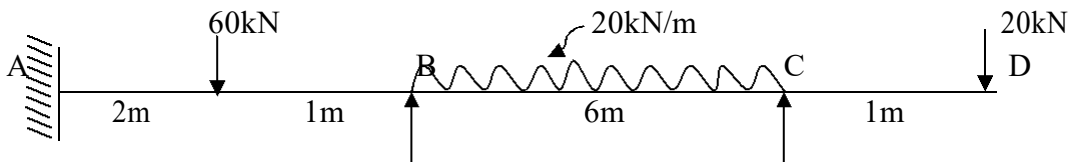
Near end moment = $\frac{4EI\theta}{l}$

For far end moment = $\frac{2EI\theta}{l}$

Where θ is rotation, clockwise +ve and anticlockwise –ve

The above should be added to the FEM

1. Analyse the continuous beam by moment distribution method. Support B yields by 9mm. Take EI = 1 x 10^{12} N-mm^2, draw BMD and EC

Fixed End Moments:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} - \frac{-6EI\delta}{l^2} = \frac{-60 \times 2 \times 1^2}{3^2} - \frac{-6 \times 1 \times 10^3 \times 0.009}{3^2} = -19.33 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} - \frac{-6EI\delta}{l^2} = \frac{-60 \times 1 \times 2^2}{3^2} - \frac{-6 \times 1 \times 10^3 \times 0.009}{3^2} = 20.67 \text{ kN-m}$$

$$M_{FBC} = \frac{-wl^2}{12} - \frac{-6EI(-\delta)}{l^2} = \frac{-20 \times 6^2}{12} - \frac{-6 \times 1 \times 10^3 \times (-0.009)}{6^2} = -58.5 \text{ kN-m}$$

$$M_{FCB} = \frac{wl^2}{12} - \frac{-6EI(-\delta)}{l^2} = \frac{20 \times 6^2}{12} - \frac{-6 \times 1 \times 10^3 \times (-0.009)}{6^2} = 61.5 \text{ kN-m}$$

$M_{FCD} = M_{CD} = -20 \times 1 = -20$ kN-m

Stiffness Factor(K)

$$K_{BA} = \frac{4EI}{l} = \frac{4EI}{3} = 1.33EI$$

$$K_{BC} = \frac{3EI}{l} = \frac{3EI}{6} = 0.5EI$$

$$K_{CB} = \frac{4EI}{l} = \frac{4EI}{6} = 0.667EI$$

Structural Analysis – II

$K_{CD}=0$

Distribution Factor(Δ)

$$BA = \frac{K_{BA}}{K_{BA}+K_{BC}} \equiv \frac{1.33EI}{1.33EI+0.5EI} \equiv 0.725 \qquad CB = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{0.67EI}{0.67EI+0} = 1$$

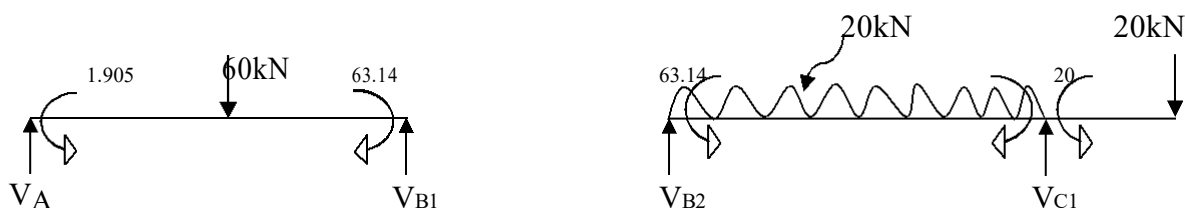$$\frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{0.5EI}{1.33EI+0.5EI} = 0.275$$

BC = = = 0.275   CD=0

Moment Distribution Table
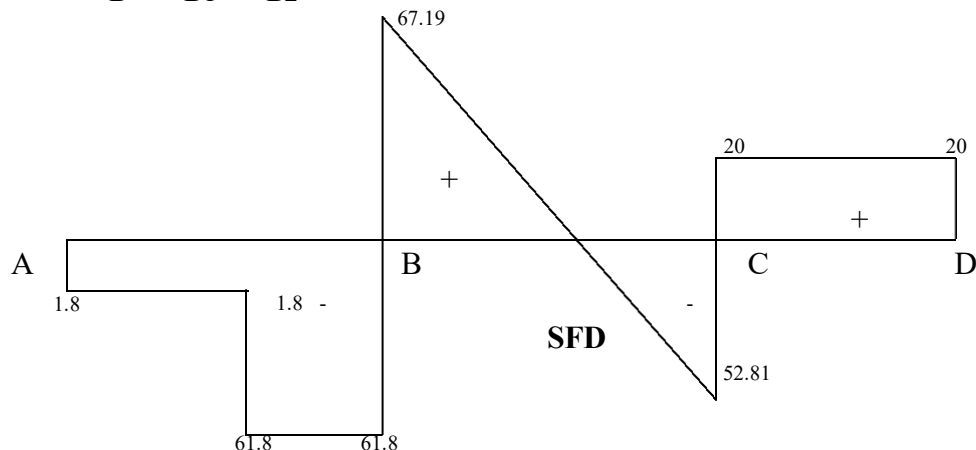
	A		B		C	D
		0.725	0.275		1	0
FEM	-19.33	20.67	-58.5		61.5	-20
BalanceOH					-41.5	
Carry over			-20.75			
Initial Moment	-19.33	20.67	-79.25		20	-20
Balance		42.47	16.11			
Carry over	21.235				0	
Final Moment	1.905	63.14	-63.14		20	-20

Support Reaction

V_A = -1.8kN, V_{B1} = 61.8kN, V_{B2} = 67.19kN, V_C = 72.81kN
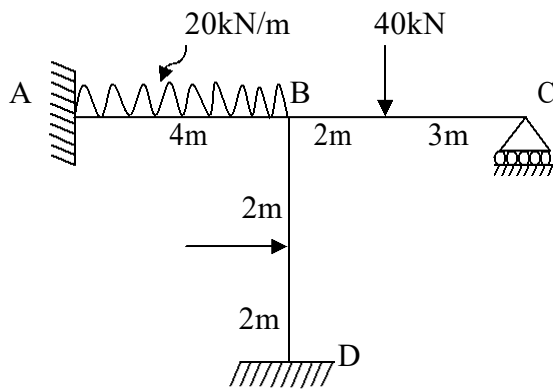
$V_B = V_{B1} + V_{B2}$ = 128.99kN

SFD

BMD
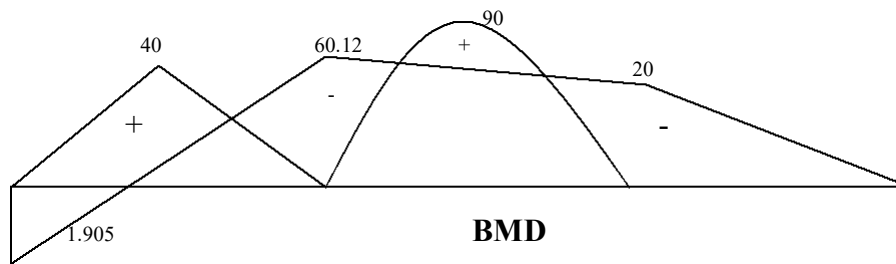
Analysis of Non – Sway frames

1. Draw BMD, EC and SFD for frame shown in the figure.

Fixed End Moments:

$$M_{FAB} = \frac{-wl^2}{12} = \frac{-20 \times 4^2}{12} = -26.67 \text{ kN-m}$$

$$M_{FBA} = \frac{wl^2}{12} = \frac{20 \times 4^2}{12} = 26.67 \text{ kN-m}$$

$$M_{FBC} = \frac{-wa\, b^2}{l^2} = \frac{-40 \times 2 \times 3^2}{5^2} = -28.8 \text{ kN-m}$$

$$M_{FCB} = \frac{wb\, a^2}{l^2} = \frac{60 \times 3 \times 2^2}{5^2} = 19.2 \text{ kN-m}$$

$$M_{FBD} = \frac{wa\, b^2}{l^2} = \frac{20 \times 2 \times 2^2}{4^2} = 10 \text{ kN-m}$$

$$M_{FDB} = \frac{-wb\, a^2}{l^2} = \frac{-20 \times 2 \times 2^2}{4^2} = -10 \text{ kN-m}$$

Stiffness Factor(K)

$$K_{BA} = \frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$\frac{4EI}{l} = \frac{4EI}{4} = EI$$

$$K_{BC} = \frac{3EI}{l} = \frac{3EI}{5} = 0.6EI$$

Distribution Factor(Δ)
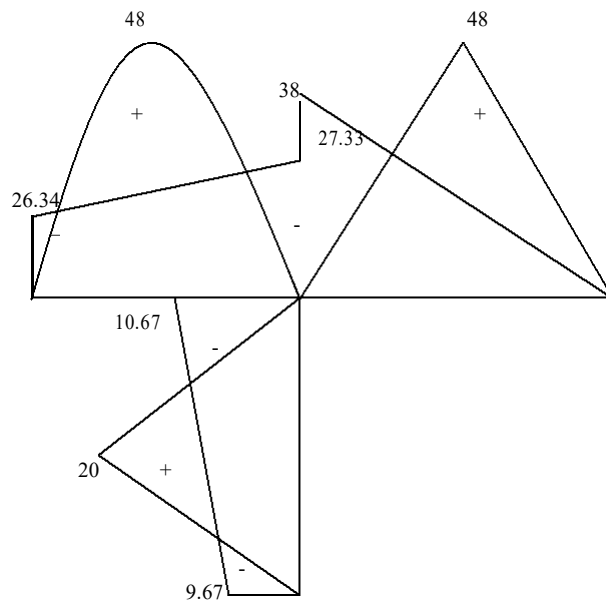
$$BA = \frac{K_{BA}}{K_{BA}+K_{BC}+K_{BD}} = \frac{EI}{EI+0.6EI+EI} = 0.385 \qquad CB = \frac{K_{BD}}{K_{BA}+K_{BC}+K_{BD}} = \frac{EI}{EI+0.6EI+EI} = 0.385$$

$$BC = \frac{K_{BC}}{K_{BA}+K_{BC}+K_{BD}} = \frac{0.6EI}{EI+0.6EI+EI} = 0.23$$

Moment Distribution Table

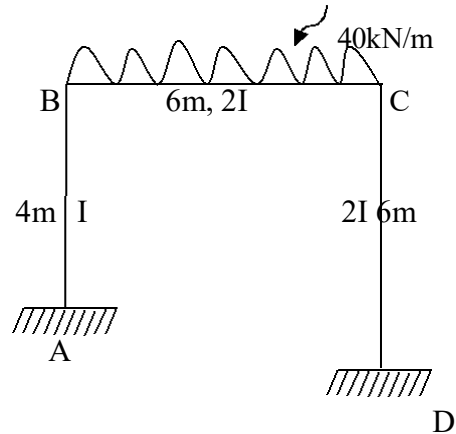|	AB	BA	BC	CB	BD	DB
		0.385	0.23		0.385	
FEM	-26.67	26.67	-28.8	19.2	10	-10
BalanceOH				-19.2		
Carry over			-9.6			
Initial Moment	-26.67	26.67	-38.4	0	10	-10
Balance		0.67	0.4		0.67	
Carry over	0.335			0		0.335
Final Moment	-26.34	27.33	-38	0	10.67	-9.67

