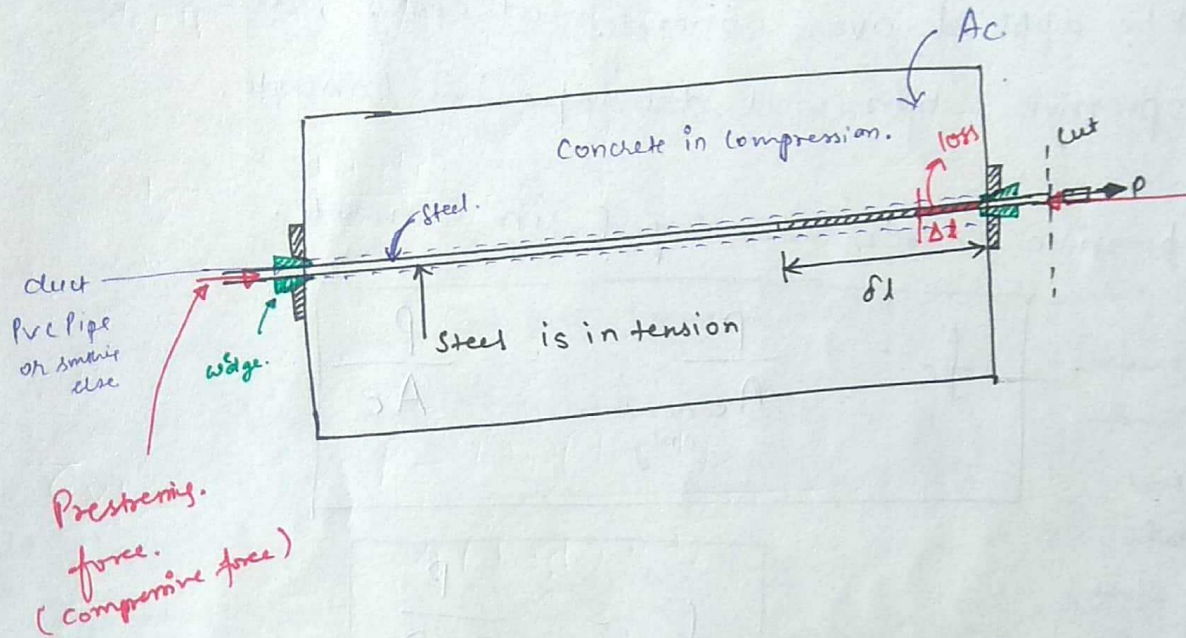


PRESTRESSED CONCRETE

⊗ Basic Concept :

Let us consider ex^{am}ple of a Post tension member.



→ If a steel reinforcement provided in a duct in tensioned at ends and fixed after providing an elongation.

→ steel Reinforcement is in tension. Elongation provided

$$= \delta l$$

→ Strain, stain etc is due to δl .

→ Strain provided in steel = $\frac{\delta l}{l} = \underline{e}$.

→ Stress developed in steel.

$$\underline{p} = e \times E_s = \left(\frac{\delta l}{l} \times E_s \right)$$

→ Total force developed in steel

$$P_s = p_s \times A_s = \frac{\sigma_s}{l} \times E_s \times A_s = P$$

This force is called prestressing force. This force will be applied over concrete at ends, due to which compressive stress will develop in concrete.

→ Compressive stress developed in concrete.

$$f_c = \frac{P}{A_{\text{concrete only}}} = \frac{P}{A_c}$$

In prestress concrete, we do not consider

area of steel.

In calculating compressive stress.

As steel is in

Tension ~~and~~ and

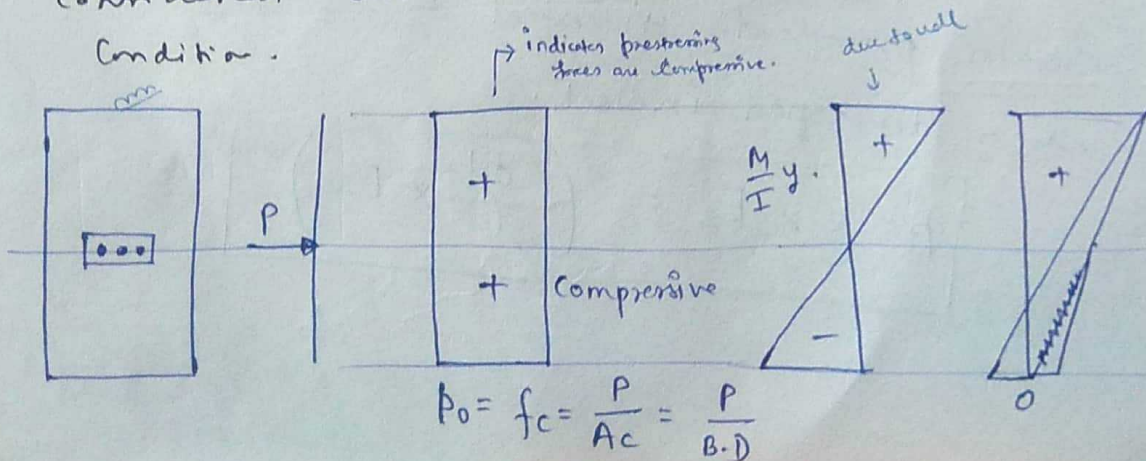
it has developed compressive force.

$$f_c = \frac{P}{B \times D}$$

Now To bear Compressive Force.

If we have to steel will go to '0' then it can bear compressive stress.

In column equivalent area of concrete and steel was considered as both were initially in 0 stress condition.



Stress developed due to Moment

$$= \pm \frac{M}{I} y.$$

Total stress at any section.

$$\text{at top} = \left(\frac{P}{A} + \frac{M}{I} y \right) \quad \text{Compressive (+ve)}$$

$$\text{at bottom} = \left(\frac{P}{A} - \frac{M}{I} y \right) \quad \begin{bmatrix} +ve \\ 0 \\ -ve \end{bmatrix}$$

Stress due to moment are counter balanced by stresses developed due to prestressing force.

(ii) If due to any reason, some part of the deformation provided is lost. say by " Δl ". This will result in loss of stress in reinfⁿ. Due to which prestressing force will reduce.

$$\text{Loss of deformation} = \Delta l$$

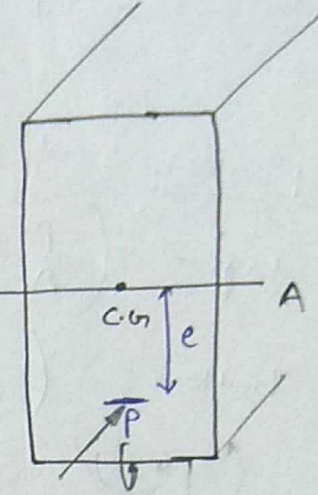
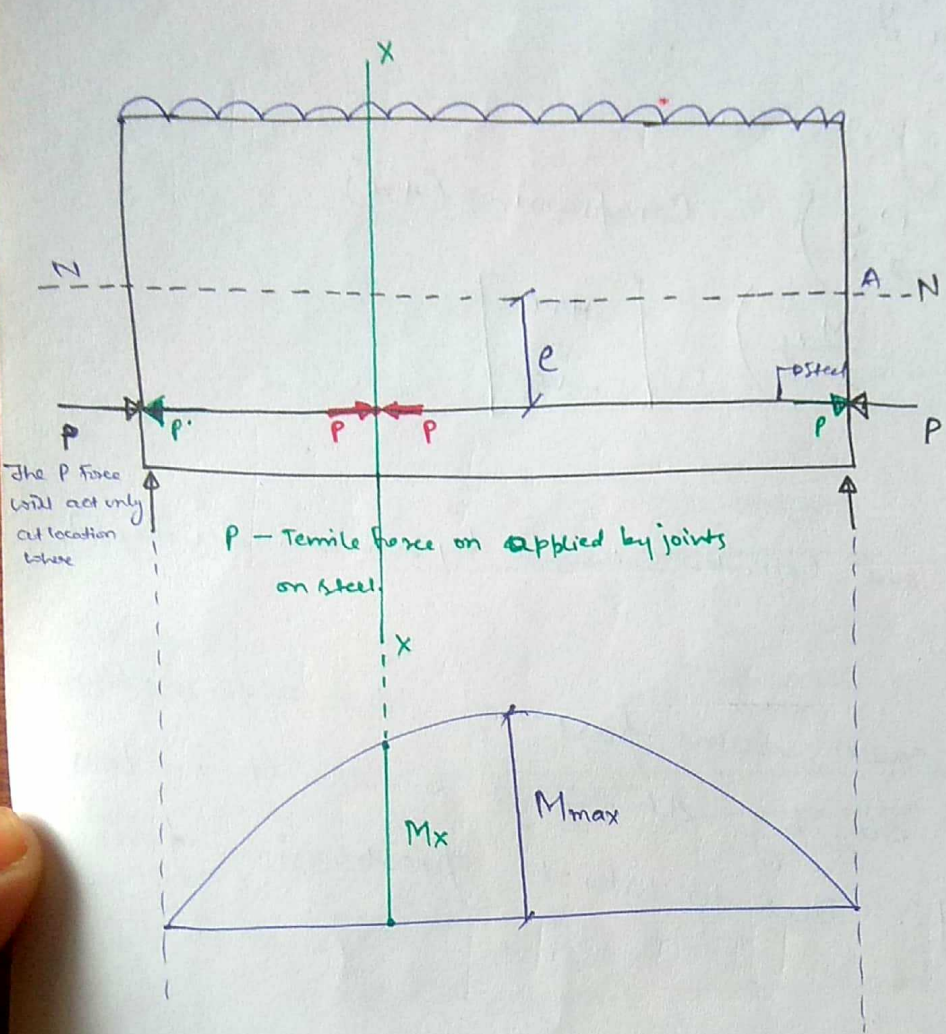
$$\text{Loss of stress} = \left(\frac{\Delta l}{l} \right) \times E_s$$

Remaining stress is reinfⁿ.

$$= \frac{(s_1 - \Delta s)}{l} \times E_s$$

Case 1

Reinfⁿ. Provided at an eccentricity.

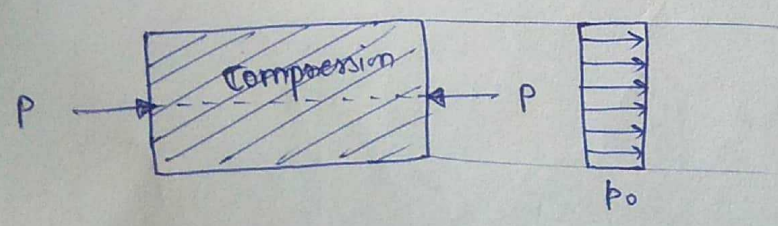


NA is C.G. of only concrete area.

Stress Concept Method.

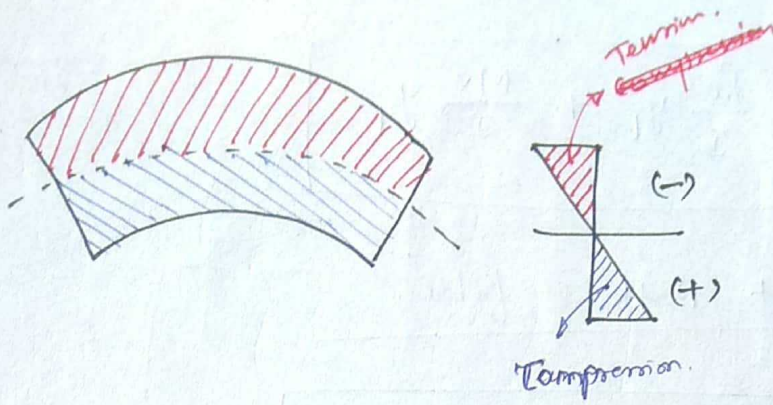
At any cross-section the beam is subjected to stresses due to three reasons.

① Direct stress due to P-force.



$$p_0 = (+) \frac{P}{A}$$

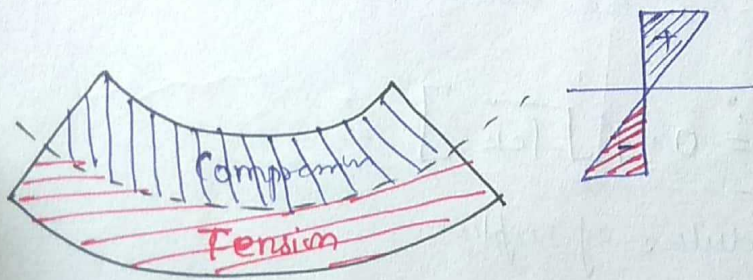
② Bending stress due to Prestressing force / eccentricity.



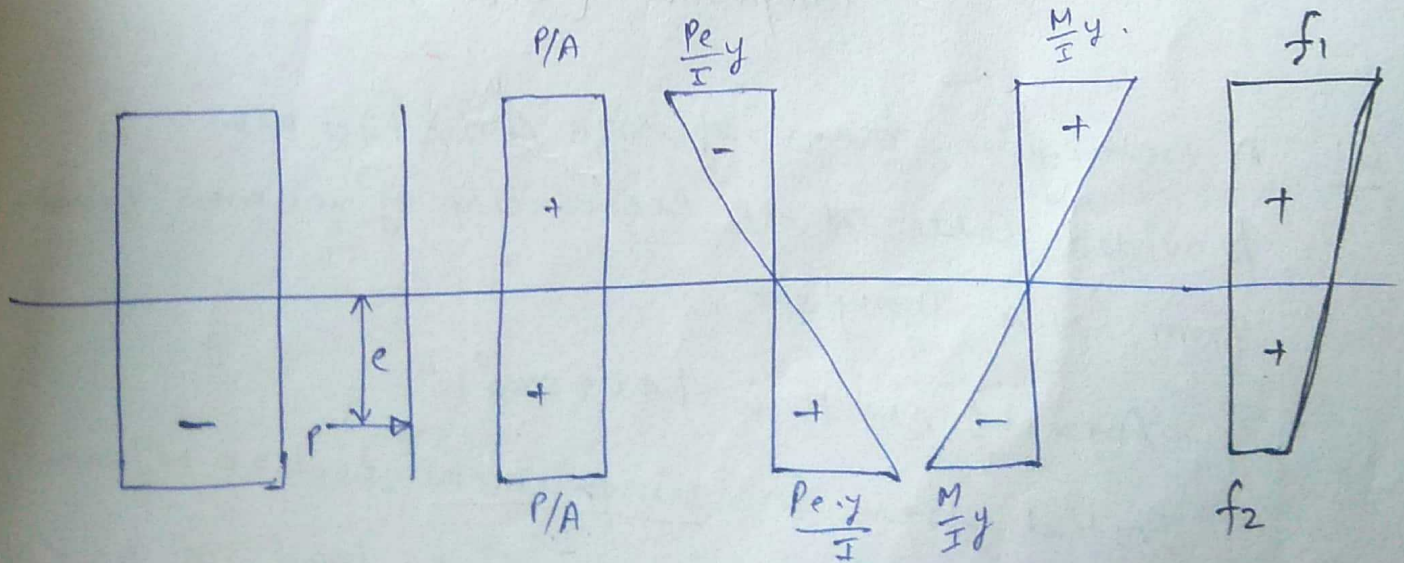
Stress

$$p_{bi} = (-) \frac{P \cdot e}{I} \cdot y$$

③ Stress due to load (Bending stresses)



Total stress



↓ is developed due to Prestressing force.
 (→ Prestressing force develops stress to stress, force due to stress, force due to stress)

Total stress at top fibre.

$$f_1 = \frac{P}{A} - \frac{Pe}{I} y_t + \frac{M_x}{I} y_t$$

y_t - dist upto top fiber and neutral axis

Total stress at Bottom fibre

$$f_2 = \frac{P}{A} + \frac{Pe}{I} y_B - \frac{M_x}{I} y_B$$

M_x = Moment at the section.

(-ve) stress may also be allowed but value.

$$< [f_{cr} = 0.7 \sqrt{f_{ck}}]$$

modulus of rupture.

(+) value $<$ permissible stress in concrete is compressive ($6\sigma_{bc}$)

Q1. A rectangular beam of size 400×720 mm is provided cable at an eccentricity of 200 mm (const.) from N.A. Throughout:

Area of steel is 1400 mm^2 .

Initial stress in reinforcement = 1500 N/mm^2 .

The beam is subjected to a live load of 60 kN/m over a simply supported effective span of 9 m .