UNIT – 4

SWAY ANALYSIS

Sway analysis by slope deflection method

1. Analyse the frame shown in the figure by slope deflection method Draw BMD and EC

Fixed End Moment:

$M_{FAB} = 0$

$M_{FBA} = 0$

$M_{FBC} = \dfrac{-wl^2}{12} = \dfrac{-40 \times 6^2}{12} = -120$ kN-m

$M_{FCB} = \dfrac{wl^2}{12} = \dfrac{40 \times 6^2}{12} = 120$ kN-m

$M_{FCD} = M_{FDC} = 0$

Slope Deflection Equation

$M_{AB} = M_{FAB} + \dfrac{2EI}{l}(2\theta_A + \theta_B - \dfrac{3\delta}{l})$

$= 0 + \dfrac{2EI}{4}(2\theta_A + \theta_B - \dfrac{3\delta}{l})$ ($\theta_A = 0$ due to fixity at support A)

$\boxed{M_{AB} = 0.5EI\,\theta_B - 0.375EI\delta}$

$M_{BA} = M_{FBA} + \dfrac{2EI}{l}(2\theta_A + \theta_B - \dfrac{3\delta}{l})$

$= 0 + \dfrac{2EI}{4}(2\theta_B + \theta_A - \dfrac{3\delta}{l})$

$\boxed{M_{BA} = EI\,\theta_B - 0.375EI\delta}$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$= -120 + \frac{2 \times 2EI}{6}(2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$\boxed{M_{BC} = -120 + 1.33EI\,\theta_B + 0.667EI\,\theta_C}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B - \frac{3\delta}{l})$$

$$= 120 + \frac{2 \times 2EI}{6}(2\theta_C + \theta_B - \frac{3\delta}{l})$$

$$\boxed{M_{CB} = 120 + 1.33EI\,\theta_C + 0.667EI\,\theta_B}$$

$$M_{CD} = M_{FCD} + \frac{2 \times 2EI}{l}(2\theta_C + \theta_D - \frac{3\delta}{l})$$

$$= 0 + \frac{2 \times 2EI}{6}(2\theta_C + \theta_D - \frac{3\delta}{l})$$

$$\boxed{M_{CD} = 1.33EI\,\theta_C - 0.33EI\delta}$$

$$M_{DC} = M_{FDC} + \frac{2 \times 2EI}{l}(2\theta_D + \theta_C - \frac{3\delta}{l})$$

$$= 0 + \frac{2 \times 2EI}{6}(2\theta_D + \theta_C - \frac{3\delta}{l})$$

$$\boxed{M_{DC} = 0.67EI\,\theta_C - 0.33EI\delta}$$

Apply the condition of Equilibrium

$M_{BA} + M_{BC} = 0$ and $M_{CB} + M_{CD} = 0$

At 'B'

$M_{BA} + M_{BC} = 0$

$EI\,\theta_B - 0.375EI\delta - 120 + 1.33EI\,\theta_B + 0.667EI\,\theta_C = 0$

$$2.33EI\,\theta_B + 0.667EI\,\theta_C - 0.375EI\delta = 120 \qquad \longrightarrow 1$$
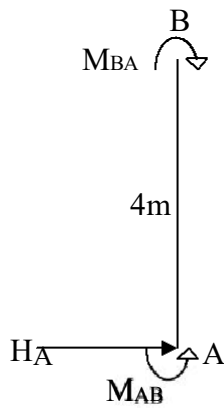
At 'c'

$M_{CB} + M_{CD} = 0$

$120 + 1.33EI\,\theta_C + 0.667EI\,\theta_B + 0.67EI\,\theta_C - 0.33EI\delta = 0$

$$0.667EI\,\theta_B + 2.33EI\,\theta_C - 0.33EI\delta = -120 \qquad \longrightarrow 2$$
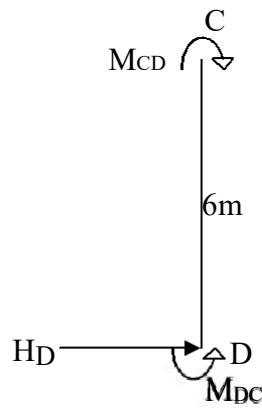
Shear condition

$$H_A = \frac{M_{AB} + M_{BA}}{4}$$

$$H_D = \frac{M_{CD} + M_{DC}}{6}$$

$$H_A = 0.375EI\theta_B - 0.1875EI\delta$$

$$H_D = 0.33EI\theta_C - 0.11EI\delta$$

$\Sigma H = 0$

$H_A + H_D = 0$

$$0.375EI\theta_B - 0.1875EI\delta + 0.33EI\theta_C - 0.11EI\delta = 0$$

$$0.375EI\ \theta_B + 0.33EI\ \theta_C - 0.296EI\delta = 0 \qquad \longrightarrow 3$$

Solving Eq 1 , 2 and 3

$$\theta_B = \frac{12.79}{EI} \quad \theta_C = \frac{-60.14}{EI} \quad \delta = \frac{24.83}{EI}$$

Substitute the above values in the S-D Equation

$M_{AB} = .5EI\ \theta_B - 0.375EI\delta$

$\boxed{M_{AB} = 27.31\ kN\text{-}m}$

$M_{BA} = EI\ \theta_B - 0.375EI\delta$

$\boxed{M_{BA} = 63.67 kN\text{-}m}$

$M_{BC} = -120 + 1.33EI\ \theta_B + 0.667EI\ \theta_C$

$\boxed{M_{BC} = -63.65\ kN\text{-}m}$

$M_{CB} = 120 + 1.33EI\ \theta_C + 0.667EI\ \theta_B$

$\boxed{M_{CB} = 88.64 kN\text{-}m}$
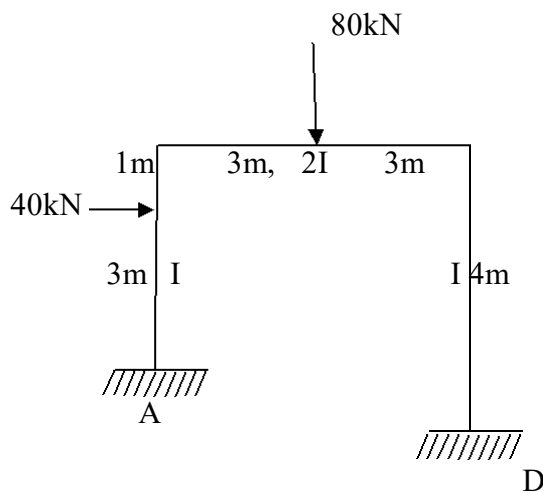
$M_{CD} = 1.33EI\ \theta_C - 0.33EI\delta$

$\boxed{M_{CD} = -88.1 kN\text{-}m}$

$M_{DC} = 0.67EI\ \theta_C - 0.33EI\delta$

$\boxed{M_{DC} = -48.29 kN\text{-}m}$
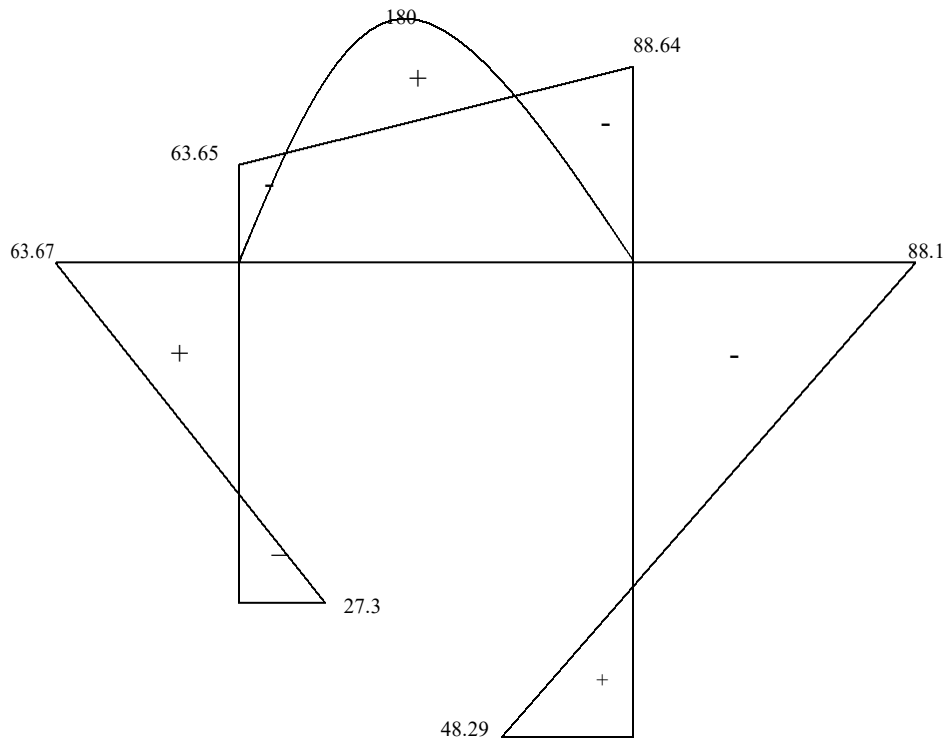
2. Analyse the frame shown in the figure by slope deflection method Draw BMD and EC

Fixed End Moment:

$$M_{FAB} = \frac{-wa\,b^2}{l^2} = \frac{-40\;X\;3\;X\;1^2}{4^2} = -7.5 \text{ kN-m}$$

$$M_{FBA} = \frac{wb\,a^2}{l^2} = \frac{40\;X\;1\;X\;3^2}{4^2} = 22.5 \text{ kN-m}$$

$$M_{FBC} = \frac{-wa\,b^2}{l^2} = \frac{-80\;X\;3\;X\;3^2}{6^2} = -60 \text{ kN-m}$$

$$M_{FCB} = \frac{wb\,a^2}{l^2} = \frac{80\;X\;3\;X\;3^2}{6^2} = 60 \text{ kN-m}$$

$M_{FCD} = M_{FDC} = 0$

Slope Deflection **Equation**

$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l}))$

$= -7.5 + \frac{2EI}{4}(2\theta_A + \theta_B - \frac{3\delta}{l}))$ ($\theta_A = 0$ due to fixity at support A)

$\boxed{M_{AB} = -7.5 + 0.5EI\theta_B - 0.375EI\delta}$

$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l}))$

$= 22.5 + \frac{2EI}{4}(2\theta_B + \theta_A - \frac{3\delta}{l})$

$\boxed{M_{BA} = 22.5 + EI\theta_B - 0.375EI\delta}$

$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$

$= -60 + \frac{2 \times 2EI}{6}(2\theta_B + \theta_C - \frac{3\delta}{l})$

$\boxed{M_{BC} = -60 + 1.33EI\theta_B + 0.667EI\theta_C}$

$M_{CB} = M_{FGB} + \frac{2EI}{l}(2\theta_C + \theta_B - \frac{3\delta}{l})$

$= 60 + \frac{2 \times 2EI}{6}(2\theta_C + \theta_B - \frac{3\delta}{l})$

$\boxed{M_{CB} = 60 + 1.33EI\theta_C + 0.667EI\theta_B}$

$M_{CD} = M_{FCD} + \frac{2 \times 2EI}{l}(2\theta_C + \theta_D - \frac{3\delta}{l})$

$= 0 + \frac{2EI}{4}(2\theta_C + \theta_D - \frac{3\delta}{l})$

$\boxed{M_{CD} = EI\theta_C - 0.375EI\delta}$

$M_{DC} = M_{FDC} + \frac{2 \times 2EI}{l}(2\theta_D + \theta_C - \frac{3\delta}{l})$

$= 0 + \frac{2EI}{4}(2\theta_D + \theta_C - \frac{3\delta}{l})$

$\boxed{M_{DC} = 0.5EI\theta_C - 0.375EI\delta}$

Apply the condition of Equilibrium

M_{BA} + M_{BC} = 0 and M_{CB}+M_{CD} = 0

At 'B'

M_{BA}+M_{BC}= 0

22.5 + EI θ_B – 0.375EIδ-60+1.33EI θ_B + 0.667EI θ_C = 0

$\quad\quad$ 2.33EI θ_B + 0.667EI θ_C – 0.375EIδ = 37.5 $\quad\longrightarrow$ 1

At 'c'

M_{CB}+M_{CD} = 0

60+1.33EI θ_C + 0.667EI θ_B+EI θ_C - 0.375EIδ = 0

$\quad\quad$ 0.667EI θ_B+ 2.33EI θ_C - 0.375EIδ= -60 $\quad\longrightarrow$ 2

Shear condition
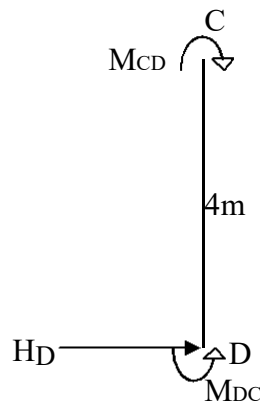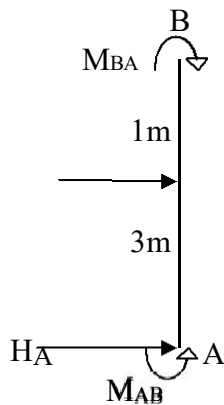
$$H_A = \frac{M_{AB} + M_{AB} -(40 \times 1)}{4} -0 \quad\quad\quad H_D = \frac{M_{CD}+ M_{CD}}{4}$$

H_A = 0.375EIθ_B – 0.1875EIδ-6.25 $\quad\quad$ H_D = 0.33EIθ_C - 0.185EIδ

ΣH=0

H_A + H_D+40 = 0

0.375EIθ_B – 0.1875EIδ-6.25 + 0.33EIθ_C - 0.185EIδ+40 = 0

$\quad\quad$ 0.375EI θ_B+ 0.375EI θ_C - 0.375EIδ= -33.75 $\quad\longrightarrow$ 3

Solving Eq 1 , 2 and 3

$$\theta_B = \frac{39.36}{EI} \quad\quad \theta_C = \frac{-19.36}{EI} \quad\quad \delta = \frac{110}{EI}$$

Substitute the above values in the S-D Equation

M_{AB} = -7.5+0.5EI θ_B - 0.375EIδ

M_{AB} = -29.07 kN-m

M_{BA} =22.5+ EI θ_B – 0.375EIδ

M_{BA} = 20.61kN-m

M_{BC} = -60+1.33EI θ_B + 0.667EI θ_C

M_{BC} = -20.62 kN-m

M_{CB} = 60+1.33EI θ_C + 0.667EI θ_B

M_{CB} = 60.62kN-m

M_{CD} = EI θ_C - 0.33EIδ

M_{CD} = -60.61kN-m

M_{DC} = 0.5EI θ_C - 0.33EIδ

M_{DC} = -50.93kN-m
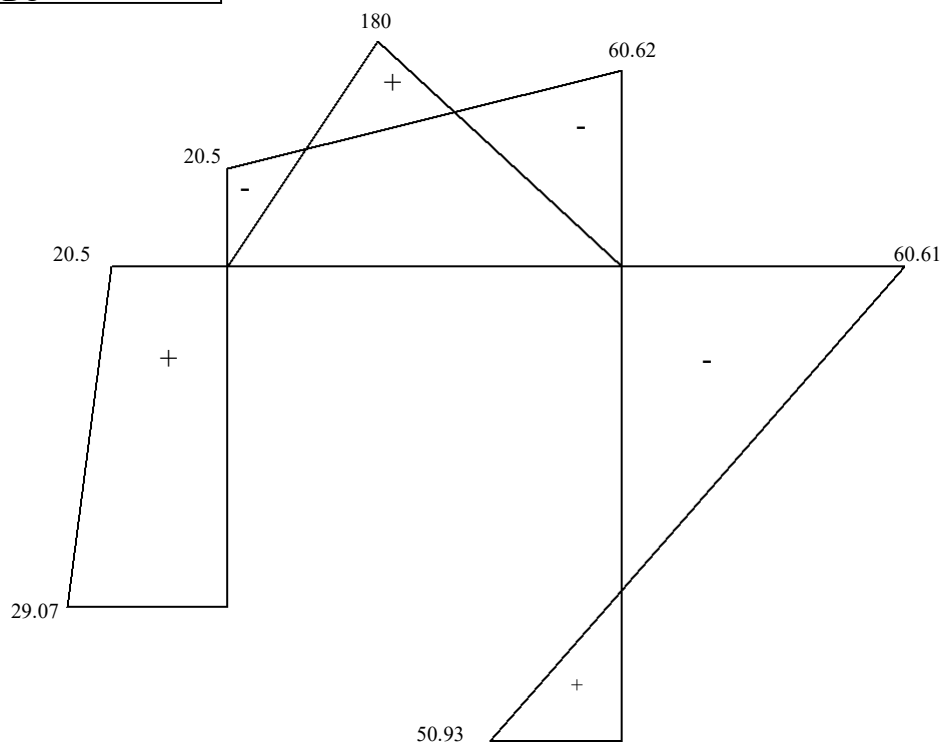
Sway analysis by Moment Distribution method

In moment distribution method of sway frames, we have to carry out two analysis to get final moments. They are

1. Non sway analysis
2. Sway analysis

Final Moment = Non sway moment + (C x Sway moment)

1. Analyse the frame by moment distribution method.
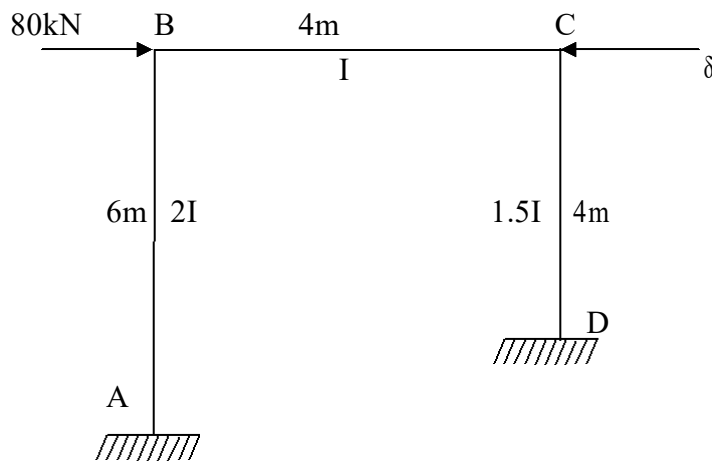
Non Sway Analysis

The sway in the frame is prevented by the force δ. As there are no loads on the members, there are no FEM's developed. Hence the final moments for non sway analysis will be absent.

Apply the shear conditions to the vertical members

$\Sigma_H = 0$,

$H_A + H_D + 80 - \delta = 0$, $\quad \delta = 80kN$ $\quad$ [$H_A = H_D = 0$]

Sway Analysis

Fixed End Moments:

$$M_{FAB} = M_{FBA} = \frac{-6EI\delta}{l^2} = \frac{-6 \times 2EI\delta}{l^2}$$

$$M_{FCD} = M_{CD} = \frac{-6EI\delta}{l^2} = \frac{-6 \times 1.5EI\delta}{l^2}.$$

Stiffness Factor(K)

$$\frac{M_{FAB}}{M_{FCD}} = \frac{2/6^2}{1.5/4^2} = \frac{16}{27}$$

$K_{BA} = \frac{4EI}{l} = \frac{4 \times 2EI}{6} = 1.33EI$

$K_{BC} = \frac{4EI}{l} = \frac{4EI}{4} = EI$

$K_{CB} = \frac{4EI}{l} = \frac{4EI}{4} = EI$

$K_{CD} = \frac{4EI}{l} = \frac{4 \times 1.5EI}{4} = 1.5EI$

Distribution Factor(Δ)

$BA = \frac{K_{BA}}{K_{BA}+K_{BC}} = \frac{1.33EI}{1.33EI+EI} = 0.57$

$CB = \frac{K_{CB}}{K_{CB}+K_{CD}} = \frac{EI}{EI+1.5EI} = 0.4$

$BC = \frac{K_{BC}}{K_{BA}+K_{BC}} = \frac{EI}{1.33EI+EI} = 0.43$

$CD = \frac{K_{CD}}{K_{CB}+K_{CD}} = \frac{1.5EI}{EI+1.5EI} = 0.6$

Moment Distribution Table

	A	B		C		D	
		0.57	0.43	0.4		0.6	
FEM	-16	-16	0	0		-27	-27
Balance		+9.12	+6.88	+10.8		+16.2	
Carry over	4.56		5.4		3.44		8.1
Balance		-3.08	-2.32	-1.38		-2.06	
Carry over	-1.54		-0.68		-1.16		-1.03
Balance		+0.39	+0.29	0.464		+0.696	
Carry over	0.194		0.232		0.146		0.348
Balance		-0.13	-0.099	-0.0584		-0.088	
Carry over	-0.066		-0.029		-0.05		-0.044
Balance		-0.0165	-0.0125	-0.02		-0.03	
Final Moment	-12.77	-9.68	-9.68	12.25		-12.25	-19.58

Shear Conditions

$\Sigma_H = 0, \quad H_A + H_D + \delta' = 0$

$H_A = \dfrac{M_{AB} + M_{AB}}{4} \qquad H_D = \dfrac{M_{CD} + M_{CD}}{4}$

$\delta' = 11.7kN$

Sway correction factor, $C = \dfrac{\delta}{\delta'} = \dfrac{80}{11.7} = 6.84$

Final Moment = Non sway moment + (C x (Sway moment)