Calculate stresses in the beam -

- (i) At ends.
- (ii) At mid span
- (iii) At 3.60 m from support

Neglect any loss of prestress in reinforcement.

$$f_1 = \frac{P}{A} - \frac{P \cdot e}{I} \cdot y_t + \frac{M \cdot y_t}{I}$$

$$I = \frac{bd^3}{12}$$

$$= \frac{900 \times 720^3}{12} = 1.24 \times 10^{10}$$

$$P = 21 \times 10^5 \text{ N}$$

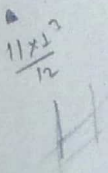
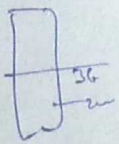
$$= \frac{21 \times 10^5}{900 \times 720} - \frac{21 \times 10^5 \times 200}{1.24 \times 10^{10}}$$

~~980~~

720

$$f_1 = -8.966$$

$$f_2 = 15.42$$

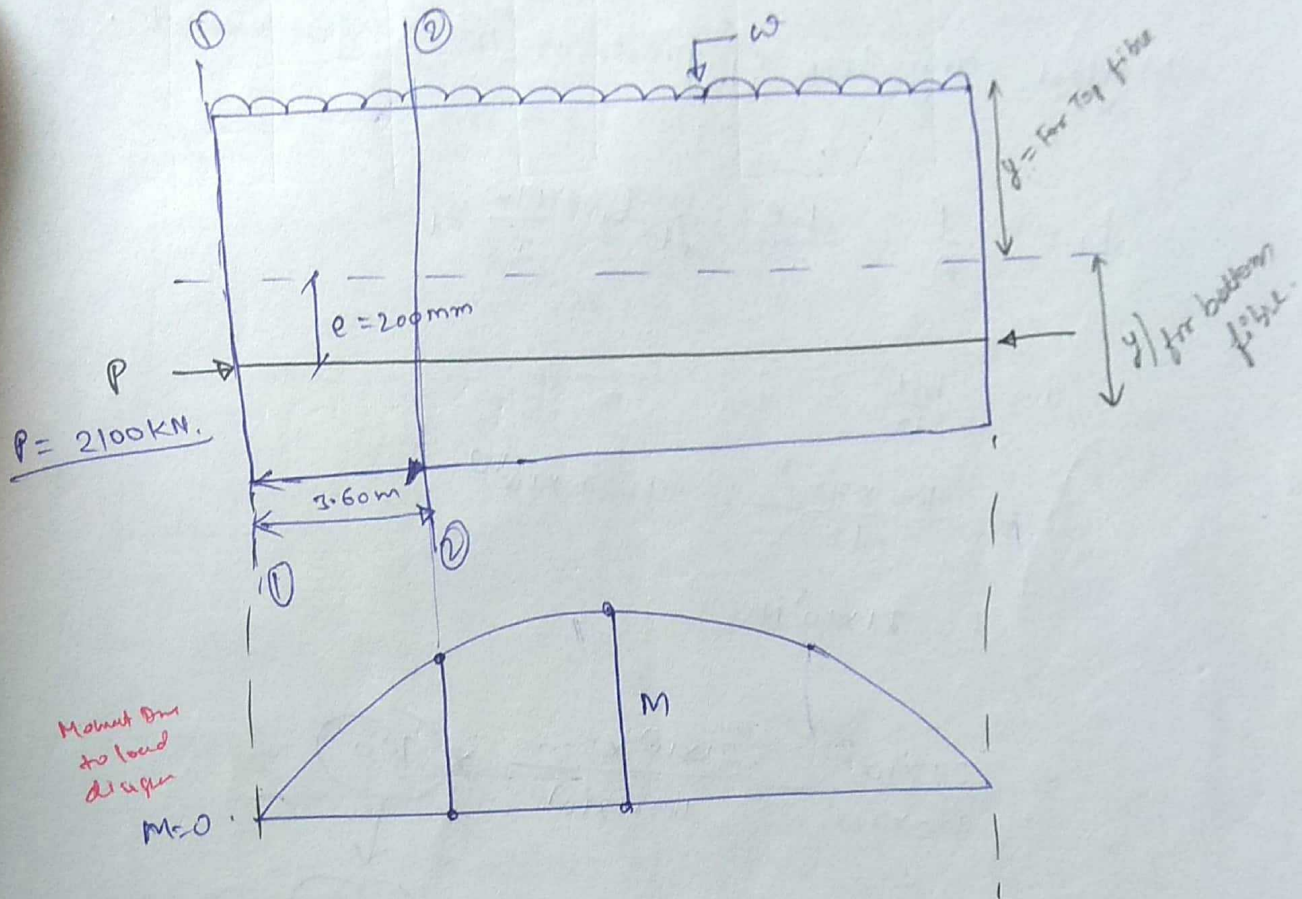


Soln
①

Prestressed forces.

$$P = A_s \cdot f_s = 1400 \times \frac{1500}{10^3} = \underline{2100 \text{ KN}}$$

$$e = 200 \text{ mm.}$$

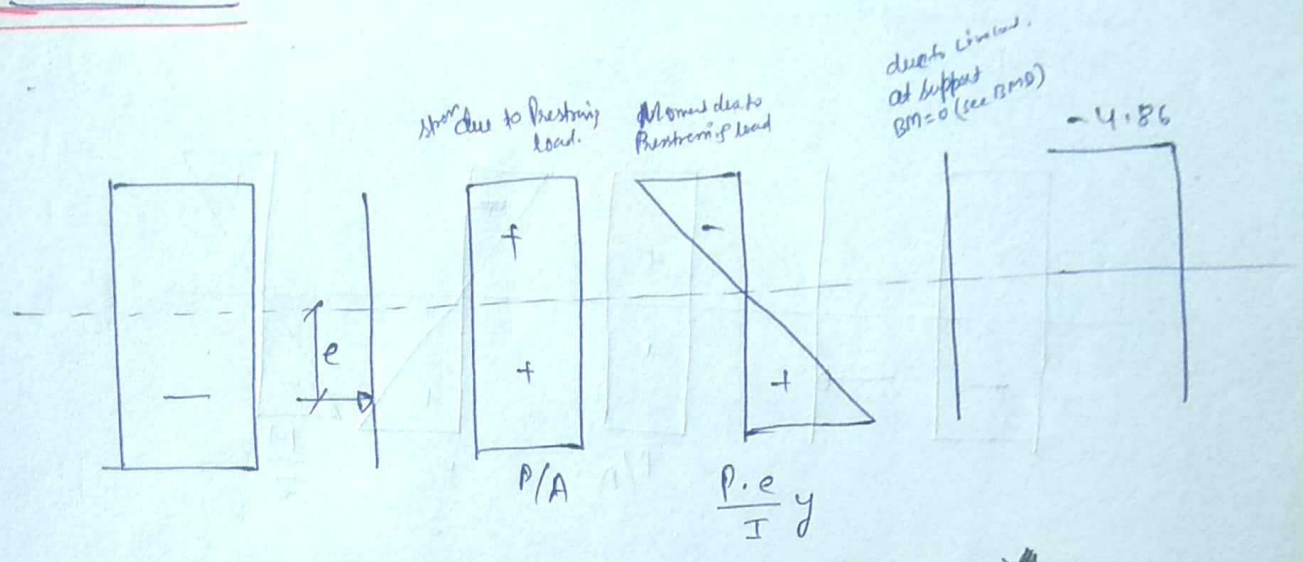


Moment due to load diagram

$$\Rightarrow A = 400 \times 720$$
$$I = 400 \times 720^3 / 12$$
$$Z = 400 \times 720^2 / 6$$

$$D.L = 0.40 \times 0.72 \times 1 \times 25 = 7.20 \text{ kN/m.}$$
$$= 60.0 \text{ kN/m}$$
$$L.L = \underline{\underline{67.20 \text{ kN/m}}}$$

① At ends.



Stress at top.

$$= \frac{P}{A} - \frac{P \cdot e}{I} y \Rightarrow \frac{2100 \times 10^3}{400 \times 720} - \frac{2100 \times 10^3 \times 200}{400 \times \frac{720^3}{12}} \times \left(\frac{720}{2}\right)$$

$$= \frac{P}{A} - \frac{P \cdot e}{Z}$$

$$= \frac{2100 \times 10^3}{400 \times 720} - \frac{2100 \times 10^3 \times 200}{400 \times 720^2 / 6}$$

$$= 7.29 - 12.15$$

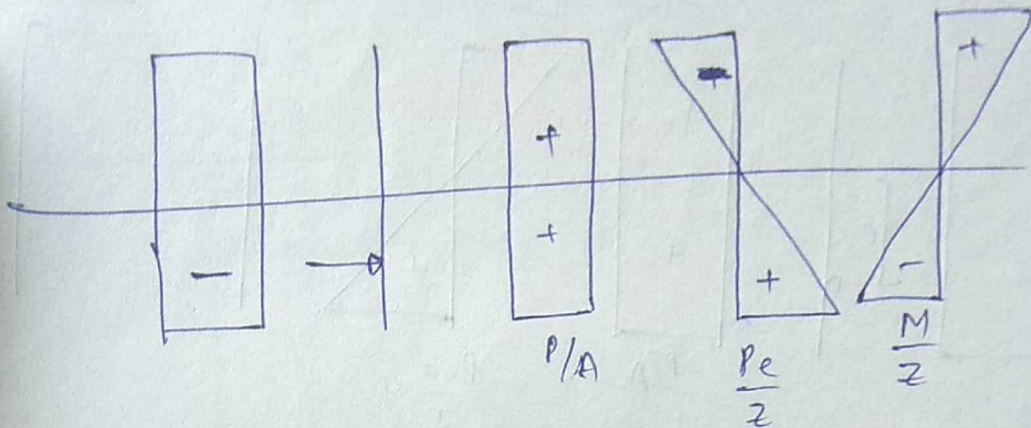
$$= -4.86$$

Stress at Bottom.

$$= \frac{P}{A} + \frac{P \cdot e}{Z}$$

$$= 7.29 + 12.15 = 19.44 \text{ N/mm}^2$$

② At mid span.



Moment at mid span.

$$= \frac{wl^2}{8} = 62.2 \times \frac{9^2}{8} = 680.40 \text{ kN-m.}$$

$$\frac{M}{z} = \frac{680.4 \times 10^6}{400 \times 220^2/6} = \pm 19.69 \text{ N/mm}^2.$$

Stress at top.

$$= \frac{P}{A} - \frac{P \cdot e}{z} + \frac{M}{z}$$

$$= 7.29 - 12.15 + 19.69 = 14.83 \text{ N/mm}^2$$

Stress at bottom.

$$= \frac{P}{A} + \frac{P \cdot e}{z} - \frac{M}{z}$$

$$= 7.29 + 12.15 - 19.69 = \frac{-0.25}{\text{N/mm}^2}.$$

③ At 3.6 m from support.

$$R_1 = R_2 = \frac{wl}{2} = 67.28 \times \frac{9}{2} = 302.4 \text{ kN}$$

$$\left\{ M_x = R_1 \cdot x - \frac{wx^2}{2} \right\} \curvearrowright$$

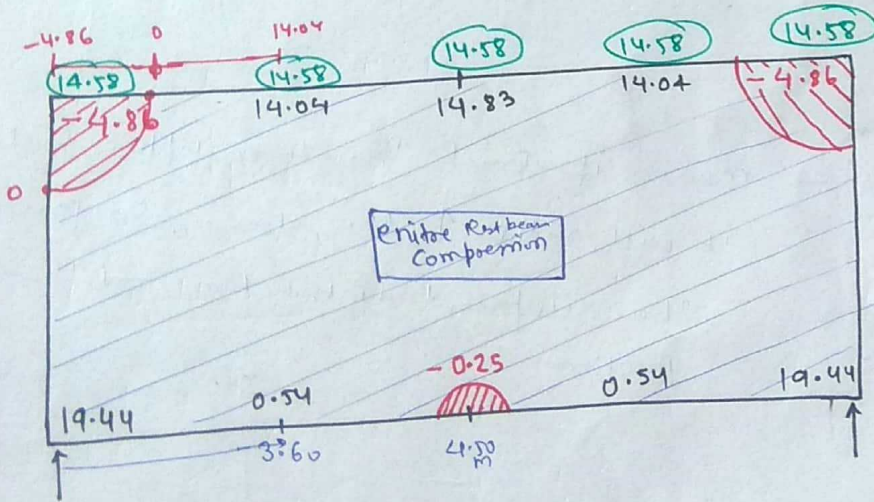
$$= 302.4 \times 3.60 - 67.2 \times \frac{3.60^2}{2}$$

$$= 653.18 \text{ kN-m}$$

$$\frac{M_x}{z} = \frac{653.18 \times 10^6}{400 \times \frac{720^2}{6}} = 18.90$$

stress at top: $= 7.29 - 12.15 + 18.90 = 14.04 \text{ N/mm}^2$

at bottom $= 7.29 + 12.15 - 18.90 = 0.54 \text{ N/mm}^2$



$$\begin{array}{r} -4.86 \\ + \\ 19.44 \\ \hline 14.58 \end{array}$$

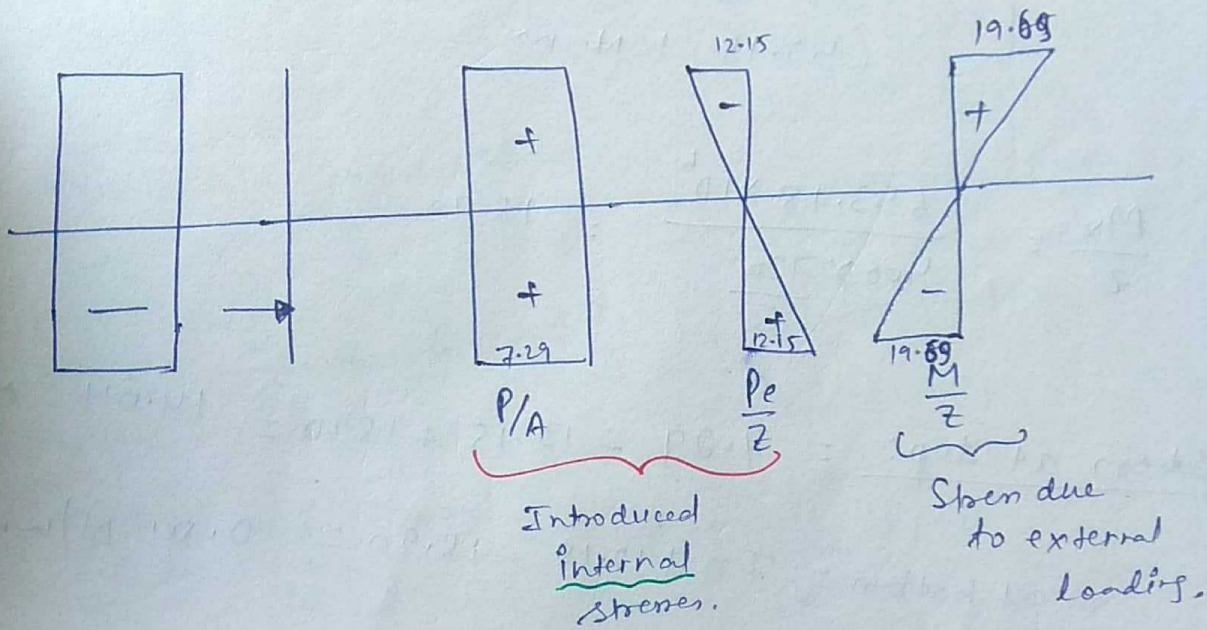
$$\begin{array}{r} 14.04 \\ + \\ 0.54 \\ \hline 14.58 \end{array}$$

$$\begin{array}{r} 14.83 \\ - \\ 0.25 \\ \hline 14.58 \end{array}$$

Prestressed Concrete.

Def:

"Prestressed Concrete is the one, in which there have been introduced internal stresses of such magnitude and distribution ^{so} that the stresses resulting from external loading can be counter balanced upto a desired degree".



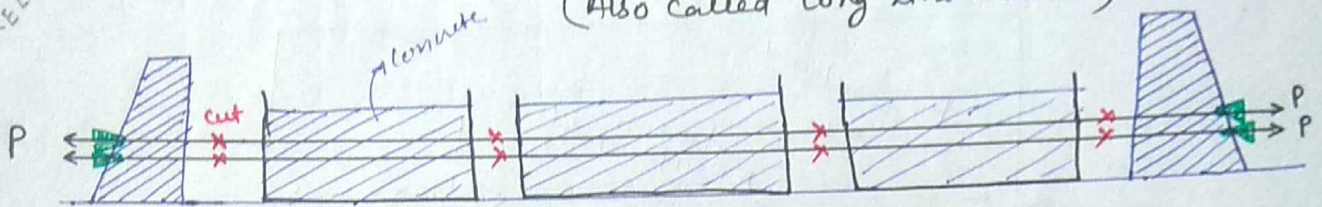
{ distribution — means if Reinf provide on upper side it will rather ↑ the stress, so provide the reinfrest distribution at lower portion.

Type of Prestressing.

① Pretensioned Prestressed Concrete.

CAPRECON SLEEPERS

used in railway sleepers
(Hoyer Method)
(Also called long line Method)



① Tensioning: :- Reinfⁿ tensioned and fixed.

② Casting: :- Concrete is casted. Reinfⁿ is kept in direct contact with concrete.

→ concrete allowed to get proper strength.

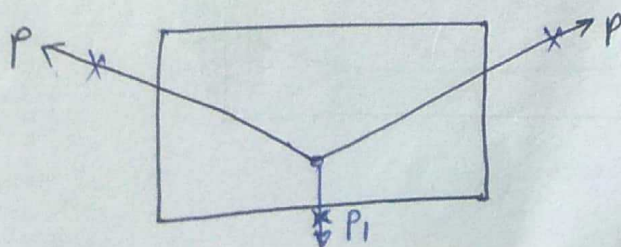
③ Transfer of prestressing force: :- By cutting the wire.

Note: In this bond is developed which provides compression in concrete.

In pretension Member

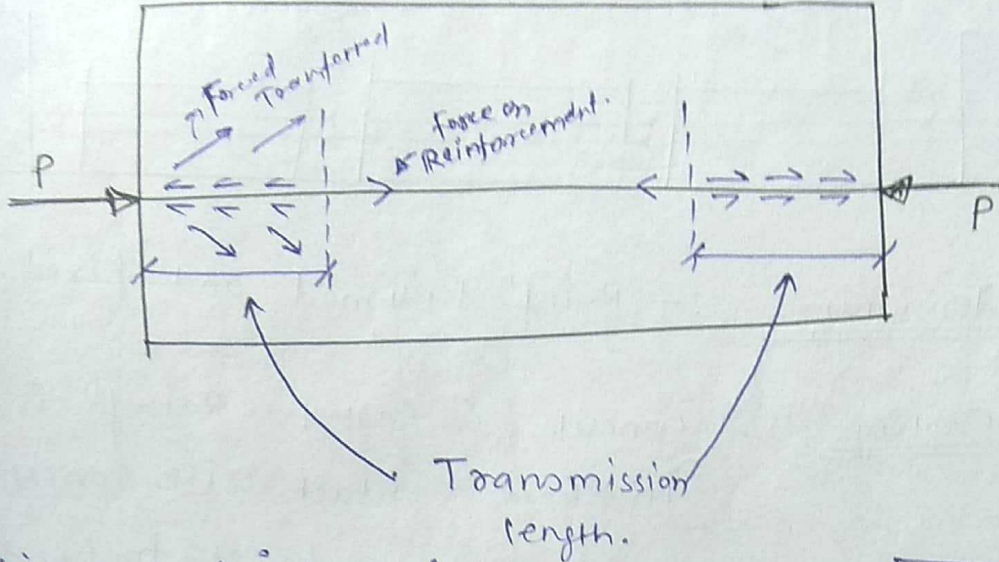
Cables may be mostly - straight

→ may be bent also in some cases.