$M_{AB} = 0 + (6.84 \times -12.77) = -87.35kN\text{-}m$

$M_{BA} = 0 + (6.84 \times -9.68) = -66.21kN\text{-}m$

$M_{BC} = 0 + (6.84 \times 9.68) = 66.21kN\text{-}m$

$M_{CB} = 0 + (6.84 \times 12.25) = -83.79kN\text{-}m$

$M_{CD} = 0 + (6.84 \times -12.25) = -83.79kN\text{-}m$

$M_{DC} = 0 + (6.84 \times -19.58) = -133.93kN\text{-}m$
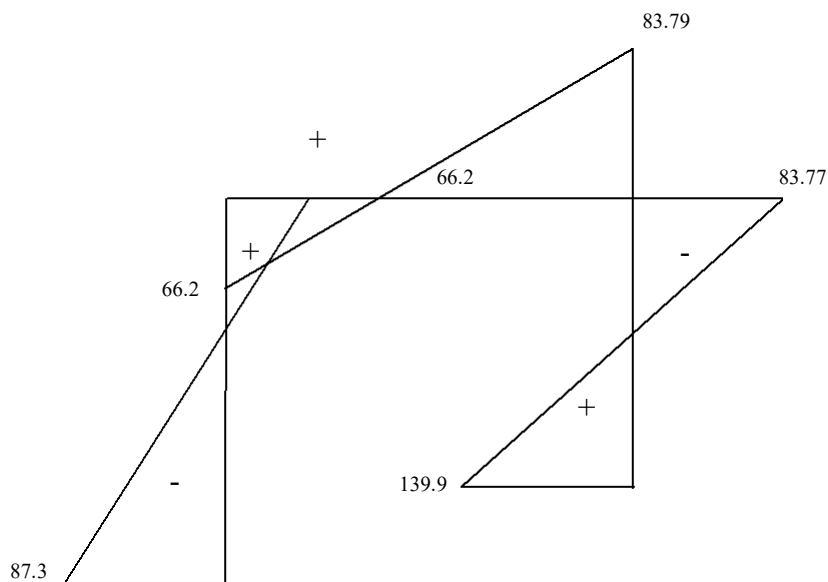
2. Analyse the frame by moment distribution method.
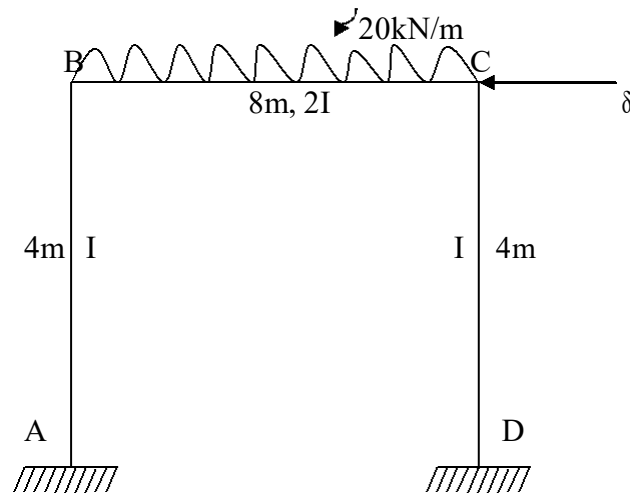
Non Sway Analysis

Fixed End Moments:

$M_{FAB} = M_{FBA} = 0$

$M_{FCD} = M_{CD} = 0$

$M_{FBC} = \dfrac{-wl^2}{12} = -106.67 \text{kN-m}$

$M_{FCB} = \dfrac{-wl^2}{12} = 106.67 \text{kN-m}$

Stiffness Factor(K)

$K_{BA} = \dfrac{4EI}{l} = \dfrac{4\,EI}{4} = EI$

$K_{BC} = \dfrac{4EI}{l} = \dfrac{4 \times 2EI}{8} = EI$

$K_{CB} = \dfrac{4EI}{l} = \dfrac{4 \times 2EI}{8} = EI$

$K_{CD} = \dfrac{3EI}{l} = \dfrac{3\,EI}{4} = 0.75EI$

Distribution Factor(Δ)

$BA = \dfrac{K_{BA}}{K_{BA}+K_{BC}} = 0.5$ $CB = \dfrac{K_{CB}}{K_{CB}+K_{CD}} = 0.57$

$BC = \dfrac{K_{BC}}{K_{BA}+K_{BC}} = 0.5$ $CD = \dfrac{K_{CD}}{K_{CB}+K_{CD}} = 0.43$

Moment Distribution Table

	A		B		C		D	
			0.5	0.5		0.57	0.43	
FEM	0		0	-106.67		106.67	0	0
Balance			53.33	53.33		-60.8	-45.87	
Carry over	26.67			-30.4		26.69		0
Balance			15.2	15.2		-15.2	-11.47	
Carry over	7.6			-7.6		7.6		0
Balance			3.8	3.8		-4.33	-3.27	
Carry over	1.9			-2.17		1.9		0
Balance			1.09	1.09		-1.083	-0.82	
Carry over	-0.54			-0.54		0.54		0
Balance			0.27	0.27		-0.31	-0.133	
Carry over	-0.135			-0.15		0.135		0
Balance			0.077	0.077		-0.077	-0.058	
Carry over	-0.039			-0.03		0.039		0
Balance			0.019	0.019		-0.02	-0.0167	
Final Moment	36.85		73.76	-73.76		61.72	-61.72	0

Apply the shear conditions to the vertical members

$\Sigma_H = 0$,

$H_A + H_D - \delta = 0$,

$H_A = \dfrac{M_{AB} + M_{AB}}{4}$ $\qquad$ $H_D = \dfrac{M_{CD} + M_{CD}}{4}$

$\delta = 12.22 \text{kN}$

Sway Analysis

Fixed End M**oments:**

$M_{FAB} = M_{FBA} = \dfrac{-6EI\delta}{l^2}$

$M_{FCD} = M_{FCD} = \dfrac{-6EI\delta}{l^2}$

$$\frac{M_{FAB}}{M_{FCD}} = \frac{\frac{-6EI\delta}{l^2}}{\frac{-6EI\delta}{l^2}} = \frac{-1}{-1} = \frac{-10}{-10}$$

Moment Distribution Table

	A	B		C	D
		0.5	0.5	0.57	0.43	
FEM	-10	-10	0	0	-10	-10
Release D						10
Carry over				5		
Initial Moment	-10	-10	0	0	-5	0
Balance		5	5	2.85	2.15	
Carry over	2.5		1.425	2.5		0
Balance		-0.72	-0.72	-1.43	-1.1	
Carry over	-0.36		-0.713	-0.36		0
Balance		0.36	0.36	0.21	0.15	
Carry over	0.18		0.11	0.18		0
Balance		-0.055	-0.055	-0.103	-0.077	
Carry over	-0.028		-0.0513	-0.028		0
Balance		0.026	0.026	0.016	0.012	
Final Moment	-7.71	-5.34	5.34	3.84	-3.84	0

Shear Conditions

$\Sigma H = 0, \quad H_A + H_D + \delta' = 0$

$H_A = \dfrac{M_{AB} + M_{AB}}{4}$

δ' = 4.23kN

$$\frac{\delta}{\delta'} = \frac{12.22}{4.23} = 2.8$$

Sway correction factor, $C = \quad = \therefore \quad = 2.89$
Final Moment = Non sway moment + (C x Sway moment)

$M_{AB} = 36.85 + (2.89 \times -7.71) = 14.66$kN-m

$M_{BA} = 73.76 + (2.89 \times -5.34) = 58.15$kN-m

$M_{BC} = -73.76 + (2.89 \times 5.34) = -58.14$kN-m

$M_{CB} = 61.70 + (2.89 \times 3.84) = 72.84$kN-m

$M_{CD} = -61.72 + (2.89 \times -3.84) = -72.82$kN-m

$M_{DC} = 0 + (2.89 \times 0) = 0$

BMD
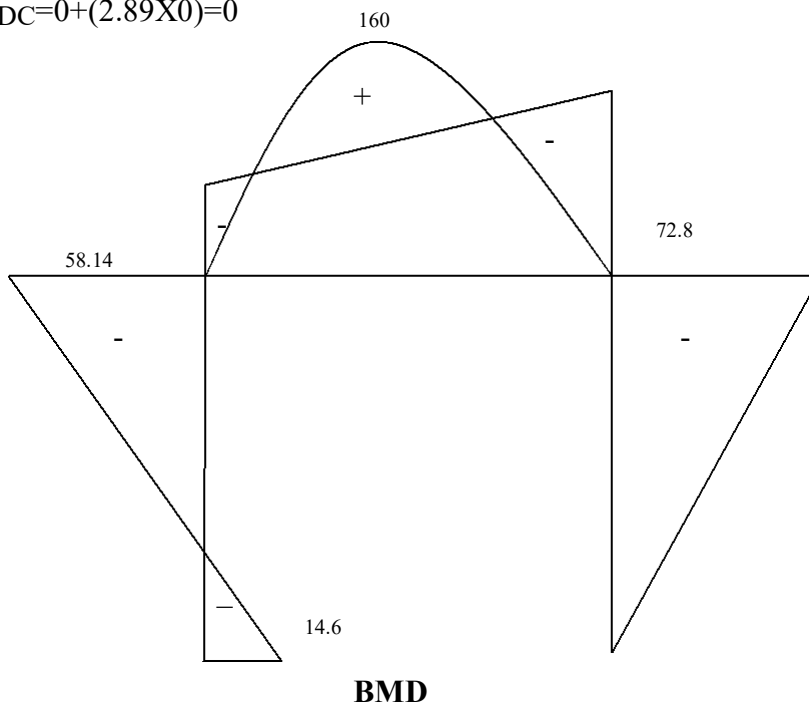
UNIT-5
FLEXIBILITY MATRIX METHOD

The systematic development of consistent deformation method in the matrix form has lead to flexibility matrix method. The method is also called force method. Since the basic unknowns are the redundant forces in the structure.

This method is exactly opposite to stiffness matrix method.

The flexibility matrix equation is given by

$$[P] [F] = \{[\Delta] - [\Delta_L]\}$$

$$[P] = [F]^{-1} \{[\Delta] - [\Delta_L]\}$$

Where,

[P] = Redundant in matrix form

[F] = Flexibility matrix

[Δ] = Displacement at supports

[Δ_L]= Displacement due to load

1. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD
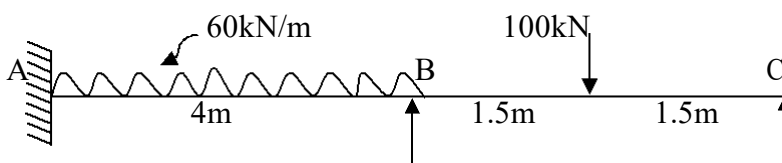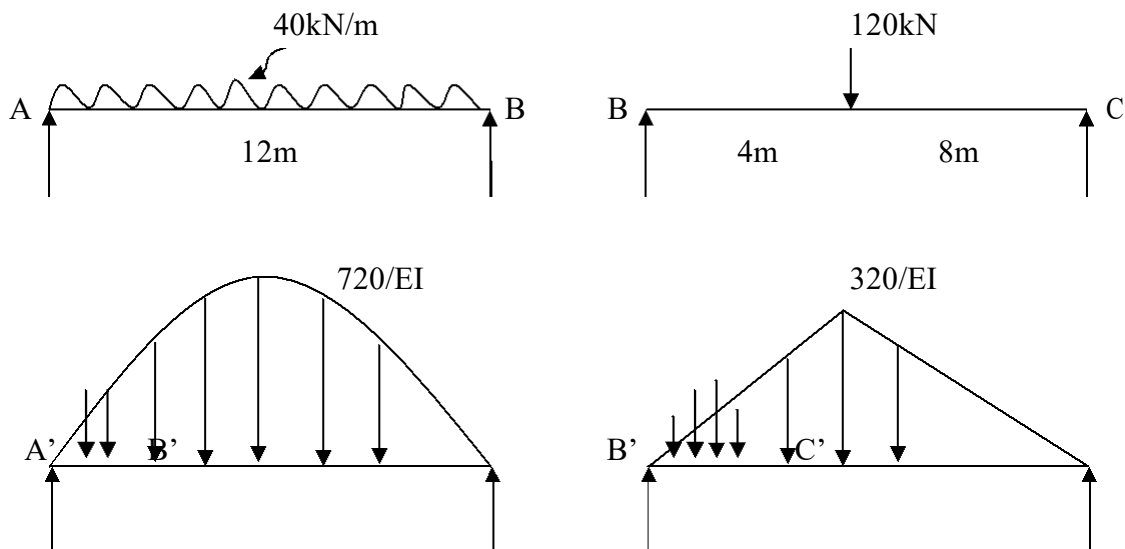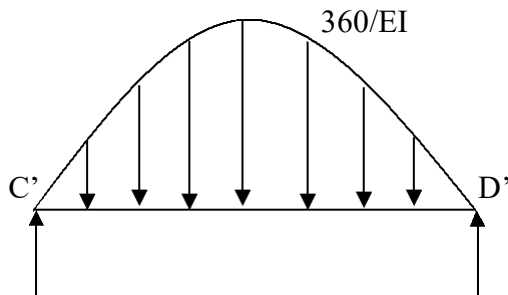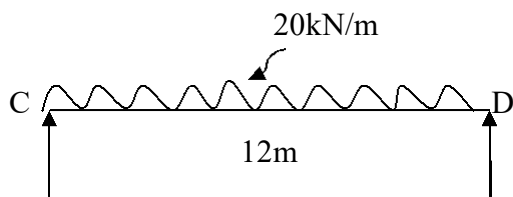

Static Indeterminacy SI = 2 (M_A and M_B)

M_A and M_B are the redundant

Let us remove the redundant to get primary determinate structure
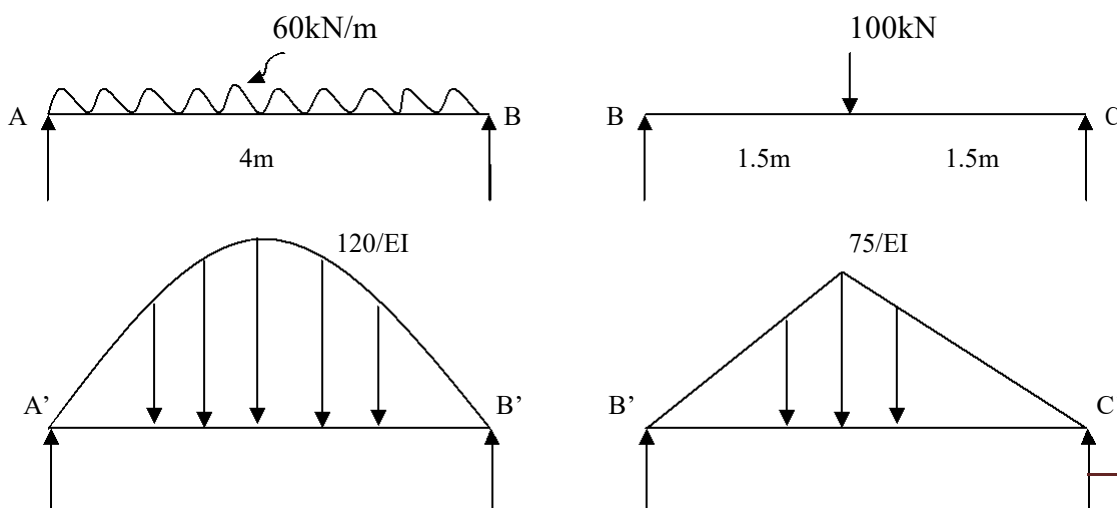
$$[\Delta_L] = \begin{pmatrix} 1L \\ \\ 2L \end{pmatrix}$$

1L = Rotation at A = SF at A'

1L = ½[2/3 X 4 X $\frac{120}{EI}$]

1L = $\frac{160}{EI}$

2L = Rotation at A = SF at B'

= V$_{B1}$' + V$_{B2}$'

2L = ½[2/3 X 4 X $\frac{120}{EI}$] + ½[2/3 X $\frac{75}{EI}$]

2L = $\frac{216.25}{EI}$ 160

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 160 \\ 216.25 \\ 216.25 \end{pmatrix}$$

Note: The rotation due to sagging is taken as positive. The moments producing due to sagging are also taken as positive.

<u>To get Flexibility Matrix</u>
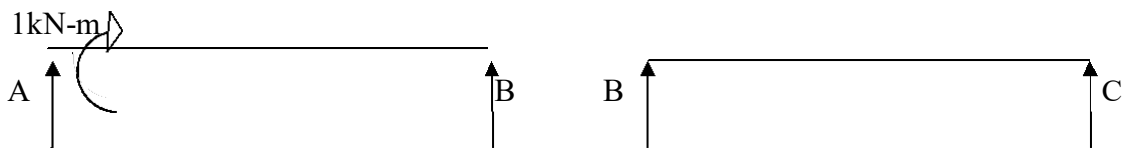
Apply unit moment to joint A

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} = \frac{1 \times 4}{3EI} = \frac{1.33}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI}$$

Apply unit moment to joint A
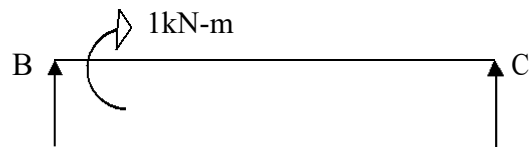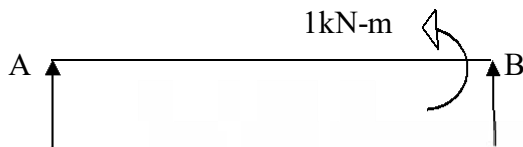
$$\delta_{12} = \frac{ml}{6EI} = \frac{1 \times 4}{6EI} = \frac{0.67}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 4}{3EI} + \frac{1 \times 3}{EI} = \frac{2.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 1.33 & 0.67 \\ \\ 0.67 & 1.33 \end{pmatrix}$$

Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix}$$

$$[P] = EI \begin{pmatrix} 1.33 & 0.67 \\ \\ 0.67 & 1.33 \end{pmatrix}^{-1} \left\{ \begin{bmatrix} 0 \\ \\ 0 \end{bmatrix} - \frac{1}{EI} \begin{bmatrix} 160 \\ \\ 216.25 \end{bmatrix} \right\}$$

$$[P] = \begin{pmatrix} M_{AB} \\ \\ M_{BA} \end{pmatrix} = \begin{pmatrix} -86.00 \\ \\ -68.08 \end{pmatrix} \text{kN-m}$$
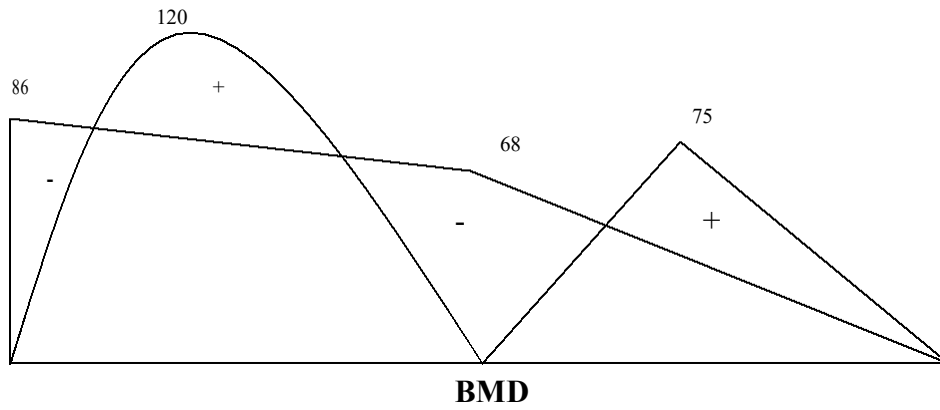
BMD

2. Analyse the continuous beam shown in the figure by flexibility matrix method, draw BMD
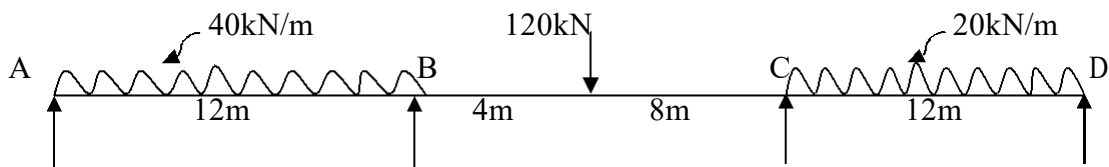
Static Indeterminacy SI = 2 (M_B and M_C)

M_B and M_C are the redundant

Let us remove the redundant to get primary determinate structure

20kN/m

C ⌒⌒⌒⌒⌒⌒⌒⌒⌒⌒⌒ D

12m

360/EI

C'   D'

$$[\Delta_L] = \begin{pmatrix} {}_{1L} \\ \\ {}_{2L} \end{pmatrix}$$

$_{1L}$ = Rotation at B = SF at B' = $V_{B1}' + V_{B2}'$

$_{1L} = \dfrac{V_{B1}' + V_{B2}'}{EI}\ 3946.67$

$_{2L}$ = Rotation at C = SF at C' = $V_{C1}' + V_{C2}'$

$_{2L} = \dfrac{V_{C1}' + V_{C2}'}{EI}\ 2293.33$

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 3946.67 \\ \\ 2293.33 \end{pmatrix}$$

<u>To get Flexibility Matrix</u>

Apply unit moment to joint A

1kN-m ◁   ▷ 1kN-m

A ↑   ↑B   B ↑   ↑C

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{3EI} = \frac{8}{EI}$$

$$\delta_{21} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}$$

Apply unit moment to joint A

1kN-m ◁   ▷ 1kN-m

B ↑   ↑C   C ↑   ↑D
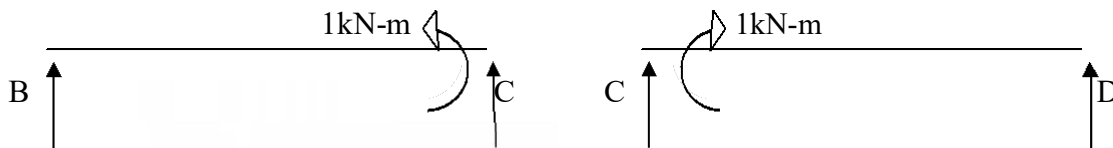
$$\delta_{12} = \frac{ml}{6EI} = \frac{1 \times 12}{6EI} = \frac{2}{EI}$$

$$\delta_{22} = \frac{ml}{3EI} + \frac{ml}{3EI} = \frac{1 \times 12}{3EI} + \frac{1 \times 12}{EI} = \frac{8}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}$$