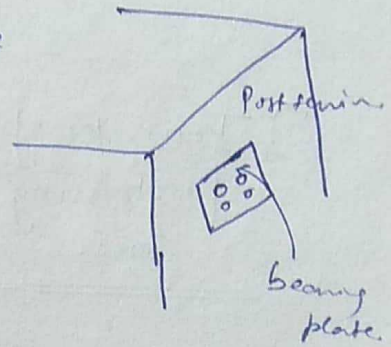


In Case of pretensioning.

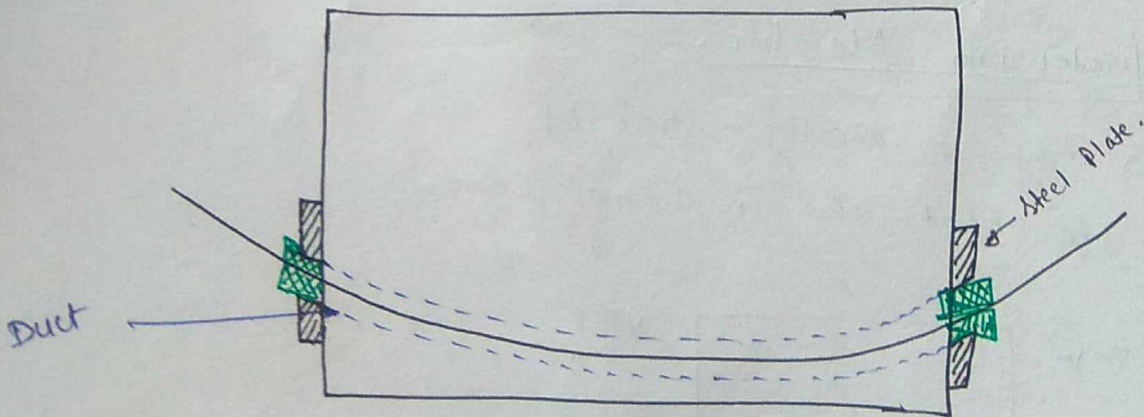
→ Transmission length is required at ends to transfer the prestressing force in concrete by bond action.



* This method is used for repetitive type member for factory production



② Post Tensioned Members.



① Casting: Beam of required shape casted by providing ducts at suitable location. Any shape is possible for cable.

② Tensioning: After getting sufficient strength in concrete, reinforcement is provided in duct and tensioned at ends.

There are two methods:-

Loss will be diff. in these two cases.

- ① Tension the Reinforcement from one end.
- ② Tensioning from both ends.

③ Anchoring: After tensioning Reinforcement are anchored at ends.

There are many methods.

See fig. in book.

- ① Fresinet Method.
- ② Magnel Blatton Method.
- ③ Le-mcall method.
- ④ Grifford Udall method.

This method

→ used for Cast in Suit Work.

→ large size members can be casted at site.

→ stress transfer over concrete is by bearing action.

Force is transferred by a steel plate fixed over concrete at ends.

In Prestressed Concrete

Very high strength steel and concrete are used :-
High tensile strength is generally achieved by slightly ↑ the % of Carbon (constant) in steel as compared to mild steel. generally - 0.6 - 0.85% Carbon.

Steel → strength = $\frac{1000 \text{ to } 2000 \text{ N/mm}^2}{\text{i.e. Higher grade steel used.}}$

Concrete → Prestensioned = min^m grade = M40 (min^m)
Post tensioned — M30 (min^m)

Generally concrete M50 to M100.

Why higher grade used :

Reason: ① Steel In steel loss of stress is very high

Total loss of stress due to all reason } = $\frac{200 \text{ to } 300 \text{ N/mm}^2}{\text{i.e. why we use higher grade.}}$

if use 250 etc. then all will be lost our purpose will not will served

Total loss of stress $\nless 10 \text{ to } 20\% \text{ of } p_0$

$200 \nless 10\% \text{ of } p_0$
 $p_0 \nless \frac{200}{0.10} = 2000 \text{ N/mm}^2$

$200 \nless 20\% \text{ of } p_0$

$p_0 \nless \frac{200}{0.2} = 1000 \text{ N/mm}^2$

② Concrete: Due to high stress value in steel, stresses in concrete due to bond stress or bearing stress will also be very high.

Due to this reason concrete is also kept of very high strength.

Analysis of prestressed concrete.

Assumptions.

① Hook's law is valid:

$$p_s = \frac{\Delta l}{l} \times E_s$$

for steel and concrete both hook's law is applicable.

② At any section, plane section before bending remains plane after bending

meaning \rightarrow Strain diagram is linear.

③ In steel any change in stresses due to prestressing force and external loading is to be neglected

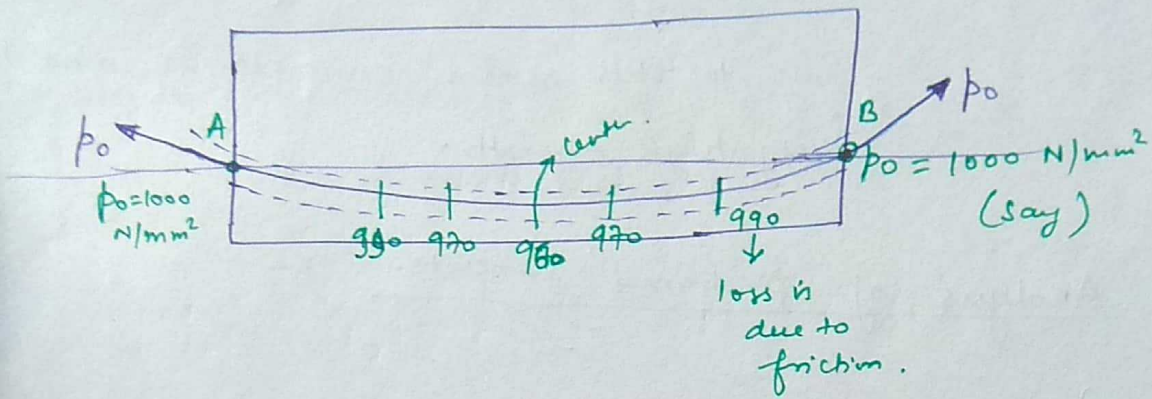
At initial stage = p_0 .

At final stage (considering all losses)

$$= k \cdot p_0$$

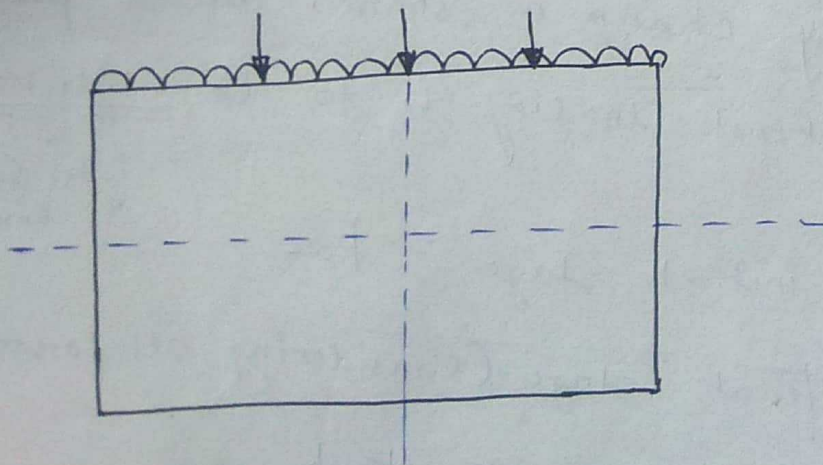
\downarrow
{ see Q6. of copy.
Result is very small
step ⑤ }

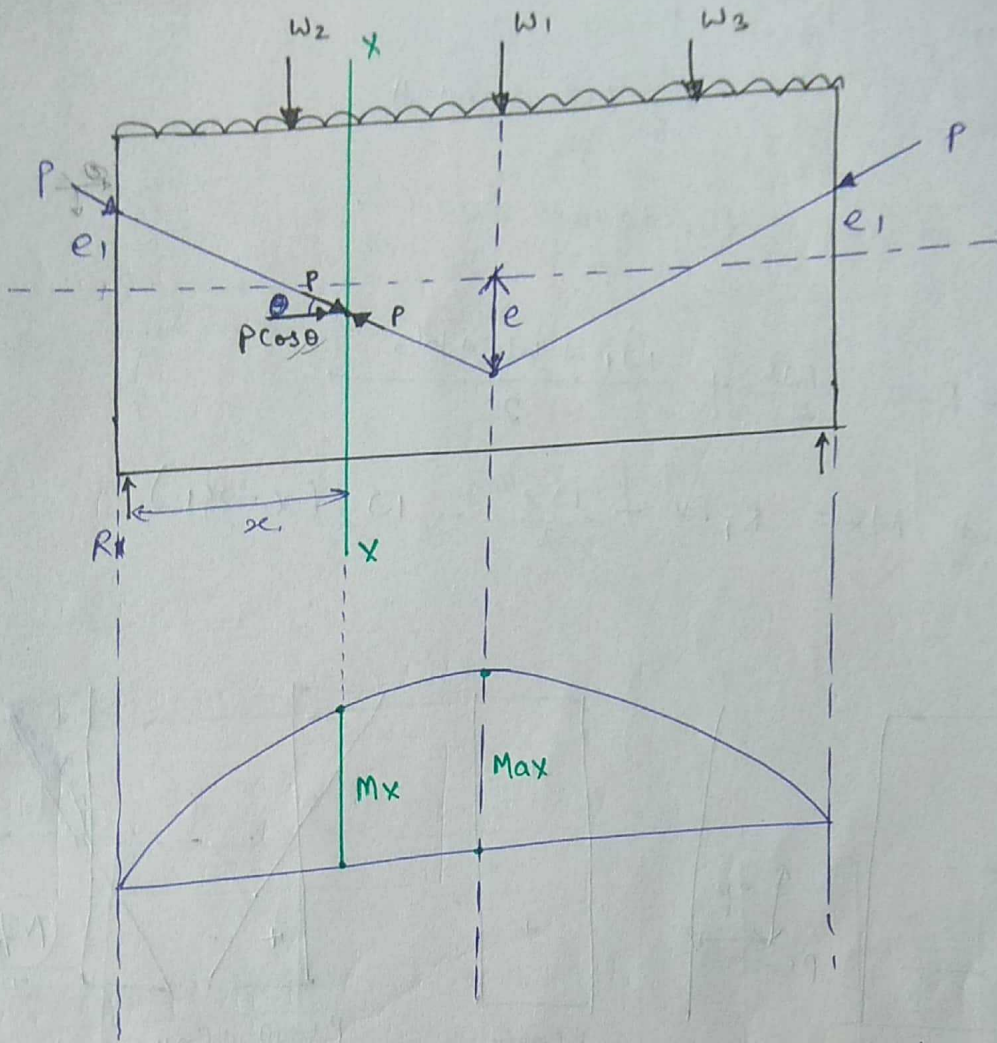
④ Any change in stress in tendⁿ along its length is also to be neglected.



Note: In case of if fix, tendⁿ at 'A' and pulled or force P_0 applied only at B then more losses occur, coz $P_0 (=1000 \text{ say})$ then at center it will be 960, but further forward A ~~is~~ ~~the~~ losses will increase and like 960, 950, 940, 930 ... and finally at A we get more losses.

Case 2 Pre-stressed Conc. Beam with bent Cable.





Stresses in beam at any section can be found by:

- (i) Stren concept Method.
 - (ii) Load balancing Concept.
 - (iii) Strength Concept. (C-Line)
- (i) Stren Concept Method.

→ NO change

→ Stresses are found due to actual prestressing force acting at the location of cable and due to actual moment at the section (which is without any load balancing), due to all loads.

means the location where you cut section and the location where you get cable line

★ ① At - x-x. {i.e. at a particular section}

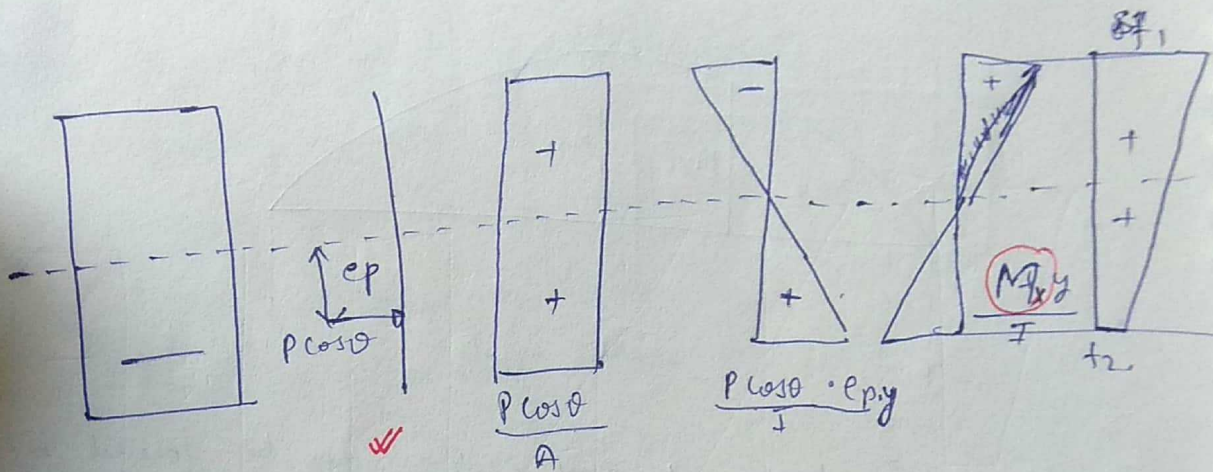
① Prestressing force = $P \cos \theta$.

② Location = eq below N.A

③ Moment at x-x due to all loads.

$$R_1 = R_2 = \frac{Wd}{2} + \frac{W_1 + W_2 + W_3}{2}$$

$$\text{Moment } M_x = R_1 \cdot x - \frac{Wx^2}{2} - W_2(x - x_1)$$



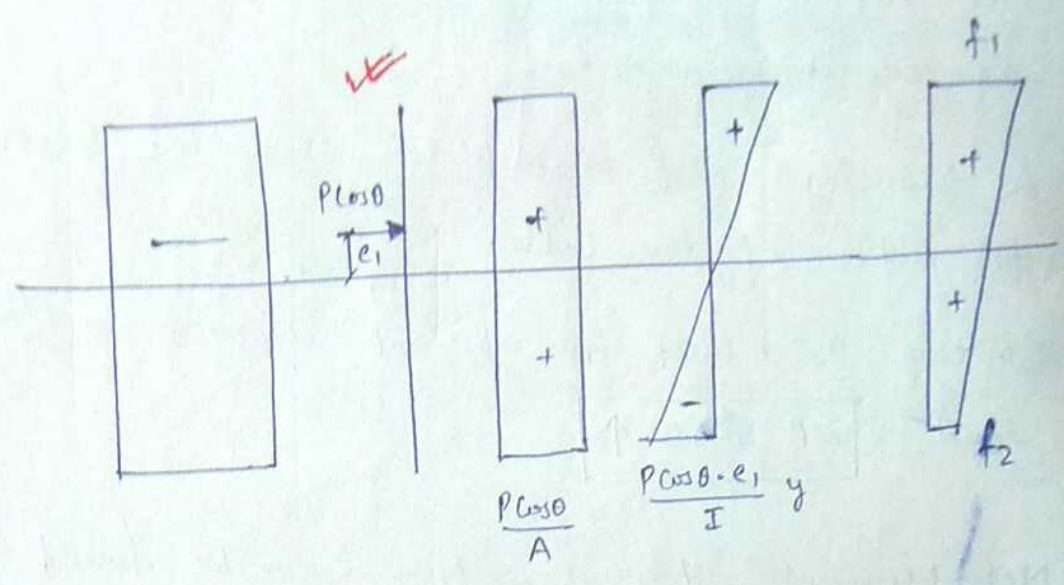
Stress at top.

$$= \frac{P \cos \theta}{A} - \frac{P \cos \theta \cdot e \cdot y}{I} + \frac{M_x \cdot y}{I} = f_1$$

Stress at Bottom.

$$= \frac{P \cos \theta}{A} + \frac{P \cos \theta \cdot e \cdot y}{I} - \frac{M_x \cdot y}{I} = f_2$$

★ (2) At ends.



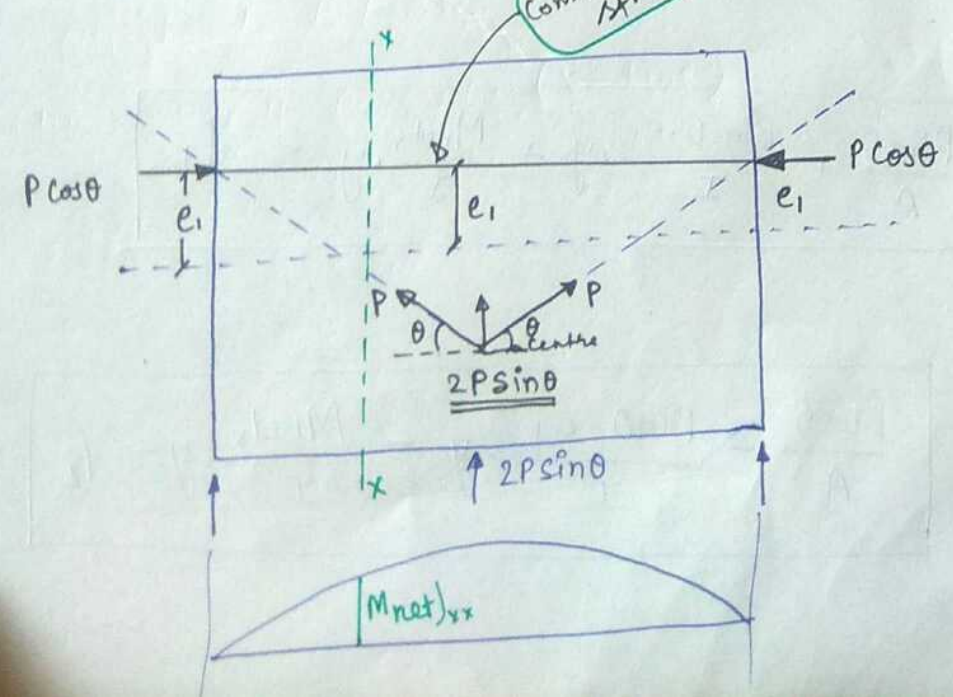
Stress at top.

$$= \frac{P \cos \theta}{A} + \frac{P \cos \theta \cdot e_1}{I} \cdot y$$

Stress at Bottom

$$= \frac{P \cos \theta}{A} - \frac{P \cos \theta \cdot e_1}{I} \cdot y$$

(2) Load balancing Concept ***



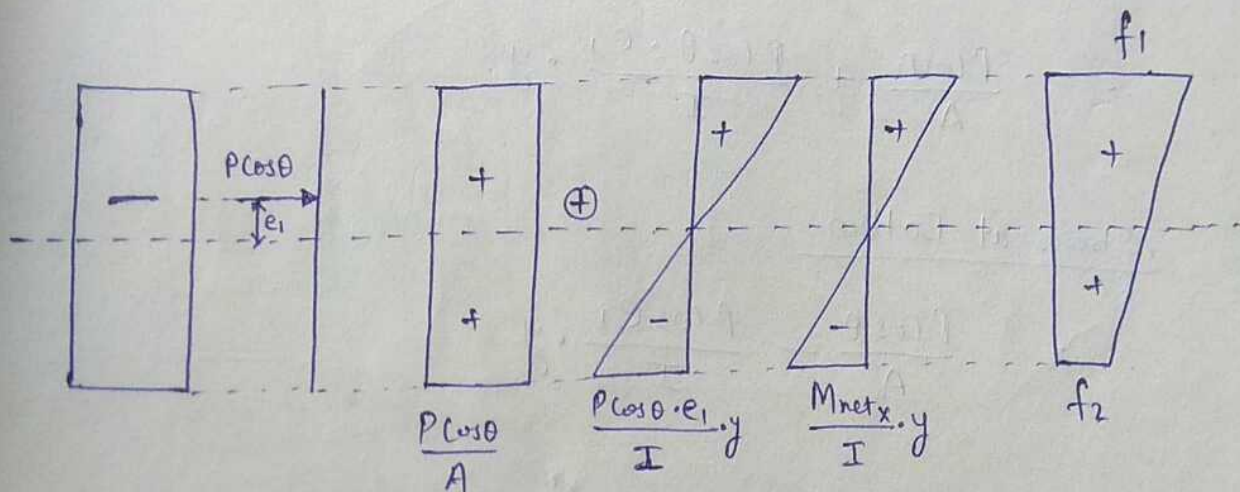
Load Balancing Concept.

(i) Cable profile is considered straight from the points where ever it is at ends.

(ii) A balancing load is applied over the beam as per profile of the cable.

In this point load in upward direction as balancing load = $\boxed{2P \sin \theta \uparrow}$

(iii) Net Moment at any section can be found due to all loads i.e balancing load.



Stresses:

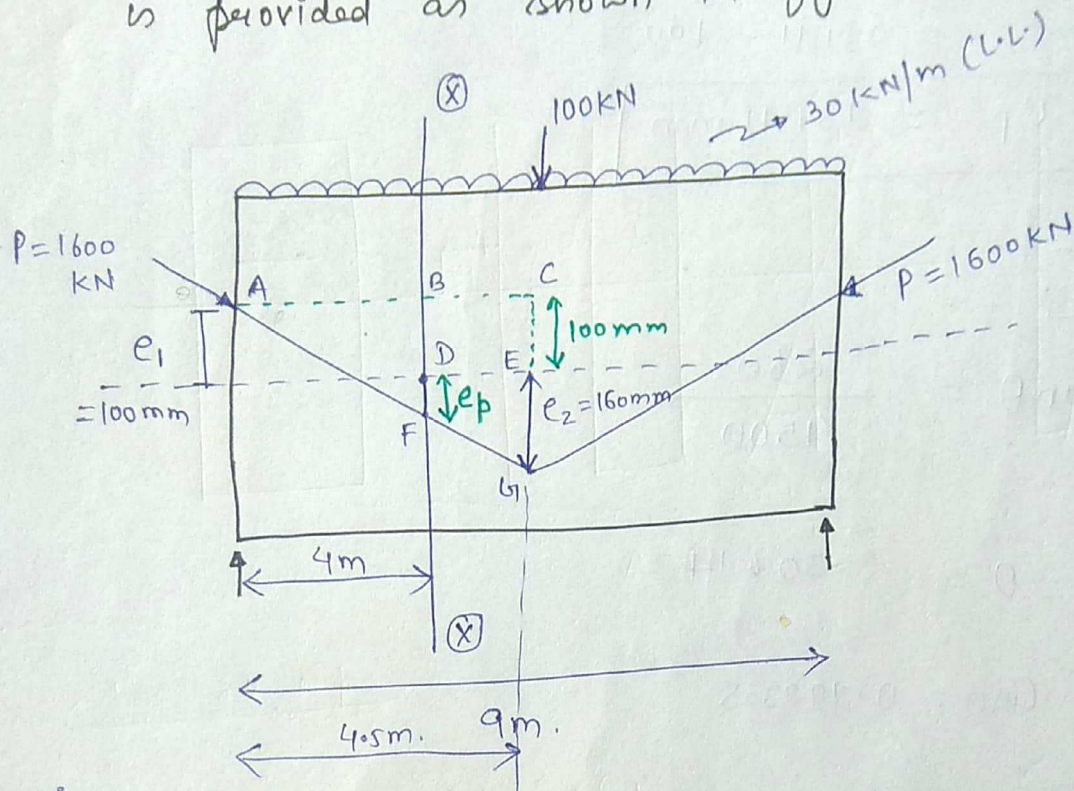
at top:

$$\boxed{\frac{P \cos \theta}{A} + \frac{P \cos \theta \cdot e_1}{I} \cdot y + \frac{M_{net_x}}{I} \cdot y = f_1}$$

at Bottom

$$\boxed{\frac{P \cos \theta}{A} - \frac{P \cos \theta \cdot e_1}{I} \cdot y - \frac{M_{net_x}}{I} \cdot y = f_2}$$

Q.2 A Rectangular beam of size 350 mm X 650 mm is provided as shown in figure.



Solution:

Calculation of load:

$$D.L = 0.35 \times 0.65 \times 1m \times 25 = 5.69 \text{ KN/m.}$$

$$L.L = 30 \text{ KN/m.}$$

$$\text{Total: } 35.69 \text{ KN/m.}$$

$$\text{Reaction: } \frac{wL}{2} = 160.59 \text{ KN} + 50 = 210.59 \text{ KN.}$$

→ due to Point load.

$$M)_{xx} = 210.59 \times \frac{4}{2} = 421.18 \text{ KN-m.}$$

$$- 35.69 \times \frac{4^2}{2} = -285.52 \text{ KN-m.}$$

$$\hline 135.66 \text{ KN-m.}$$

★ From Shear Concept Method.
from similar Δ

$$\frac{BF}{AB} = \frac{CG}{AC}$$

$$BF = 4m \times \frac{260}{4.50}$$

$$= 231.11$$

$$\therefore FD = BF - BD$$

$$= 231.11 - 100$$

$$e_p = 131.11 \text{ mm}$$

(b) P cos θ

$$\tan \theta = \frac{260}{4500}$$

$$\theta = 3.30674^\circ$$

$$\cos \theta = 0.998335$$

$$\therefore P \cos \theta = 1600 \times 0.998335$$

$$= 1597.34 \text{ kN}$$

(c) Moment

$$DL = 0.35 \times 0.65 \times 1.0 \times 25 = 5.69 \text{ kN/m}$$

$$LL = \frac{30}{35.69} \text{ kNm}$$

$$R_1 = R_2 = \frac{WL}{2} + \frac{W}{2} = \frac{35.69 \times 9}{2} + \frac{100}{2}$$

$$= 210.605 \text{ kN}$$

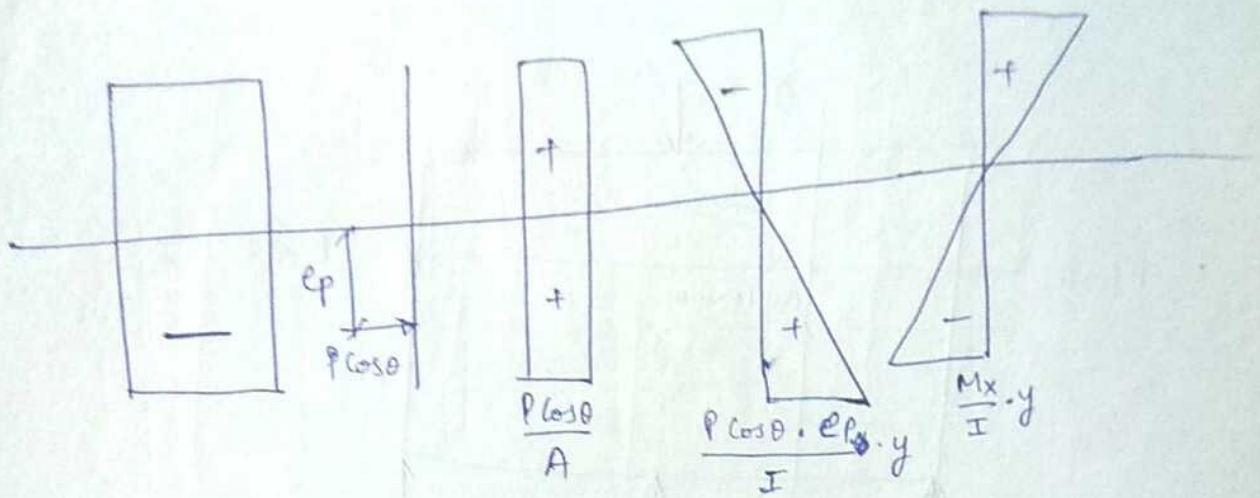
Made

$$M_x = R_1 \cdot x - \frac{Wx^2}{2}$$

$$= 210.605 \cdot 4 - \frac{35.69 \times 4^2}{2}$$

$$= \underline{\underline{556.9 \text{ kN-m}}}$$

Stress Concept Mtd.



$$\frac{P \cos \theta}{A} = \frac{1597.34 \times 10^3}{350 \times 650} = 7.02 \text{ N/mm}^2$$

$$\frac{P \cos \theta \cdot e_p}{Z} = \frac{1597.34 \times 10^3 \times 131.11}{350 \times 650^2 / 6} = 8.50 \text{ N/mm}^2$$

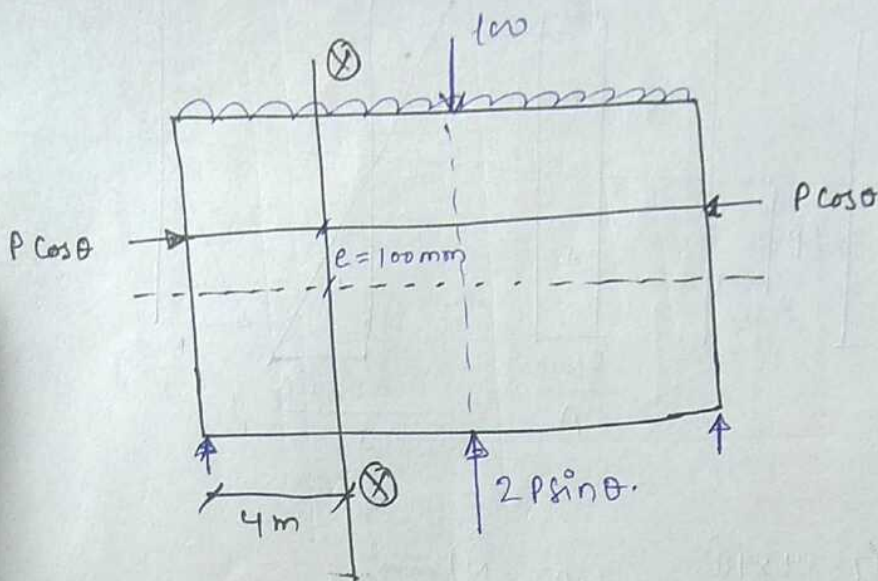
$$\frac{M_x}{Z} = \frac{556.9 \times 10^6}{350 \times 650^2 / 6} = 22.60 \text{ N/mm}^2$$

$$\begin{aligned} \therefore \text{Stress at top} &= 7.02 - 8.50 + 22.60 \\ &= 21.12 \text{ N/mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Stress at Bottom} &= 7.02 + 8.50 - 22.60 \\ &= -7.08 \text{ N/mm}^2 \end{aligned}$$

Method II.

Now from load balancing method.



(a) $e = 100 \text{ mm}$ above N.A.

(b) $P \cos \theta$

From similar Δ in initial fig. $\tan \theta \rightarrow \theta$

then $\cos \theta \rightarrow$ $P \cos \theta = 1597.34 \text{ kN}$

(c) Upward balancing Load.

$$= 2P \sin \theta$$

$$= 2 \times 1600 \times \sin 3.30675$$

$$= 184.581 \text{ kN.}$$

$\tan \theta = \frac{e}{h}$

$\theta = 3.30675$

$(c) 2 \times 1600 \times \frac{260}{\sqrt{100^2 + 260^2}}$

184.581

(d) Moment

$$w = 35.69 \text{ kN/m (calculated earlier)}$$

$$W = 100 \text{ kN} \downarrow$$

$$2P \sin \theta = 184.581 \text{ kN} \uparrow$$

$$R_1 = R_2 = \frac{wL}{2} + \frac{W}{2} - \frac{2P \sin \theta}{2}$$

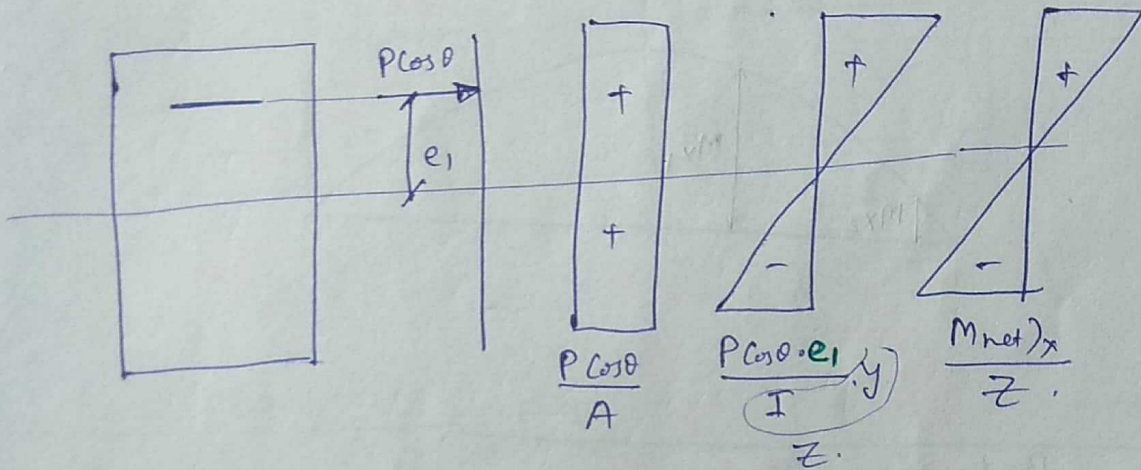
$$= \frac{35.69 \times 9}{2} + \frac{100}{2} - \frac{184.581}{2}$$

$$= 118.31 \text{ KN.}$$

$$M_{x \text{ net}} = R_1 \cdot x - \frac{w x^2}{2}$$

$$= 118.31 \times 4 - \frac{35.69 \times 4^2}{2}$$

$$= 187.738 \text{ KN-m.}$$



$$\frac{P \cos \theta}{A} = \frac{1597.34 \times 10^3}{350 \times 650} = +7.02 \text{ N/mm}^2$$