Apply the flexibility equation

$$[P] = [F]^{-1}\{[\Delta] - [\Delta L]\}$$

$$[\Delta] = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$[P] = EI \begin{pmatrix} 8 & 2 \\ 2 & 8 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \frac{1}{EI} \begin{pmatrix} 33946 \\ 22993 \end{pmatrix} \right\}$$

$$[P] = \begin{Bmatrix} M_{AB} \\ M_{BA} \end{Bmatrix} = \begin{Bmatrix} -449.97 \\ -174.22 \end{Bmatrix} \text{kN-m}$$


BMD
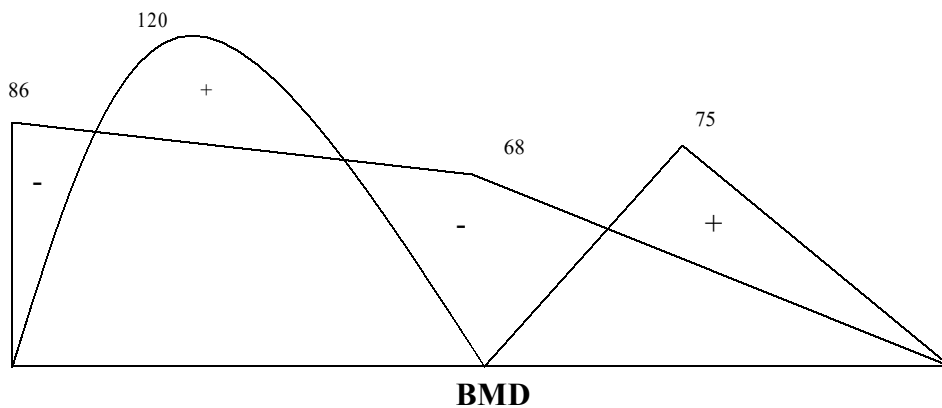
SINKING OF SUPPORT

1. Analyse the continuous beam by flexibility method, support B sinks by 5mm. Sketch the BMD and EC given EI = 15 X 10^3 kN-m^2
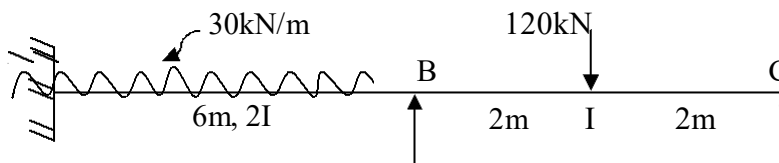
NOTE: In this case of example with sinking of supports, the redundant should be selected as the vertical reaction.

Static indeterminacy is equal to 2. Let V_B and V_C be the redundant, remove the redundant to get the primary structure.

B'   C'

A'
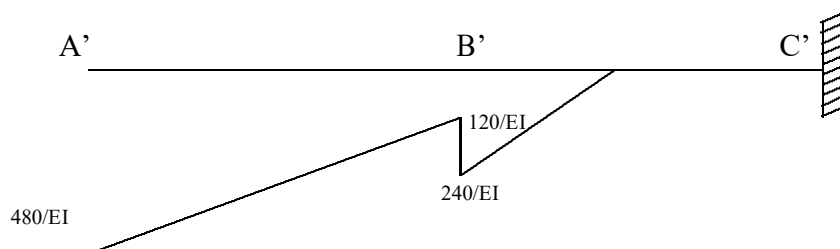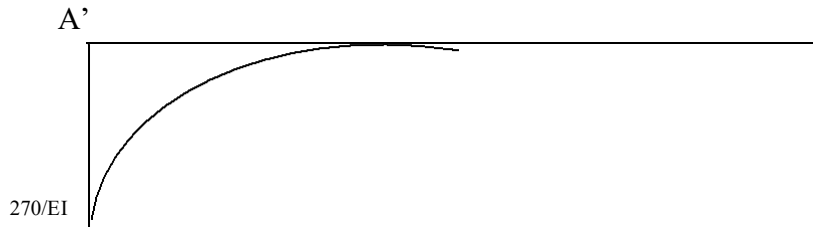
270/EI

$$[\Delta_L] = \begin{pmatrix} 1L \\ \\ 2L \end{pmatrix}$$

$1L$ = Displacement at B in primary determinate structure = BM at B' in conjugate beam

$1L = [\frac{1}{2} X 6 X \frac{360}{EI} X (2/3 X 6)] + (6 X \frac{120}{EI} X 6/2) + [\frac{1}{3} X 6 X \frac{270}{EI} X (3/4 X 6)]$

$1L = \frac{8910}{EI}$

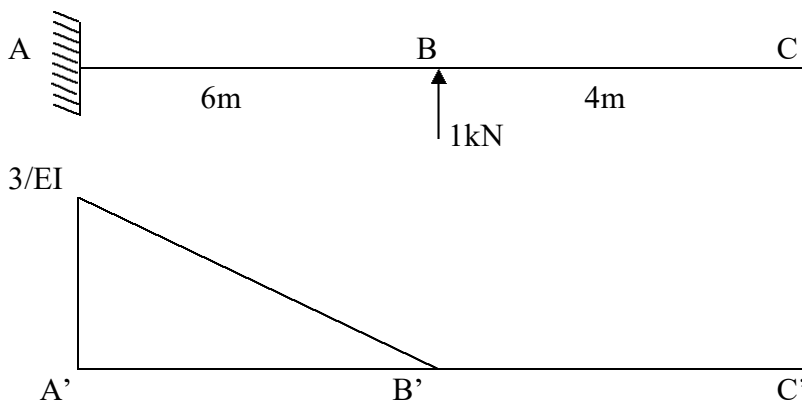$2L$ = Displacement at C in primary determinate structure = BM at C' in conjugate beam

$2L = [\frac{1}{2} X 6 X \frac{360}{EI} X (2/3 X 6 + 4)] + (6 X \frac{120}{EI} X 6/2 + 4) + [\frac{1}{3} X 6 X \frac{270}{EI} X (3/4 X 6 + 4)]$

$2L = \frac{19070}{EI}$

$$[\Delta_L] = \frac{1}{EI} \begin{pmatrix} 8910 \\ \\ 19070 \end{pmatrix}$$

To get Flexibility Matrix

Apply unit Load at B

A   B   C

6m   4m

1kN

3/EI

A'   B'   C'

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix}$$

$$\delta_{11} = -\frac{1}{2} \times 6 \times \frac{3}{EI}(2/3 \times 6) \times 6) \quad \frac{-36}{EI}$$

$$\delta_{21} = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6 + 4) = \quad \frac{-72}{EI}$$

Apply unit load at C
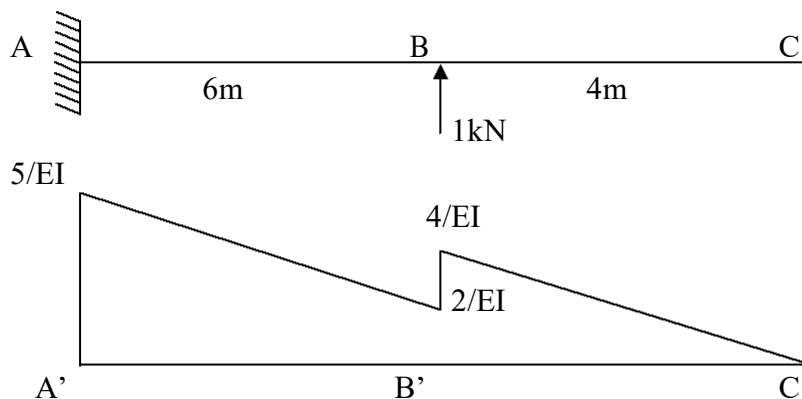
$$\delta_{12} = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6) + [6 \times \quad \frac{2}{EI} \times (6/2)] = \frac{-72}{EI}$$

$$\delta_{22} = -\frac{1}{2} \times 6 \times \frac{3}{EI} \times (2/3 \times 6 + 4) - [6 \times \frac{2}{EI} \times (6/2 + 4)] - \frac{1}{2} \times 4 \times \frac{4}{EI} \times (2/3 \times 4) = \frac{-177.33}{EI}$$

$$[F] = \begin{pmatrix} \delta_{11} & \delta_{12} \\ \delta_{21} & \delta_{22} \end{pmatrix} = \frac{1}{EI} \begin{pmatrix} -36 & -72 \\ -72 & -177.33 \end{pmatrix}$$

Apply the flexibility equation
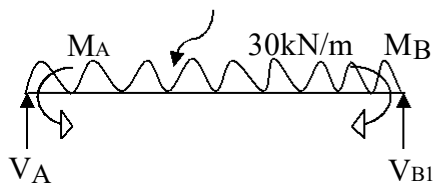
$$[P] = [F]^{-1}\{[\Delta] - [\Delta_L]\}$$

$$[\Delta] = \begin{pmatrix} 0.005 \\ 0 \end{pmatrix}$$

Structural Analysis – II

$$[P] = EI \begin{pmatrix} -36 & -72 \\ \\ -72 & -177.33 \end{pmatrix}^{-1} \left\{ \begin{pmatrix} 0.005 \\ \\ 0 \end{pmatrix} \frac{1}{EI} \begin{pmatrix} 88900 \\ \\ 190070 \end{pmatrix} \right\}$$

$$[P] = \begin{pmatrix} V_B \\ \\ V_C \end{pmatrix} = \begin{pmatrix} 161.43 \\ \\ 41.98 \end{pmatrix} \text{kN-m}$$
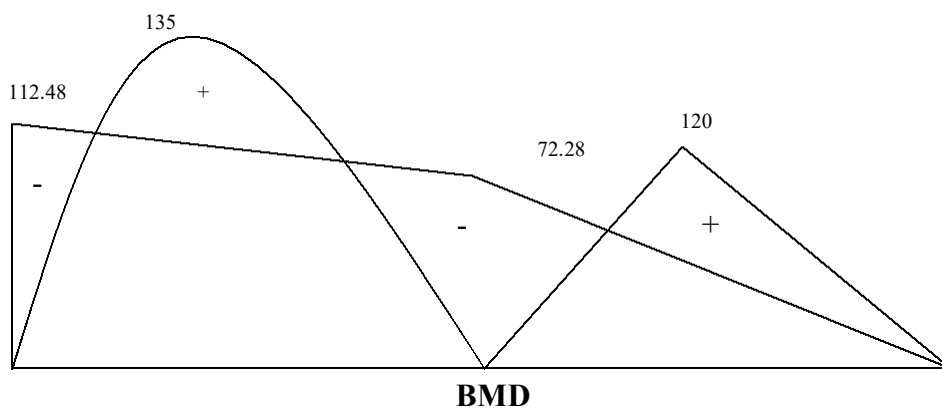
Support Reaction
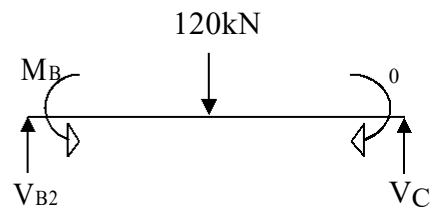
$V_A = 96.64$kN, $V_{B1} = 83.36$kN, $V_{B2} = 78.07$kN, $V_C = 41.98$kN

$V_B = V_{B1} + V_{B2} = 161.43$kN

$$\begin{pmatrix} M_A \\ \\ M_B \end{pmatrix} = \begin{pmatrix} 112.48 \\ \\ 72.28 \end{pmatrix} \text{kN-m}$$

BMD

UNIT-6
STIFFNESS MATRIX METHOD

The systematic development of slope deflection method in the matrix form has lead to Stiffness matrix method. The method is also called Displacement method. Since the basic unknowns are the displacement at the joint.