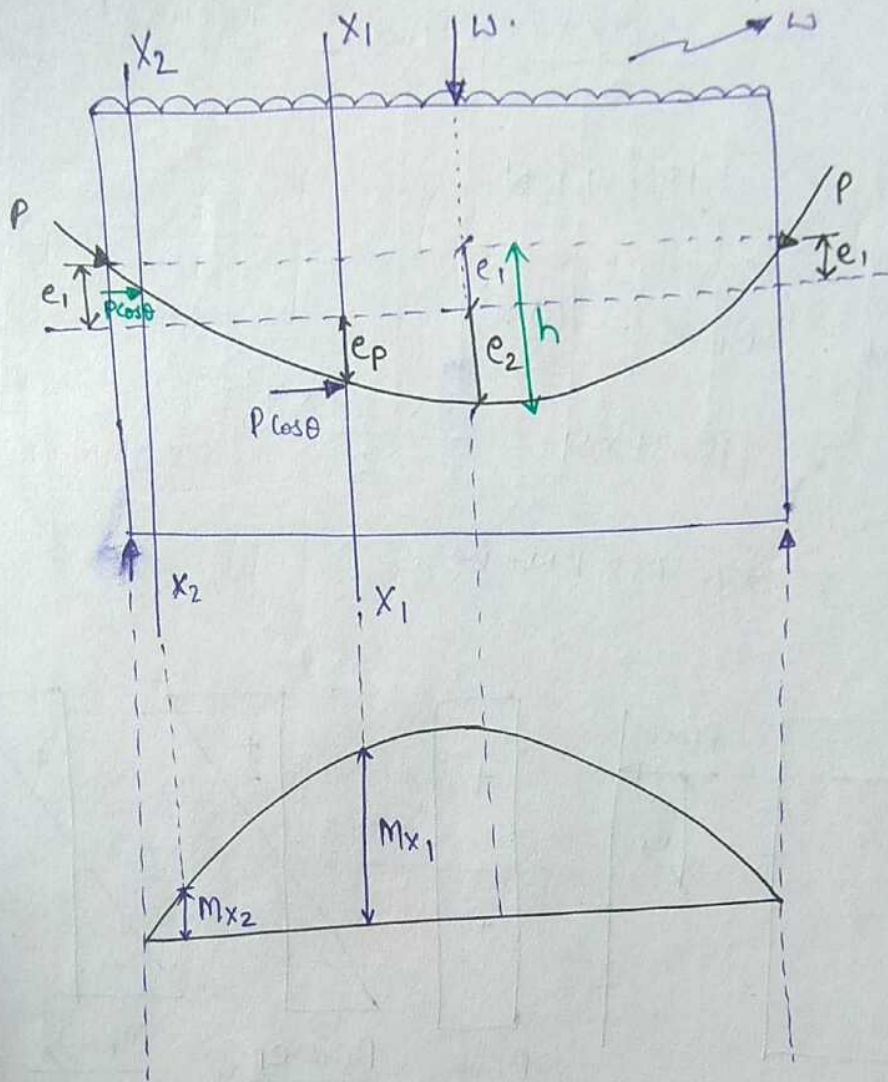
$$\frac{P \cos \theta \cdot e_1}{Z} = \frac{1597.34 \times 10^3 \times 100}{350 \times 650^2 / 6} = \pm 6.48 \text{ N/mm}^2$$

$$\frac{M_{x \text{ net}}}{Z} = \frac{187.738 \times 10^6}{350 \times 650^2 / 6} = \pm 7.62 \text{ N/mm}^2$$

$$\text{Stress at top} = 7.02 + 6.48 + 7.62 = \underline{\underline{21.12 \text{ N/mm}^2}}$$

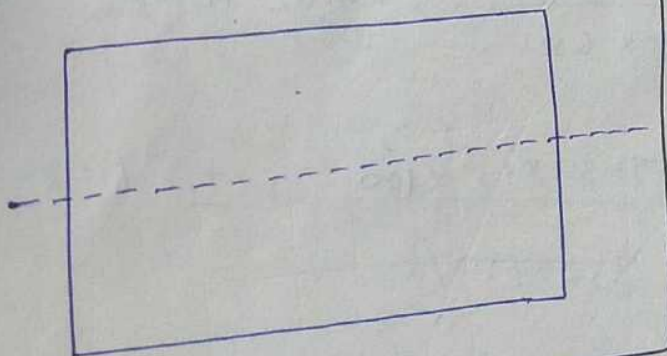
$$\text{Stress at bottom} = 7.02 - 6.48 - 7.62 = \underline{\underline{-7.08 \text{ N/mm}^2}}$$

Case 3 Beam with parabolic profile of cable.



Load Balancing.

left



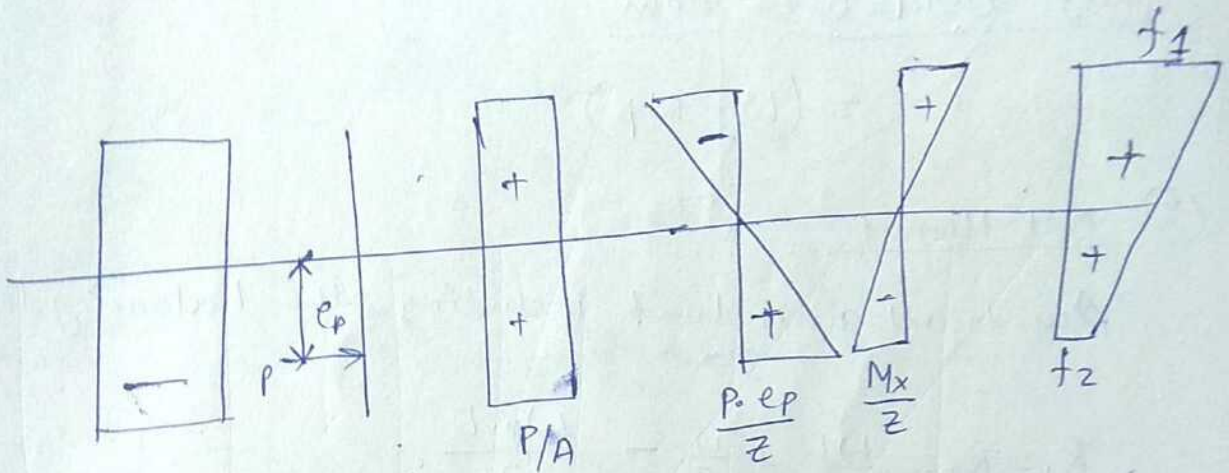
① Stress Concept. at any section

① Consider P force = $P \cos \theta \approx P$

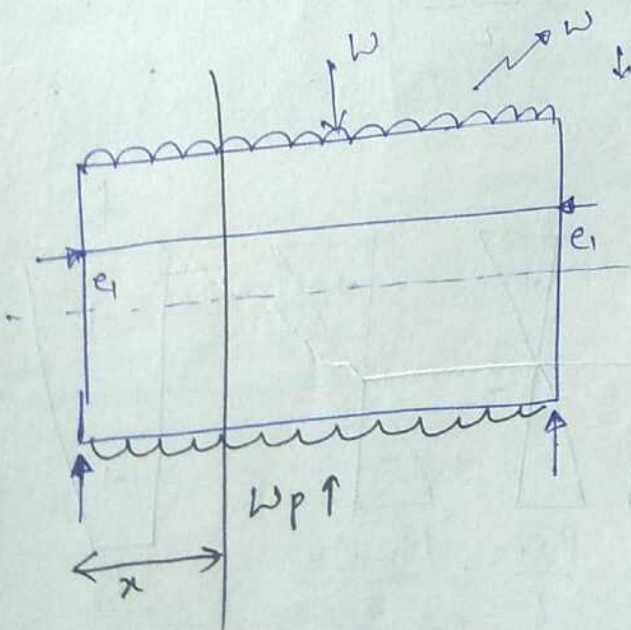
② Eccentricity of P -force = e_p

③ Moment at section = M_x

Say at $x_1 - x_1$



② Load balancing.



① Cable profile made straight

② Balancing load is a U.d.L

$$w_p = \frac{8P \cdot h}{l^2}$$

here
 $h = e_1 + e_2$

h is like central height.

Net U.d.L over beam.

$$= (W - w_p)$$

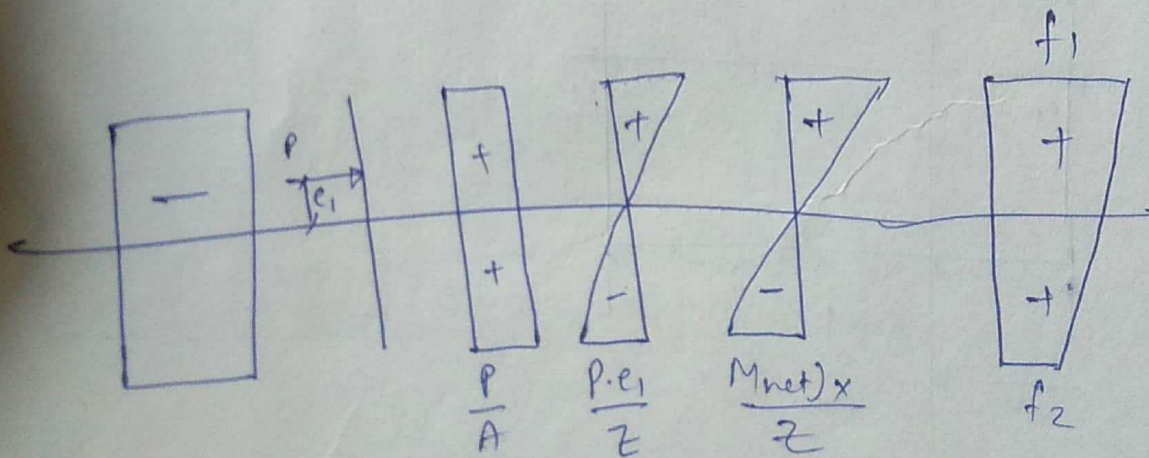
③ Net Moment

due to all given load including the balancing load.

$$R_1 = R_2 = \frac{WL}{2} + \frac{W}{2} - \frac{w_p \cdot L}{2}$$

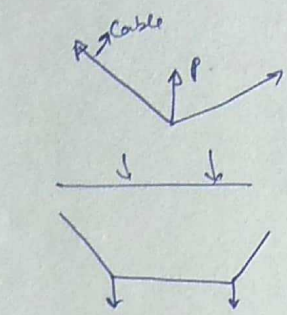
Moment

$$M_x = R_1 \cdot x - \frac{Wx^2}{2} + \frac{w_p \cdot x^2}{2}$$

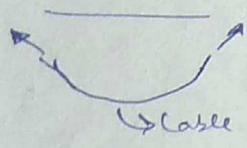


$$\text{Stresses: } \frac{P}{A} \pm \frac{P \cdot e_1}{Z} \pm \frac{M_{net} \cdot x}{Z}$$

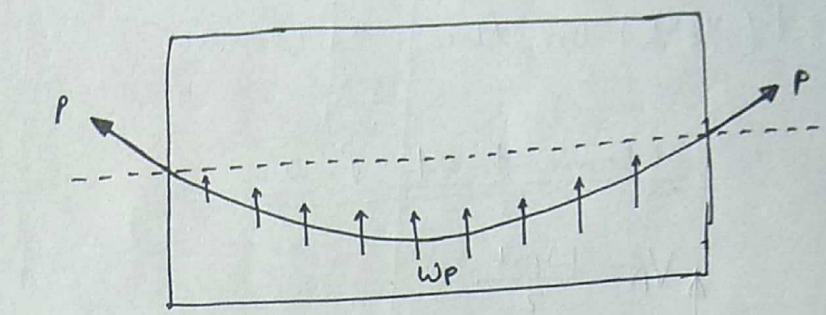
Properties of Parabolic profile of Cable.



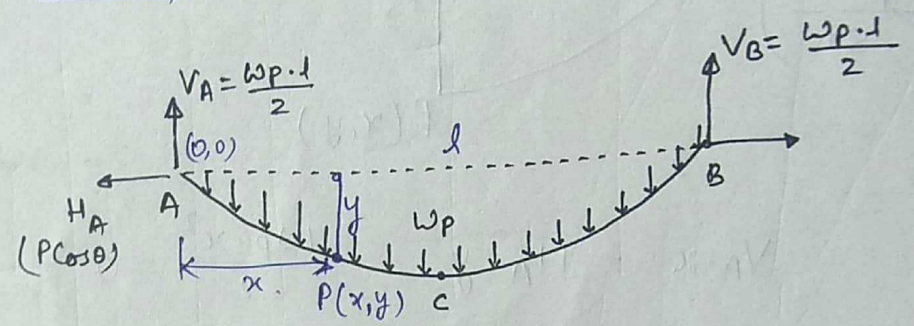
if udl



Point load at each point called udl



~~Redundant~~



Property of Cable

Important property of a Cable:

It cannot take any Moment.

* Moment at each and every point of Cable = 0

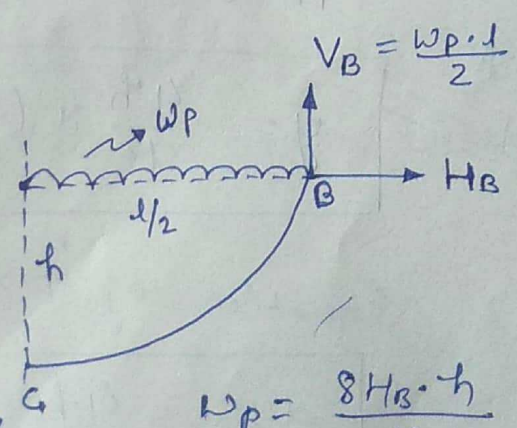
① Moment at C = 0

$$V_B \cdot \frac{l}{2} - H_B \cdot h - w_p \cdot \frac{l}{2} \cdot \frac{l}{4} = 0$$

$$\frac{w_p \cdot l}{2} \cdot \frac{l}{2} - H_B \cdot h = w_p \cdot \frac{l^2}{8} = 0$$

$$H_B \cdot h = \frac{w_p \cdot l^2}{4} - \frac{w_p \cdot l^2}{8}$$

$$H_B \cdot h = \frac{w_p \cdot l^2}{8}$$



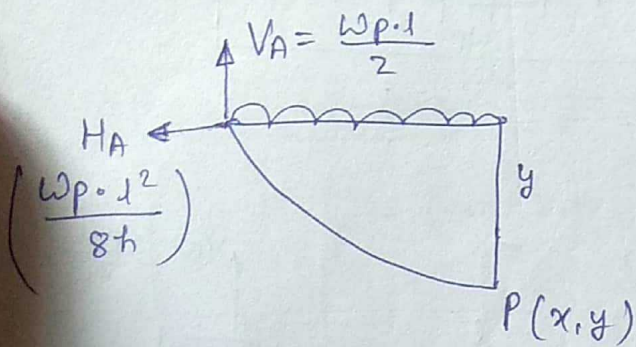
$$w_p = \frac{8 H_B \cdot h}{l^2}$$

$$= \frac{8 \cdot P \cos \theta \cdot h}{l^2} = \frac{8 P h}{l^2}$$

① Equation of parabolic Cable.

Consider A Point as origin Consider any point $P(x, y)$ on the cable.

Moment at $P=0$.



$$V_A \cdot x - H_A \cdot y - \frac{w_p \cdot x^2}{2} = 0$$

$$\frac{w_p \cdot l}{2} \cdot x - \frac{w_p \cdot l^2}{8h} \cdot y - \frac{w_p \cdot x^2}{2} = 0$$

$$l \cdot x - \frac{l^2}{4h} \cdot y - x^2 = 0$$

$$\frac{l^2}{4h} \cdot y = x(l-x)$$

$$y = \frac{4h}{l^2} x(l-x) \rightarrow \textcircled{A}$$

Equation of parabolic cable.

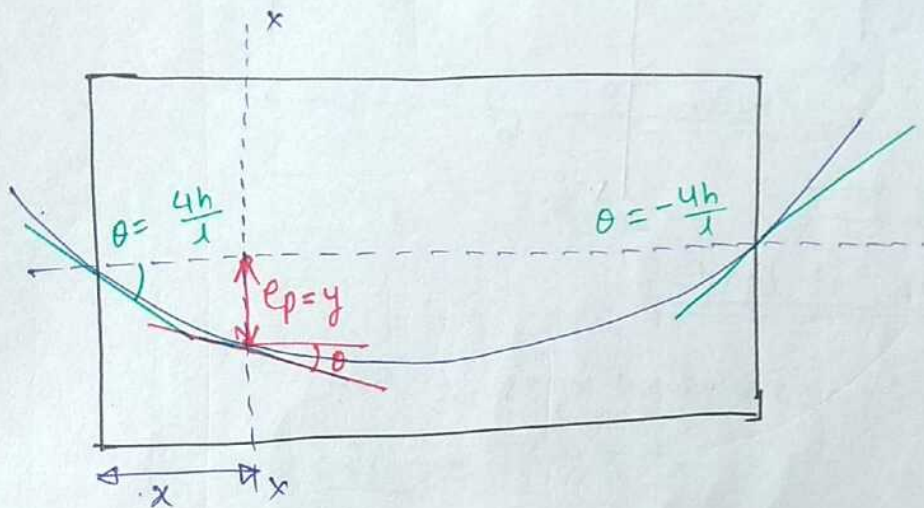
$$\frac{dy}{dx} = \frac{4h}{l^2} (l-2x) \rightarrow \text{slope equation.}$$

$= \theta = \tan \theta$

slope at ends $[x=0 \text{ and } l]$

$$\boxed{x=0} \quad \theta = \tan \theta = \frac{dy}{dx} = \frac{4h}{l}$$

$$\boxed{x=l} \quad \theta = \tan \theta = \frac{dy}{dx} = -\frac{4h}{l}$$

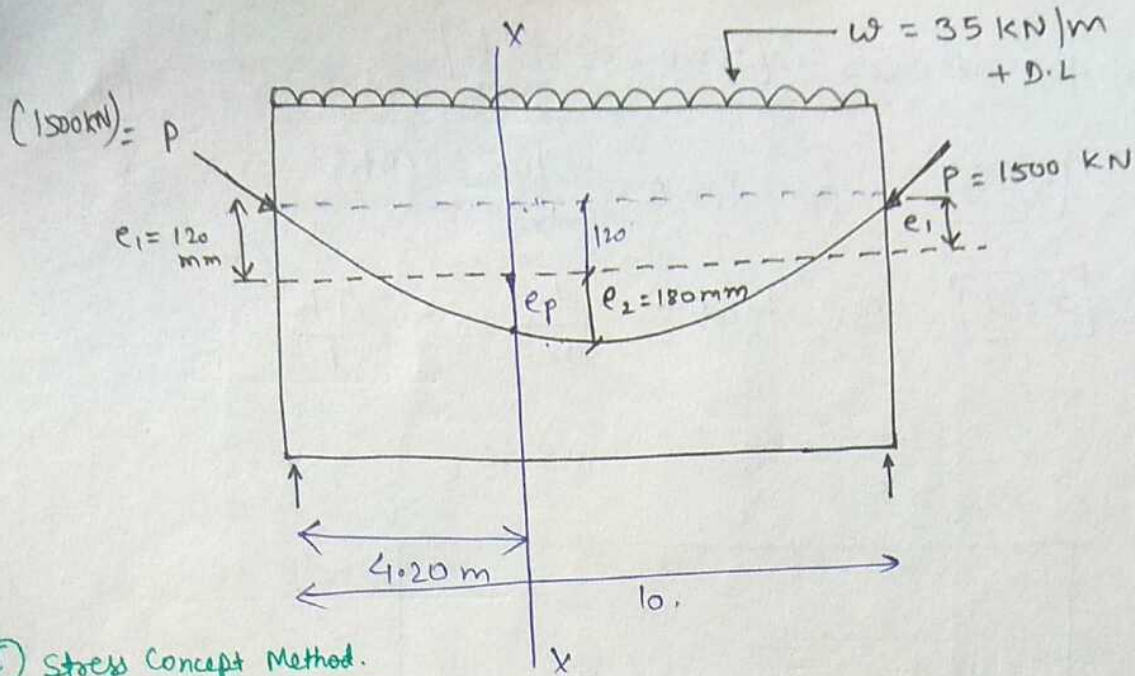


Q3. A prestressed Concrete beam of size $320\text{mm} \times 800\text{mm}$ is used for a span of 10m . Prestressing cable of parabolic profile is used with $P = 1500\text{KN}$

eccentricity = 120mm above NA
at
ends

eccentricity at mid span = 180mm below N.A

The beam is subject to a live load of 35KN/m .
Calculate stresses in the Beam at a distance 4.20m from support.



① Stress Concept Method.

① Calculation of load.

$$D.L = 0.32 \times 0.8 \times 1 \times 25 = 6.4$$

$$L.L = \frac{35}{41.4 \text{ kN/m}}$$

② Consider.
P/Ko/s

deflection at X-X

$$y = \frac{4h}{l^2} x(l-x)$$

$$= \frac{4 \times (120 + 180)}{10^2} \times 4.2 \times (10 - 4.2)$$

$$= \frac{4 \times 3}{10^2} \times 4.2 \times (5.8)$$

$$= 0.29232 \text{ m}$$

$$= 292.32 \text{ mm}$$

$$\therefore e_p = 292.32 - 120 = 172.32 \text{ mm}$$

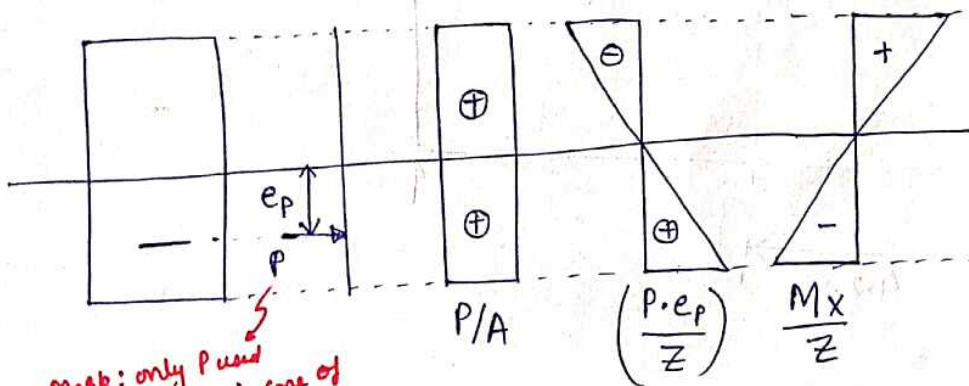
② $P = 1500 \text{ kN}$ ✓

(Neglect $\cos \theta$)

③ DL = 6.40
 L.L = 35
 $w = \frac{41.4 \text{ kN/m}}{}$

$R_1 = R_2 = \frac{wL}{2} = \frac{41.4 \times 10}{2}$
 $= \underline{207 \text{ kN}}$

$M_x = R_1 \cdot 4.2 - w \cdot \frac{4.2^2}{2}$
 $= 207 \times 4.2 - 41.4 \times \frac{4.2^2}{2}$
 $= \underline{504.252 \text{ kN-m}}$



Mark: only P used
 no P cos θ in case of
 parabolic profile

$\frac{P}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$ ✓

$\frac{P \cdot e_p}{Z} = \frac{1500 \times 10^3 \times 172.32}{320 \times \frac{800^2}{6}} = \mp 7.57$ ✓

$$\frac{M_x}{Z} = \frac{504.252 \times 10^6}{320 \times \frac{800^2}{6}} = \pm 14.77$$

Stress at top.

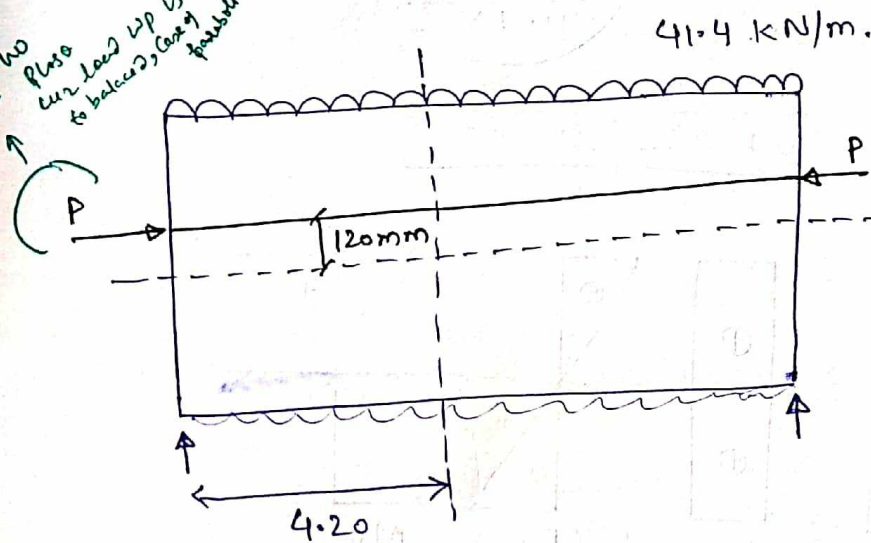
$$= 5.86 - 7.57 + 14.77 = \underline{13.06} \text{ N/mm}^2$$

Stress at bottom.

$$= 5.86 + 7.57 - 14.77 = \underline{-1.34} \text{ N/mm}^2$$

② Load Balancing Concept:

Remark: No plus or minus sign is used to balance, only parabolic



Bal. U.d.L

$$w_p = \frac{8Ph}{l^2} = \frac{8 \times 1500 \times 0.30}{10^2}$$

$$= 36 \text{ kN/m}$$

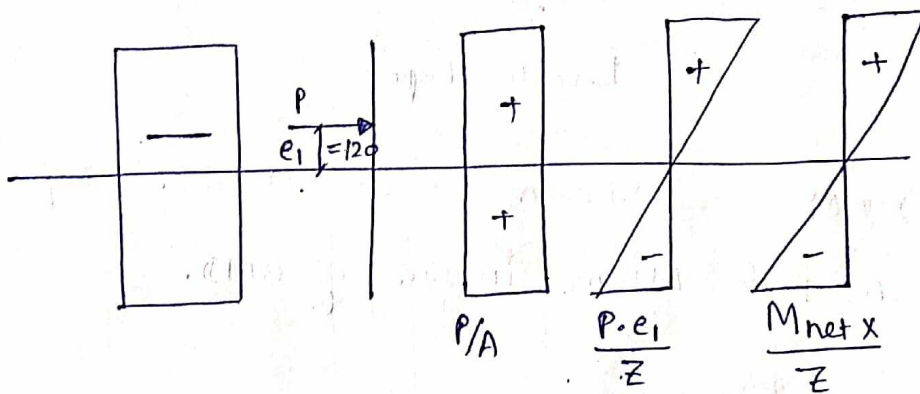
Net udl

Net udl.

$$(w - w_p) = 41.4 - 36 \\ = 5.4 \text{ kN/m}$$

$$R_1 = R_2 = \frac{w_{\text{net}} \cdot l}{2} = \frac{5.4 \times 10}{2} = 27 \text{ kN}$$

$$M_{\text{net } x-x} = R_1 \cdot x - \frac{w_{\text{net}} \cdot x^2}{2} \\ = 27 \times 4.2 - \frac{5.4 \times 4.2^2}{2} \\ = 65.772 \text{ kN-m}$$



$$\frac{P}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$$

$$\frac{P \cdot e_1}{Z} = \frac{1500 \times 10^3 \times 120}{320 \times 800^2 / 16} = \pm \overline{5.27}$$

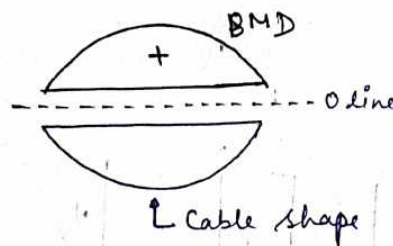
$$\frac{M_{x \text{ net}}}{Z} = \frac{65.772 \times 10^6}{320 \times \frac{800^2}{6}} = \pm \overline{1.93}$$

$$\text{Stress at top} = 5.86 + 5.27 + 1.93 = \underline{13.06} \text{ N/mm}^2$$

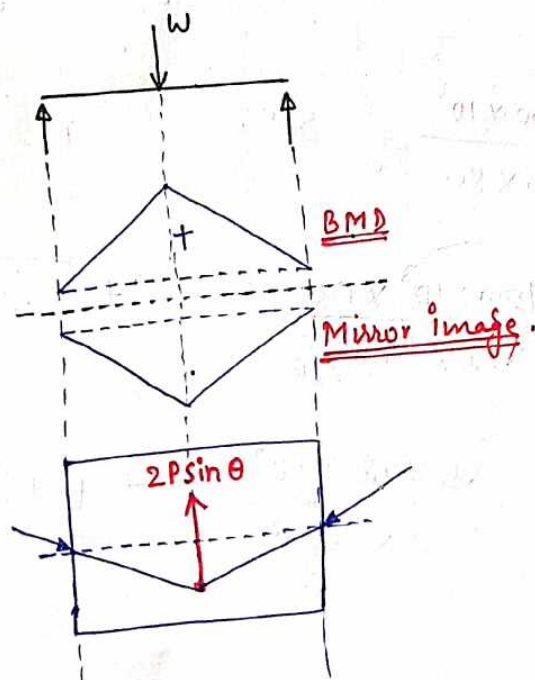
$$\text{Stress at bottom} = 5.86 - 5.27 - 1.93 = \underline{-1.34} \text{ N/mm}^2$$

Load Balancing Profile of Cable for different loading

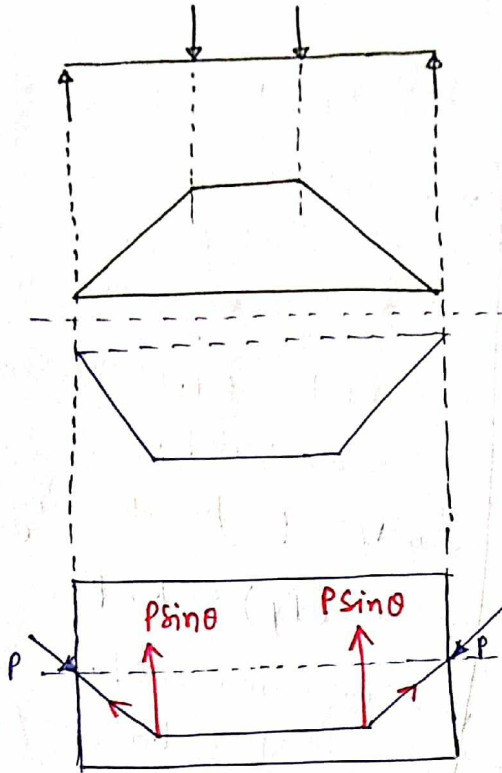
→ The shape of cable for balancing a particular load is similar as the BMD due to that load as per type of beam.



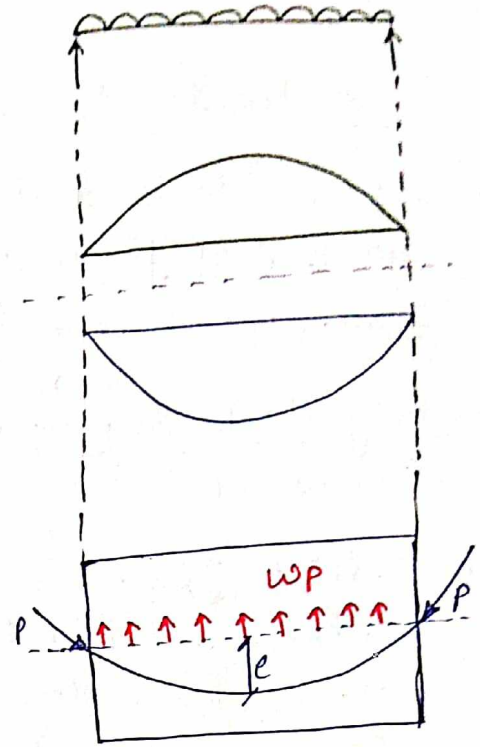
if (+ve) BM are shown above O line, shape of cable shall be mirror image of BMD.



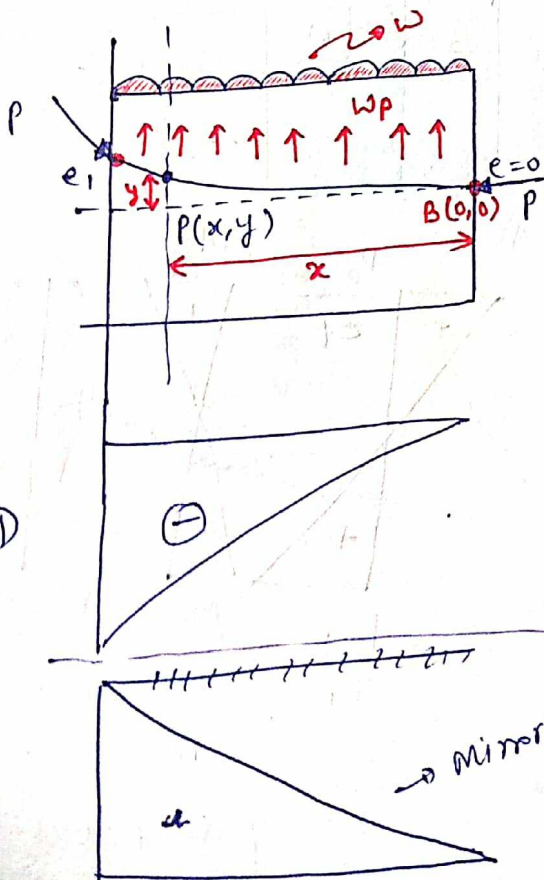
#



#



#



$$P e_1 = w \cdot l \cdot \frac{l}{2}$$

$$e_1 = \frac{w l^2}{2P}$$

Moment about P

$$P \cdot y = \frac{w \cdot x^2}{2}$$

$$* y = \frac{w x^2}{2P}$$

Equation of Cable

stresses at different stage of loading.

① At transfer. ✓

→ stress calculated just after transfer of prestressing force in the member.

✓ only dead load ^(DL) may be considered

→ Initial prestressing force is considered (i.e. P_0 value)

→ No losses are considered.

(Elastic shortening losses neglected)

② Final stage of loading

→ stress calculated at final stage, when all loads are working over member.

→ DL and L.L all loads are considered.

→ Final prestressing force

$$(P_f) = K \cdot P_0$$

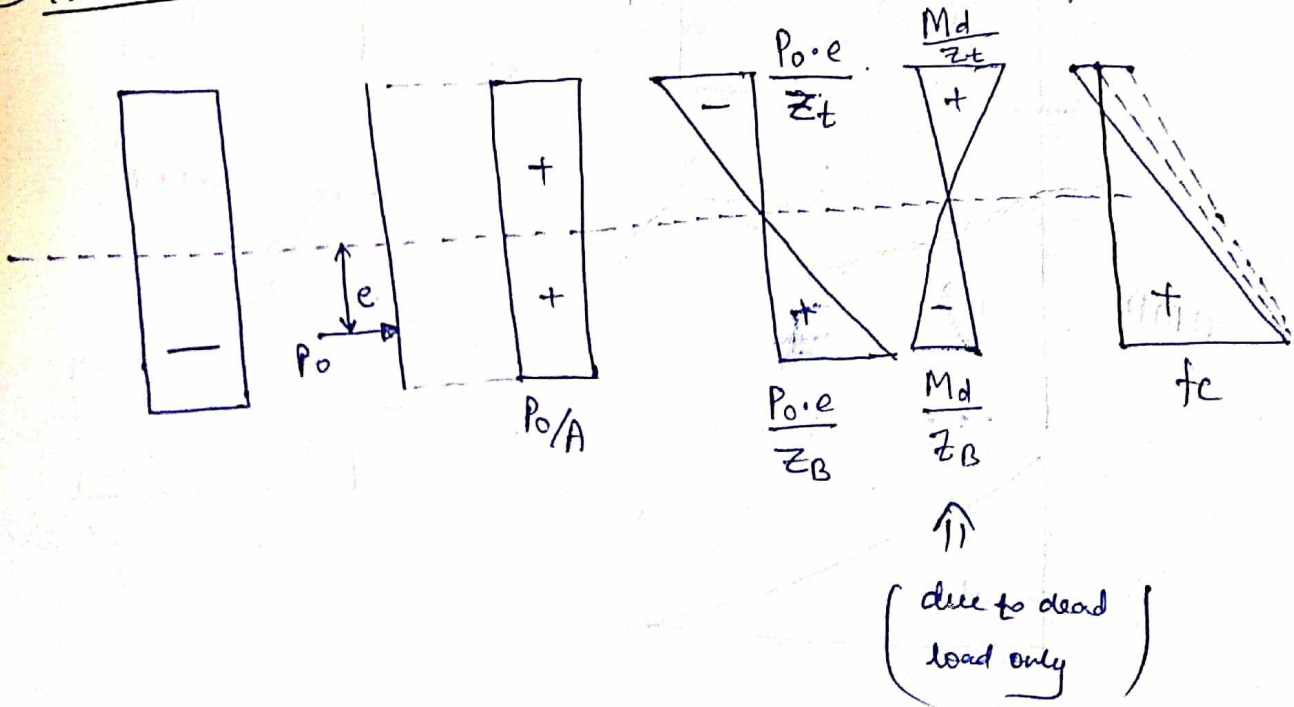
is considered.