The stiffness matrix equation is given by

$$[\Delta]\,[K] = \{[P] - [P_L]\}$$

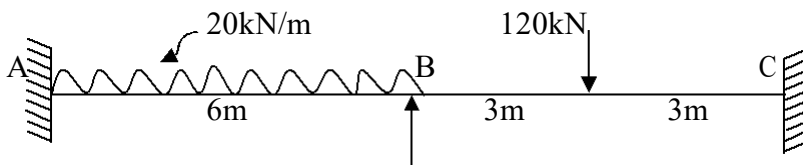$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

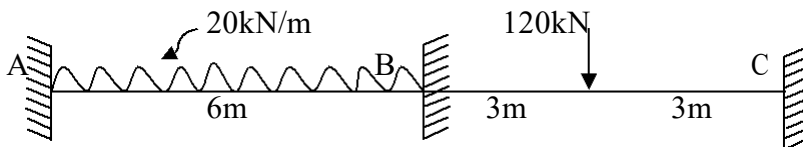Where,

[P] = Redundant in matrix form
[F] = Stiffness matrix
[P] = Final force at the joints in matrix form
[P_L]= force at the joints due to applied load in matrix form

1. Analyse the continuous beam by Stiffness method Sketch the BMD
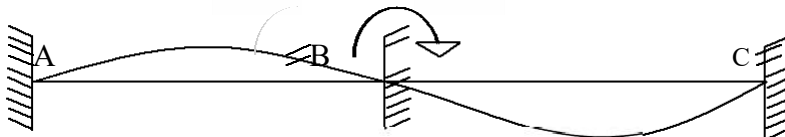

Kinematic Indeterminacy KI = 1 (θ_B)

$$[P_L] = M_{FBA} + M_{FBC}$$
$$= \quad + (- \quad) = -$$
$$[P_L] = -30\text{kN-m} \quad \frac{wl}{12} + \frac{wl}{8} = \frac{20 \times 6^2}{12} - \frac{120 \times 6}{8}$$

Apply unit displacement at joint B.

$$[K] = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{6} + \frac{4EI}{6} = 1.33EI \qquad (\theta=1)$$

By condition of equilibrium at joint B

$$[P]=0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$= \frac{1}{K}\{[P] - [P_L]\}$$

$$\frac{1}{1.33EI}\{[0] - [-30]\} \quad \frac{22.56}{EI}$$

$$\theta_B = \quad \{[0] - [-30]\} = \quad$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -60 + \frac{2\,EI}{6}(2\theta_A + \frac{22.5}{EI}) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

$$M_{AB} = -52.5\text{kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

$$= 60 + \frac{2\,EI}{6}(2 \times \frac{22.5}{EI} + \theta_A)$$

$$M_{BA} = 75.04\text{kN-m}$$

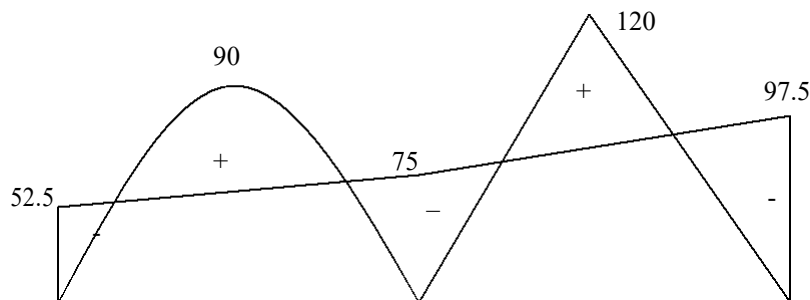$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -90 + \frac{2EI}{6}(2 \times \frac{22.5}{EI} + 0)$$

$$M_{BC} = -75\text{kN-m}$$

$$M_{CB} = M_{FCB} + \frac{2EI}{l}(2\theta_C + \theta_B)$$

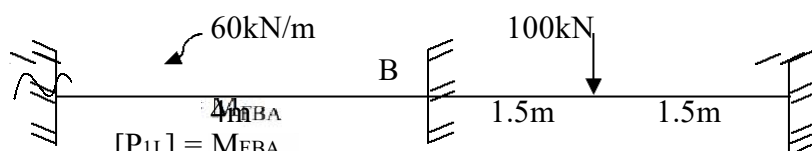$$= 90 + \frac{2EI}{6}(0 + \frac{22.5}{EI})$$

$$M_{CB} = 97.52\text{kN-m}$$

BMD

2. Analyse the continuous beam by Stiffness method Sketch the BMD
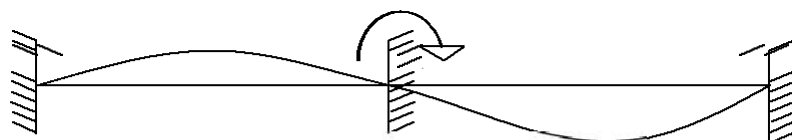

Kinematic Indeterminacy KI = 2 (θ_B & θ_C)

$[P_{1L}] = M_{FBA}$
$+ -M_{FBC}$
$= \frac{wl^2}{12} + ((- M_{FBC})$

$[P_{2L}] = M_{FCB} = \frac{wl}{8} = \frac{60 \times 4^2}{12} - \frac{100 \times 3}{8} = 42.5 \text{kN-m}$

$\frac{wl}{8} = \frac{100 \times 3}{8} = 37.5 \text{kN-m}$

$$[P_L] = \begin{pmatrix} P_{1L} \\ P_{2L} \end{pmatrix} = \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix} \text{kN-m}$$
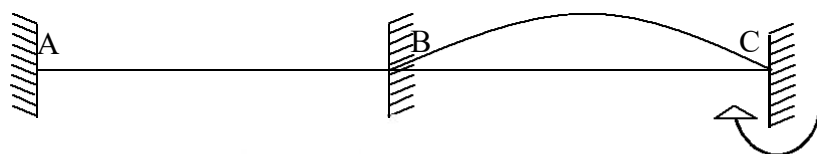
Apply unit displacement at joint B.

$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{4} + \frac{4EI}{3} = 2.33EI$ ($\theta = 1$)

$K_{21} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$

Apply unit displacement at joint B.

$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$

$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{pmatrix} 2.33 & 0.67 \\ 0.67 & 1.33 \end{pmatrix}^{-1}\left\{\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} 42.5 \\ 37.5 \end{pmatrix}\right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI}\begin{bmatrix} -11.88 \\ -22.19 \end{bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$= -80 + \frac{2EI}{4}(2\theta_A - \frac{11.88}{EI}) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

$$M_{AB} = -85.94 \text{ kN-m}$$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_B + \theta_A)$$

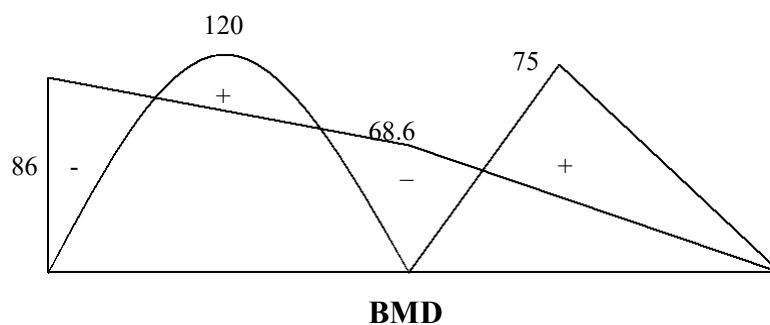$$= 80 + \frac{2EI}{4}(2 \times \frac{-11.88}{EI} + \theta_A)$$

$$M_{BA} = 68.12 \text{ kN-m}$$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -37.5 + \frac{2EI}{6}(2 \times \frac{-11.88}{EI} + \frac{-22.9}{EI})$$
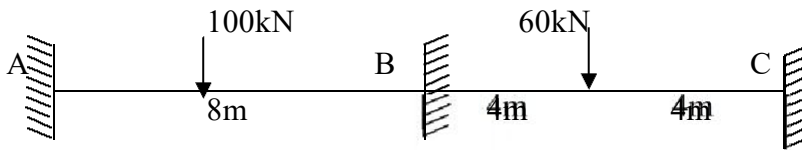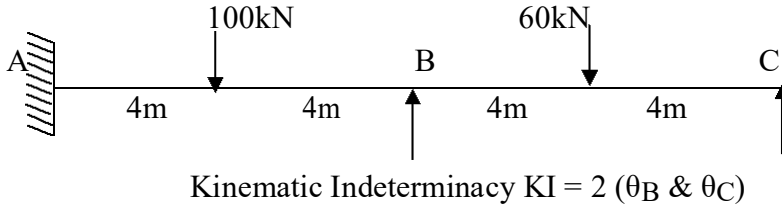
$$M_{BC} = -68.6 \text{ kN-m}$$

$$M_{CB} = 0$$

BMD

Sinking of support

1. Analyse the continuous beam shown in figure by stiffness method. Support B sinks by 300/EI units and support C sinks by 200/EI units
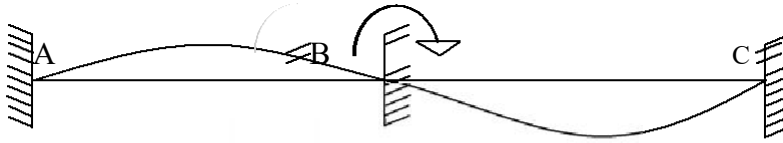

Kinematic Indeterminacy KI = 2 (θ_B & θ_C)

$$[P_{1L}] = M_{FBA} + M_{FBC} - \frac{6EI\delta}{l^2} - \frac{6EI\delta}{l^2}$$

$$= \frac{100 \, X \, 8}{8} - \frac{60 \, X8}{8} - \frac{6 \, X \, 300}{8^2} + \frac{6 \, X \, 100}{8^2} = 21.25 \text{kN-m}$$

$$[P_{2L}] = M_{FCB} - \frac{6EI\delta}{l^2} = \frac{60 \, X8}{8} + \frac{6 \, X \, 100}{8^2} = 69.38 \text{kN-m}$$

$$[P_L] = \begin{Bmatrix} P_{1L} \\ \\ P_{2L} \end{Bmatrix} = \begin{Bmatrix} 21.25 \\ \\ 69.38 \end{Bmatrix} \text{kN-m}$$
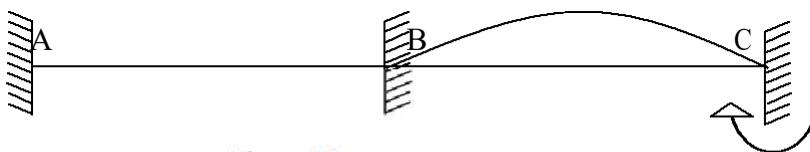
Apply unit displacement at joint B.