$K =$ Loss factor.

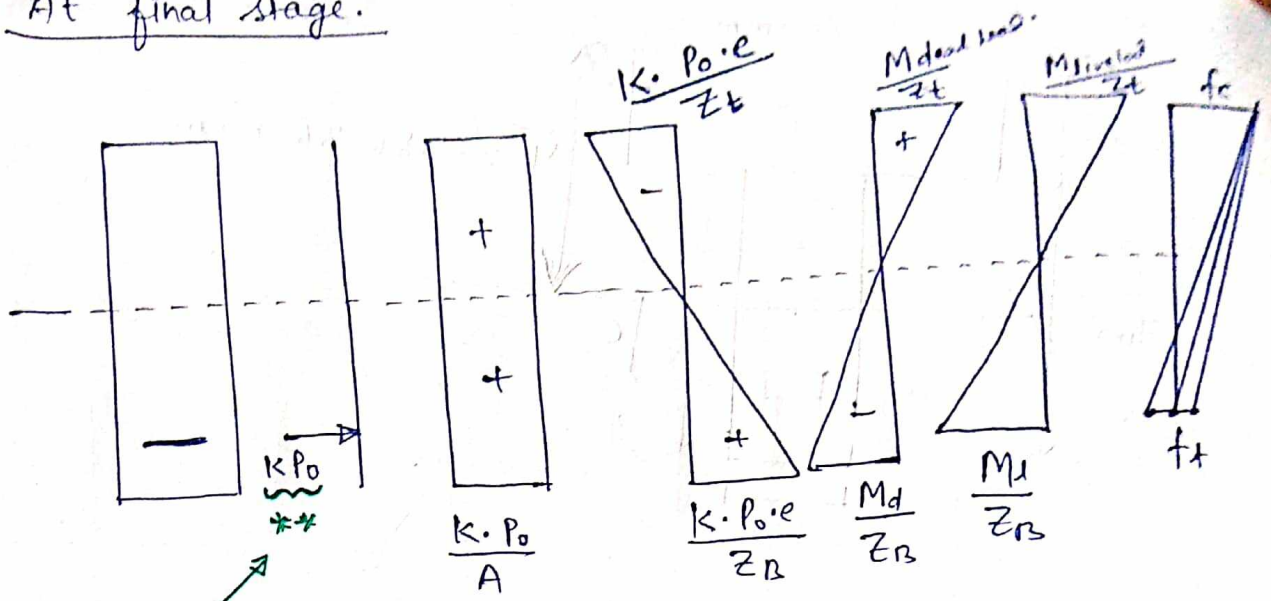
→ All losses are considered.

Consider a post tensioned beam.

① stress at transfer.



(ii) At final stage.



k signifies losses considered

Mark $\left\{ \begin{array}{l} R=0.8 \\ \text{means} \\ \text{loss is } 20\% \text{ of } P \end{array} \right.$

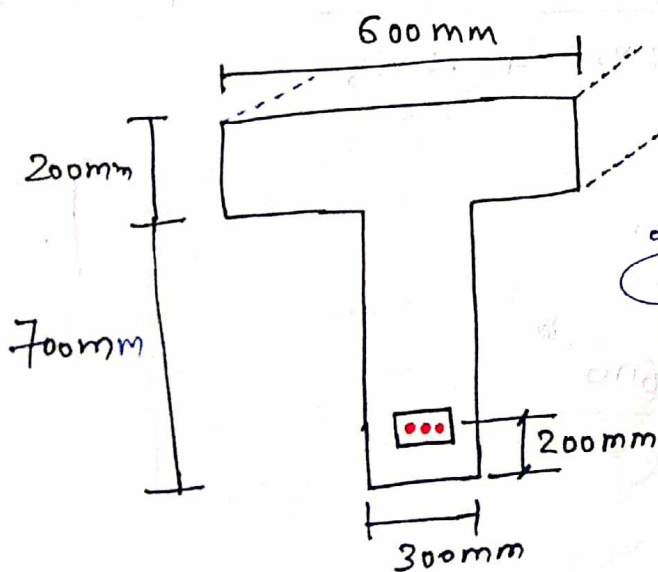
$M_l \rightarrow$ Moment due to live load.

$M_d \rightarrow$ " " " " Dead "

$Z_t \rightarrow$ at top

$Z_B \rightarrow$ at Bottom

Q4. A Post tensioned Prestressed Concrete beam as shown in fig:



Live load = 40 kN/m

Span of beam = 12 m

Area for steel

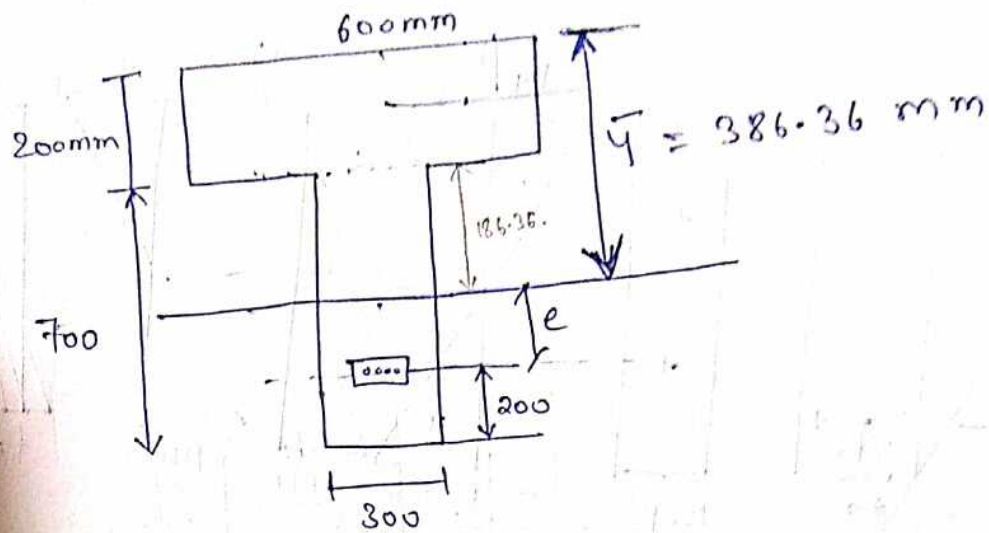
Area = 1600 mm²

stress (initial) = 1400 N/mm²

loss of stress = 20%

$\therefore k = 0.8$

Calculate max^m stresses in beam at mid span at two stage of loading.



Ans

Solⁿ: (1) Dead load = $0.6 \times 2 \times 25 + 0.7 \times 3 \times 25$
 $= 8.25$
 L.L = 40
 $\omega = \underline{48.25}$

(2) $\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$ (Varignon's Th.)
 $= \frac{0.6 \times 200 \times 100 + 0.700 \times 300 \times (350 + 2)}{0.6 \times 2 + 0.7 \times 3}$

$\bar{y} = 386.36 \text{ mm}$

(3) $I_{NA} = \frac{(200 \times 600^3)}{12}$

$$\begin{aligned}
 \textcircled{3} \quad I_{NA} &= \frac{600 \times 200^3}{12} + 600 \times 200 \times (386.36 - 100)^2 \\
 &\quad + \frac{300 \times 700^3}{12} + 300 \times 700 \times (550 - 386.36)^2 \\
 &= 4.58 \cdot 2.45 \times 10^{10} \text{ mm}^4
 \end{aligned}$$

$$\textcircled{4} \quad P_0 = A_s \times p_0 = 1600 \times \frac{1400}{10^3} = \underline{2240 \text{ kN}}$$

$$\begin{aligned}
 e &= y_B - 200 = (900 - 386.36) - 200 \\
 e &= \underline{313.64 \text{ mm}}
 \end{aligned}$$

$$y_t = 386.36 \text{ mm (Dist. of top fibre from NA)}$$

$$y_B = 513.64 \text{ (Dist. of bottom fibre from NA)}$$

$$\begin{aligned}
 \textcircled{5} \quad y_B &= 900 - 386.36 \\
 &= 513.64 \text{ mm}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{6} \quad DL &= (0.6 \times 0.2 + 0.3 \times 0.7) \times 1 \times 25 \\
 w_d &= \underline{8.25 \text{ kN/m}}
 \end{aligned}$$

$$M_d = \frac{w_d \cdot l^2}{8}$$

$$= 8.25 \times \frac{12^2}{8} = 148.5 \text{ ✓}$$

$$M_{cl} = \frac{40 \times 12^2}{8} = 720 \text{ kNm, ✓}$$

Calculate.

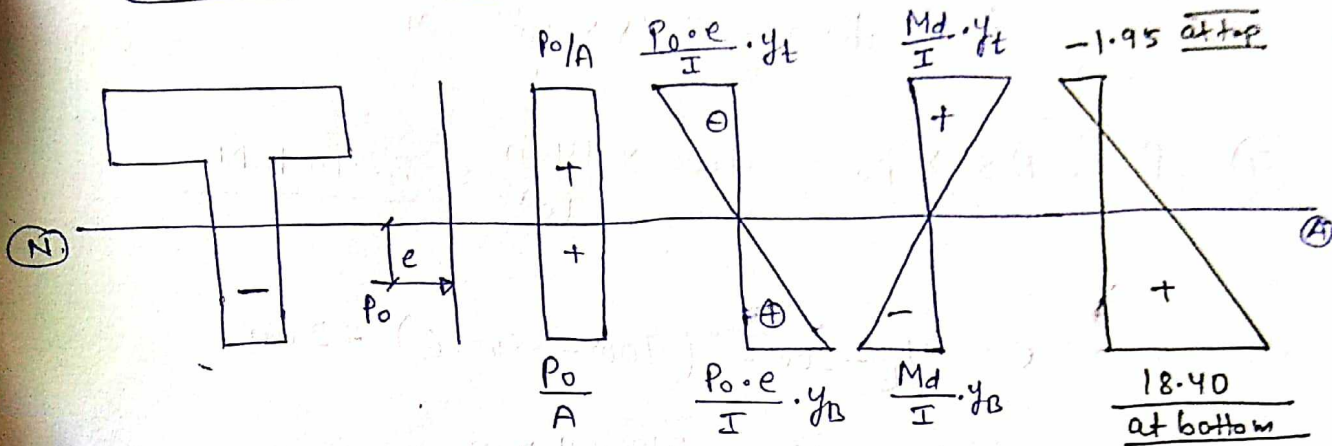
M_{dead} &

M_{cl} separately

⑦ $K = 0.80$

cur loss of stress = 20%

⑦ At transfer. (only dead load considered)



Stress value at top:

$$= \frac{P_0}{A} - \frac{P_0 \cdot e}{I} y_t + \frac{M_d}{I} \cdot y_t$$

$$= \frac{2240 \times 10^3}{330000} - \frac{2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 386.36 +$$

$$\frac{148.5 \times 10^6}{2.45 \times 10^{10}} \times 386.36$$

$$= 6.79 - 11.08 + 2.34 = \underline{\underline{-1.95 \text{ N/mm}^2}}$$

Stress value at bottom:

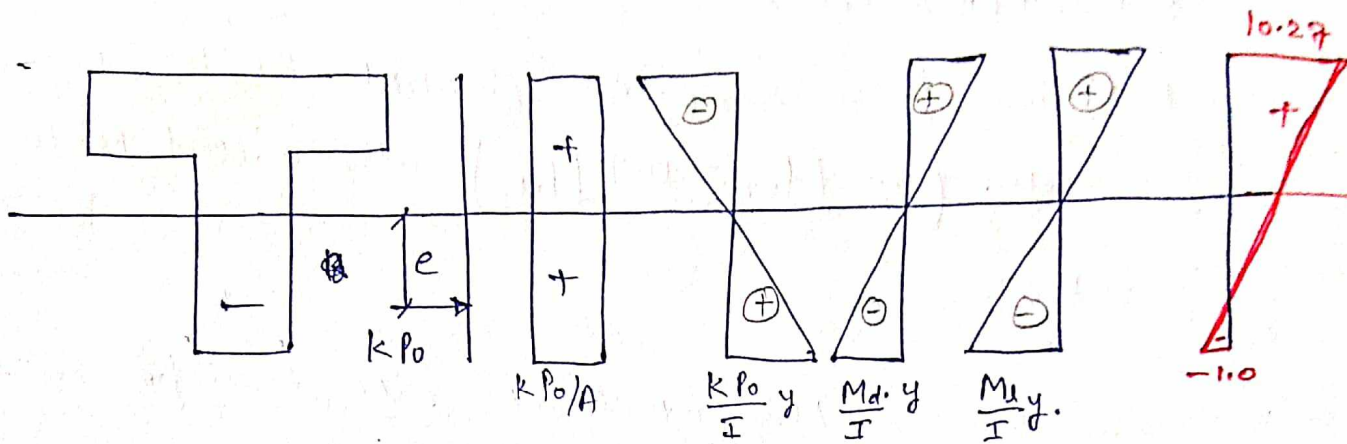
$$= \frac{P_0}{A} + \frac{P_0 \cdot e}{I} \cdot y_b - \frac{M_d}{I} y_b$$

$$= \frac{2240 \times 10^3}{330000} + \frac{2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 513.64 -$$

$$\frac{148.50 \times 10^6}{2.45 \times 10^{10}} \times 513.64$$

$$= 6.79 + 14.72 - 3.11 = \underline{18.40}$$

At final stage.



at top.

$$= k \cdot \frac{P_0}{A} - k \frac{P_0 \cdot e}{I} y_t + \frac{M_a + M_u}{I} y_t$$

$$= \frac{0.80 \times 2240 \times 10^3}{330000} - \frac{0.80 \times 2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 386.36 + \frac{868.50 \times 10^6}{2.45 \times 10^{10}} \times 386.36$$

$$= 5.43 - 8.86 + 13.70 = \underline{+10.27 \text{ N/mm}^2}$$

at bottom

$$= k \cdot \frac{P_0}{A} + \frac{k P_0 e}{I} y_b - \frac{M_a + M_u}{I} y_b$$

$$= \frac{0.8 \times 2240 \times 10^3}{330000} + \frac{0.8 \times 2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} * 513.64 - \frac{868.50 \times 10^6}{2.45 \times 10^{10}} \times 513.64$$

$$= 5.43 + 11.78 - 18.21$$

$$= \underline{-1.0}$$

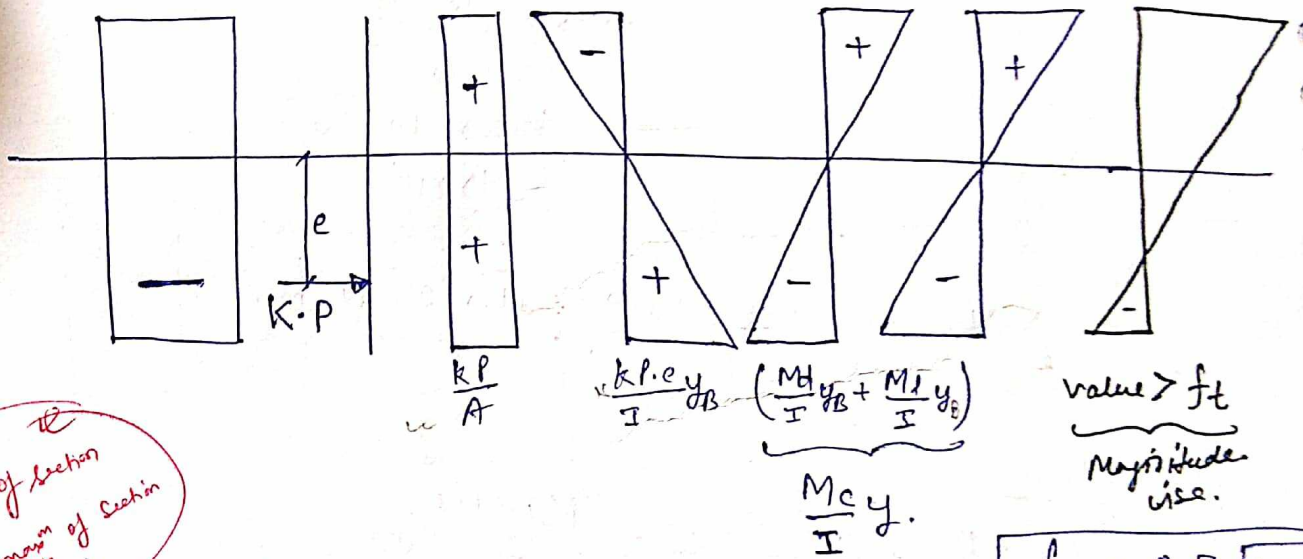
★ Cracking Moment

→ Cracking Moment is the value of Moment due to (DL+LL) all loads, for which stress in concrete at tension fibre reach to flexural tensile strength of concrete. (i.e. $f_{cr} = 0.7 \sqrt{f_{ck}}$) also called Modulus of Rupture.

→ Beyond Cracking Moment cracks will develop in the member.

At final stage.

{ Mark: Bottom (y_B) dist. considered (as Tension of cracks to be checked. }



I is of section
 y_B is max. dist. from N.A.