$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} = \frac{4EI}{8} + \frac{4EI}{8} = EI \qquad (\theta = 1)$$

$$K_{21} = \frac{2EI\theta}{8} = \frac{2EI}{3} = 0.25EI$$

Apply unit displacement at joint B.

$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{8} = 0.25EI$$

Structural Analysis – II

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{8} = 0.50EI$$

By condition of equilibrium at joint B

$$[P] = 0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI}\begin{bmatrix} 1 & 0.25 \\ 0.25 & 0.50 \end{bmatrix}^{-1} \left\{ \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 21.25 \\ 69.38 \end{Bmatrix} \right\}$$

$$\begin{Bmatrix} \theta_B \\ \theta_C \end{Bmatrix} = \frac{1}{EI}\begin{Bmatrix} 15.36 \\ -146.44 \end{Bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B = \frac{3(+\delta)}{l})$$

$$= -100 + \frac{2EI}{8}(2\theta_A + \frac{15.36}{EI} - \frac{3EI(300/EI)}{8})$$ ($\theta_A = 0$ due to fixity at support A)

$M_{AB} = -124.29 \text{ kN-m}$

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B - \frac{3\delta}{l})$$

$$= 100 + \frac{2EI}{8}(\theta_A + 2 \times \frac{15.36}{EI} - \frac{3EI(300/EI)}{8})$$
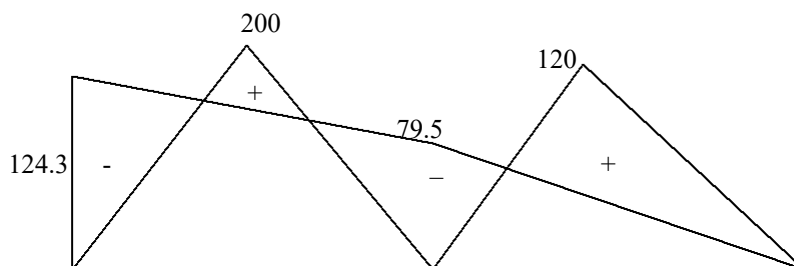
$M_{BA} = 79.55 \text{ kN-m}$

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C - \frac{3\delta}{l})$$

$$= -60 + \frac{2EI}{8}(2 \times \frac{15.36}{EI} + \frac{-146.44}{EI} - \frac{3EI(-100/EI)}{5})$$
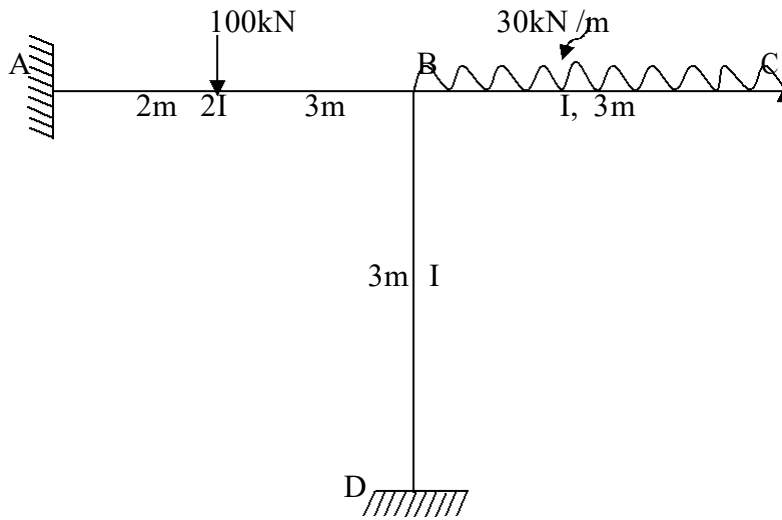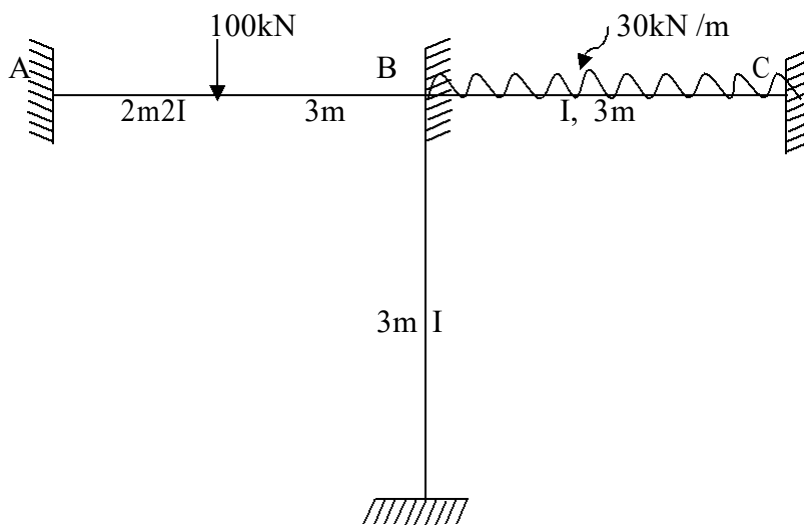
$M_{BC} = -79.55 \text{ kN-m}$

$M_{CB} = 0$

BMD

Analysis of frames

1. Analyse the frame by stiffness method

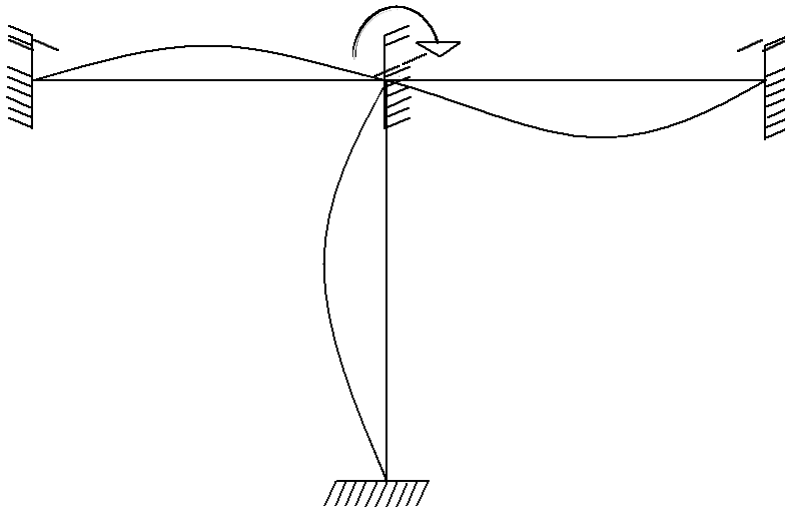
Kinematic Indeterminacy KI = 2 (θ_B & θ_C)

$[P_{1L}] = M_{FBA} - M_{FBC} + M_{FCD}$

$$= \frac{100 \times 2 \times 3^2}{5^2} - \frac{30 \times 3^2}{12} = 25.5 \text{kN-m}$$

$[P_{2L}] = M_{FCB} = \frac{30 \times 3^2}{12} = 22.5 \text{kN-m}$

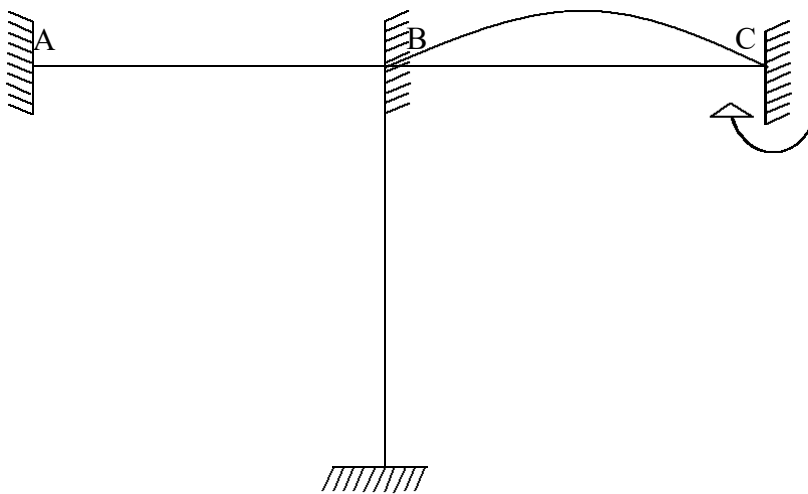$$[P_L] = \begin{Bmatrix} P_{1L} \\ P_{2L} \end{Bmatrix} = \begin{Bmatrix} 25.5 \\ 22.5 \end{Bmatrix} \text{kN-m}$$

Apply unit displacement at joint B.

$$K_{11} = \frac{4EI\theta}{l} + \frac{4EI\theta}{l} + \frac{4EI\theta}{l} \equiv \frac{4 \times 2EI}{5} + \frac{4EI}{3} + \frac{4EI}{3} \equiv 4.267EI \qquad (\theta=1)$$

$$K_{21} = \frac{2EI\theta}{l} \equiv \frac{2EI}{3} \equiv 0.67EI$$

Apply unit displacement at joint C.

$$K_{12} = \frac{2EI\theta}{l} = \frac{2EI}{3} = 0.67EI$$

$$K_{22} = \frac{4EI\theta}{l} = \frac{4EI}{3} = 1.33EI$$

By condition of equilibrium at joint B

$$[P]=0$$

$$[\Delta] = [K]^{-1}\{[P] - [P_L]\}$$

$$[\Delta] = \frac{1}{EI} \begin{bmatrix} 4.267 & 0.67 \\ 0.67 & 1.33 \end{bmatrix}^{-1} \left\{ \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} 25.5 \\ 22.5 \end{Bmatrix} \right\}$$

$$\begin{bmatrix} \theta_B \\ \theta_C \end{bmatrix} = \frac{1}{EI} \begin{bmatrix} -3.604 \\ -15.01 \end{bmatrix}$$

Slope deflection equation

$$M_{AB} = M_{FAB} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$=-72+ \frac{2 \times 2EI}{5}(2\theta_A + \frac{-3.604}{EI}) \qquad (\theta_A = 0 \text{ due to fixity at support A})$$

M_{AB} = -74.88kN-m

$$M_{BA} = M_{FBA} + \frac{2EI}{l}(2\theta_A + \theta_B)$$

$$=72+ \frac{2 \times 2EI}{5}(\theta_A + 2 \times \frac{-3.604}{EI})$$

M_{BA} = 42.23kN-m

$$M_{BC} = M_{FBC} + \frac{2EI}{l}(2\theta_B + \theta_C)$$

$$= -22.5 + \frac{2EI}{3}(2 \times \frac{-3.604}{EI} + \frac{-15.1}{EI})$$

M_{BC} = -37.37kN-m

$$M_{BD} = M_{FBD} + \frac{2EI}{l}(2\theta_B + \theta_D)$$

$$= 0 + \frac{2EI}{3}(2 \times \frac{-3.604}{EI} + 0)$$

M_{BD} = -4.81kN-m

$$M_{DB} = M_{FDB} + \frac{2EI}{l}(2\theta_D + \theta_B)$$

$$= 0 + \frac{2EI}{3}(2 \times 0 + \frac{-3.604}{EI})$$

M_{DB} = -2.402kN-m

M_{CB}=0