At Cracking Moment.

$$\frac{K.P.}{A} + \frac{K \cdot P \cdot e}{I} y_B - \frac{M_c \cdot y_B}{I} = -f_t = f_{cr}$$

Mark: Tension fibre

$M_c = \frac{w_c \cdot l^2}{8}$ → For simply support. (Cracking moment)

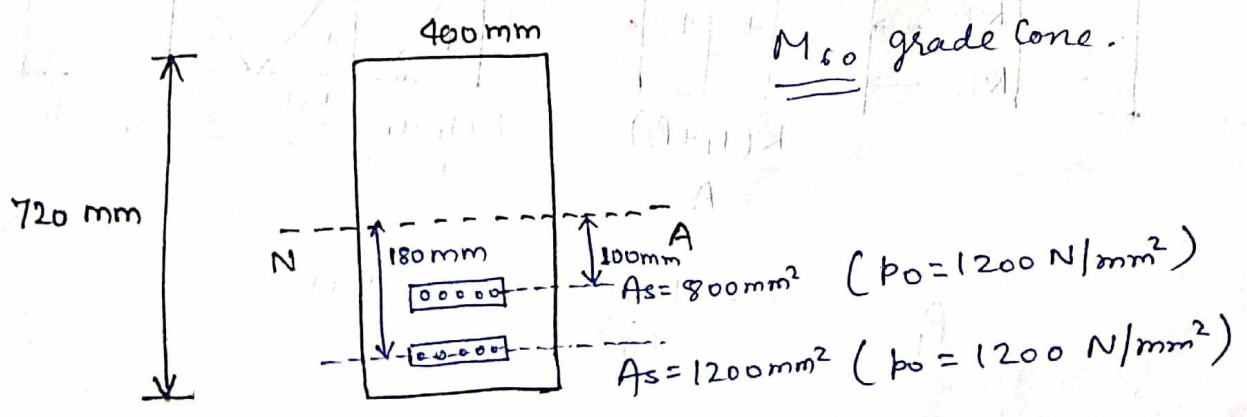
To be found →
 Mc As a whole to be found which include (Mmax)

Actual load on structure is kept less than Cracking moment.

Factor of safety against cracking = $\frac{\text{Cracking load}}{\text{Working load}}$

$\therefore \text{F.O.S} = \frac{\text{Cracking Moment}}{\text{Working Moment}}$

Q5 A rectangular Beam is provided with cables as shown in fig. The beam is simply supported over a span of 8m and is subjected to a live load of 40 kN/m.



- Calculate:
- ① Cracking Moment
 - ② Load factor against cracking.
 - ③ Stresses at Mid span due to working load.

$$① \quad I = \frac{BD^3}{12} = 1.24 \times 10^{10} \text{ mm}^4$$

$$P_{01} = 960 \text{ kN.}$$

$$P_{02} = 1440 \text{ kN.}$$

② Dead load.

$$D_L = 0.4 \times 0.22 \times 17 \times 25 = 7.2 \text{ kN.}$$

$$L.L = \underline{\underline{40 \text{ kN.}}}$$

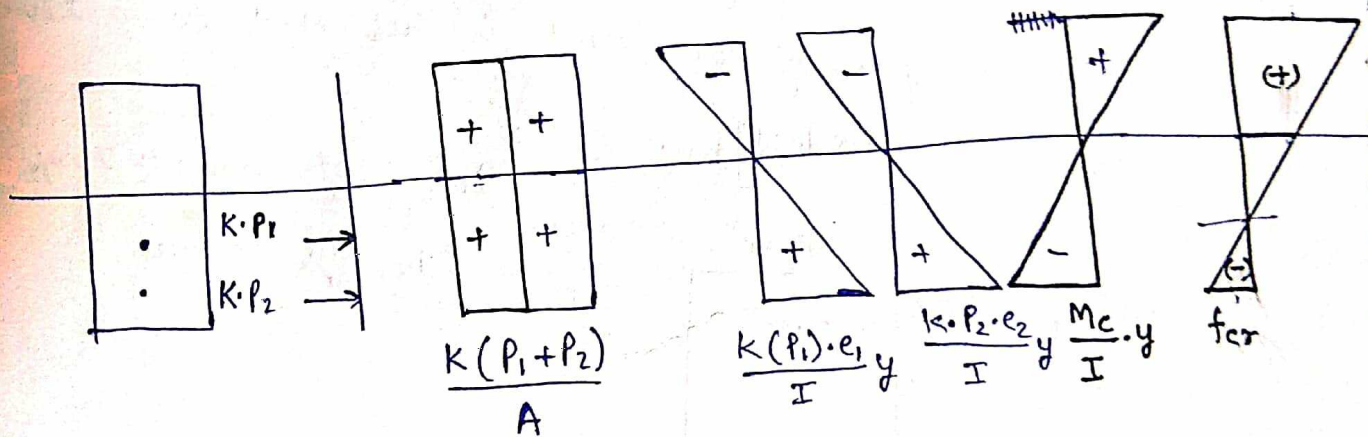
$$\underline{\underline{47.2 \text{ kN.}}}$$

∴

$$K = 0.80$$

$$A_{\text{area}} = 288000 \text{ mm}^2$$

At final stage.



$$P_1 = 800 \times \frac{1200}{10^3} = 960 \text{ kN.}$$

$$e_1 = 100 \text{ mm}$$

$$P_2 = 1200 \times \frac{1200}{10^3} = \underline{1440 \text{ KN}}$$

$$e_2 = 180 \text{ mm}$$

$$f_{cr} = 0.7 \sqrt{f_{ck}} = 0.7 \sqrt{60}$$

$$= 5.42 \text{ N/mm}^2$$

① For Cracking Moment (at final stage)

$$\frac{k \cdot P_1 + k \cdot P_2}{A} + \frac{k \cdot P_1 \cdot e_1}{I} \cdot y + \frac{k \cdot P_2 \cdot e_2}{I} \cdot y - \frac{M_c}{I} \cdot y = \underline{\underline{-5.42}}$$

$$\frac{0.8(960+1440) \times 10^3}{400 \times 720} + \frac{0.8 \times 960 \times 10^3 \times 100}{400 \times 720^2 / 6} + \frac{0.8 \times 1440 \times 10^3 \times 180}{400 \times 720^2 / 6}$$

$$- \frac{M_c}{400 \times 720^2 / 6} = -5.42$$

~~$$\frac{M_c}{I} =$$~~

$$6.67 + 2.22 + 6.00 - \frac{M_c}{400 \times 720^2 / 6} = -5.42$$

$$M_c = 20.33 \times 400 \times 720^2 / 6 \times \frac{1}{10^6}$$

$$= \underline{\underline{702.605 \text{ KN-m}}}$$

② Factor of safety. (Working Moment)

$$D.L = 0.4 \times 0.72 \times 1 \times 25 = 7.2$$

$$L.L = \frac{40.0}{47.2} \text{ kN.}$$

$$B.M = 47.2 \times \frac{8^2}{8} = 377.6 \text{ kN-m.}$$

$$\text{Factor of safety} = \frac{M_c}{\underbrace{M_w}_{\text{Working Moment}}} = \frac{702.605}{377.6} = 1.86.$$

③ Stresses at bottom fibre due to working moment.

$$\frac{k \cdot P_1 + k \cdot P_2}{A} + \frac{k \cdot P_1 \cdot e_1}{I} \cdot y + \frac{k \cdot P_2 \cdot e_2}{I} \cdot y - \frac{M_w}{I} \cdot y.$$

$$\Rightarrow \frac{0.8(960 + 1440) \times 10^3}{400 \times 720} + \frac{0.8 \times 960 \times 10^3 \times 100}{400 \times 720^2 / 6} + \frac{0.8 \times 1440 \times 10^3 \times 180}{400 \times 720^2 / 6} - \frac{377.6 \times 10^6}{400 \times 720^2 / 6}$$

$$\Rightarrow 6.67 + 2.22 + 6.00 - 10.92 = +3.97 \text{ N/mm}^2$$

Stress at top fibre

$$= 6.67 - 2.22 - 6.02 + 10.92$$

$$= 9.35 \text{ N/mm}^2.$$

2nd Method.

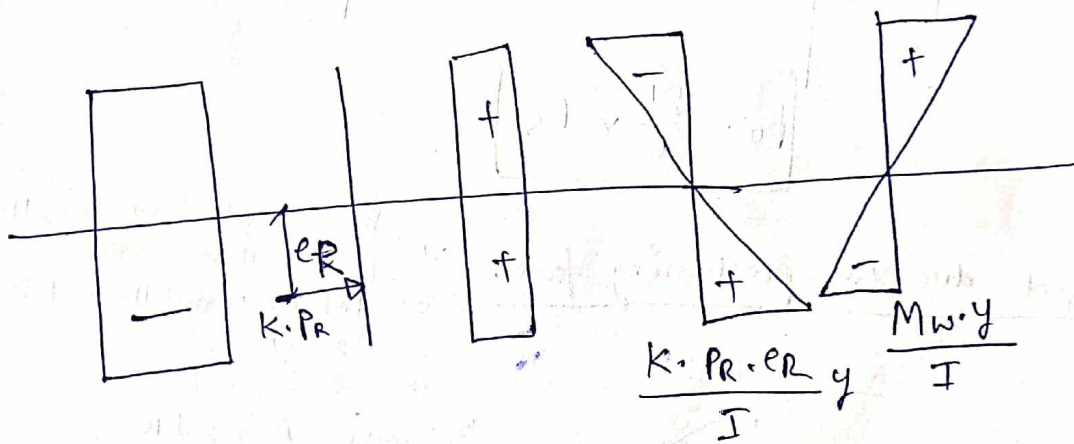
$$P_R = 960 + 1440 = 2400 \text{ kN.}$$

$$e_R = \frac{P_1 \cdot e_1 + P_2 \cdot e_2}{P_1 + P_2}$$

$$= \frac{960 \times 100 + 1440 \times 180}{2400}$$

$$= 148 \text{ mm.}$$

Stress at bottom due to Working.



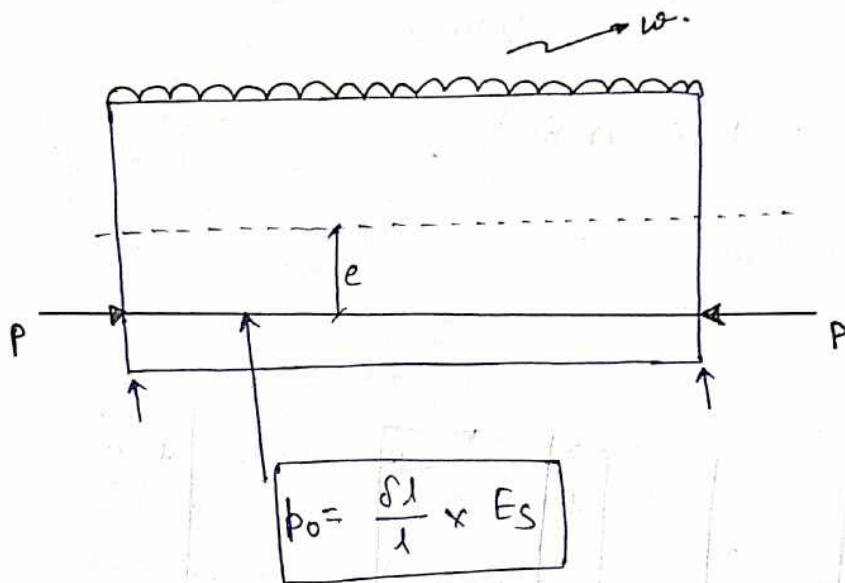
$$\text{Stress} = \frac{0.8 \times 2400 \times 10^3}{400 \times 720} + \frac{0.8 \times 2400 \times 10^3 \times 148}{400 \times 720^2 / 6} - 10.92$$

$$= 6.67 + 8.29 - 10.92$$

$$= +3.97 \text{ N/mm}^2$$

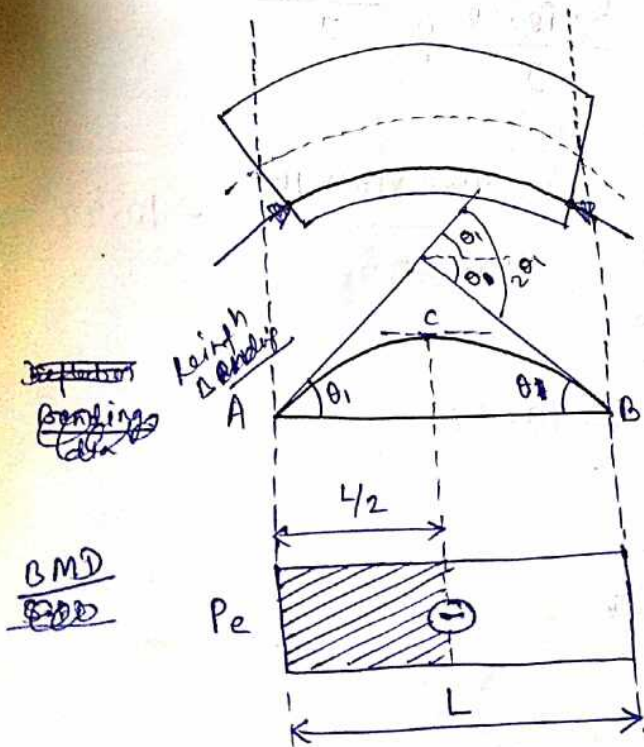
Slope and deflection.

⑦ Effect of prestressing force and loading over stresses in reinforcement.



① Effect due to prestressing force.

here due to prestressing ^{only} Hogging effect will be seen
 ⇒ let angle subtended is θ_1 at each end.



Between A and B

$\theta_1 = (-\theta_1) = \frac{P \cdot e \cdot L}{EI}$ → Area of BMD.

$2\theta_1 = \frac{P \cdot e \cdot L}{E_c \cdot I_c}$

B/w A and C

$$\theta_1 - 0 = \frac{P_0 \cdot \frac{L}{2}}{EI}$$

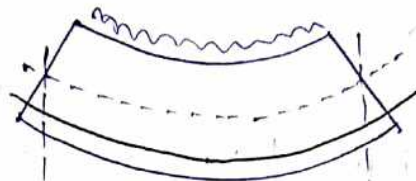
$$\theta_1 = \frac{P_0 \cdot L}{2EcIc}$$

→ cam.
 $\frac{WL}{2Ec}$

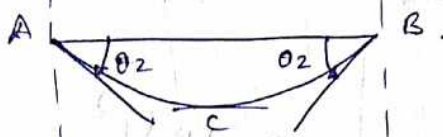
② Due to u.d.l. (i.e. effect of loading)

⇒ here due to load sagging will be seen, let θ_2 be the angle subtended at ends

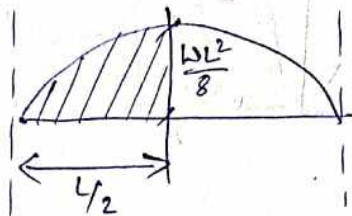
Difference of slope b/w A and C



$$\theta_2 - 0 = \frac{\frac{2}{3} \cdot \frac{WL^2}{8} \cdot \frac{L}{2}}{EcIc}$$



$$\theta_2 = \frac{WL^2}{24EcIc} \rightarrow \text{①}$$



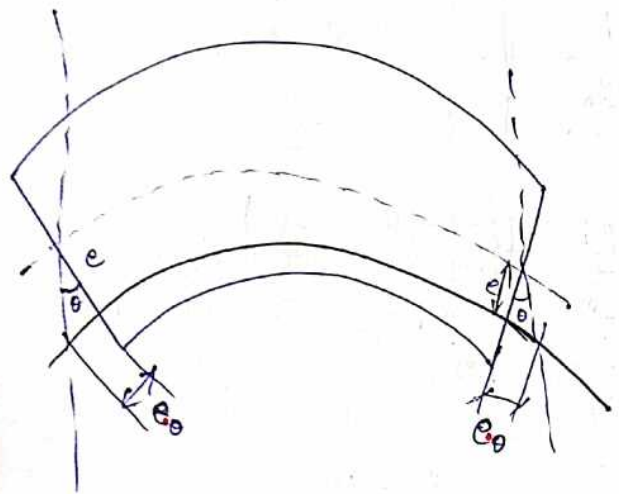
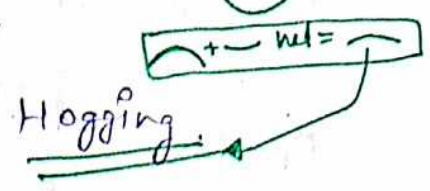
③ Net slope of the Beam at ends

① if $\theta_1 > \theta_2$ $\theta = (\theta_1 - \theta_2)$

② if $\theta_2 > \theta_1$ $\theta = (\theta_2 - \theta_1)$
(if mean sagging effect due to load is more than hogging effect due to prestress)

Case 1 When $\theta_1 > \theta_2$ i.e. Prestressing force $>$ loading effect/bene.

Net slope $\theta = (\theta_1 - \theta_2)$

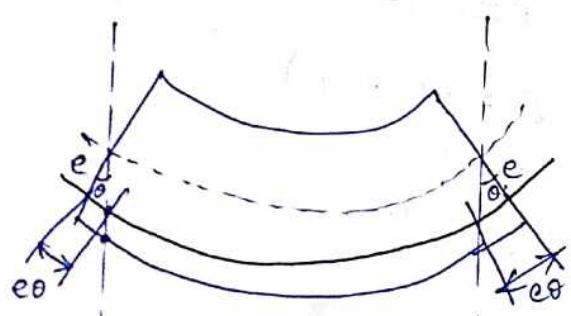


Total reduction of length in reinforcement.
 $= e\theta + e\theta = (2e\theta)l$

Loss of stress in steel
 $= \frac{(2e\theta)l}{l} \times E_s$ ($\theta = \theta_1 - \theta_2$)

Case 2. if $\theta_2 > \theta_1$

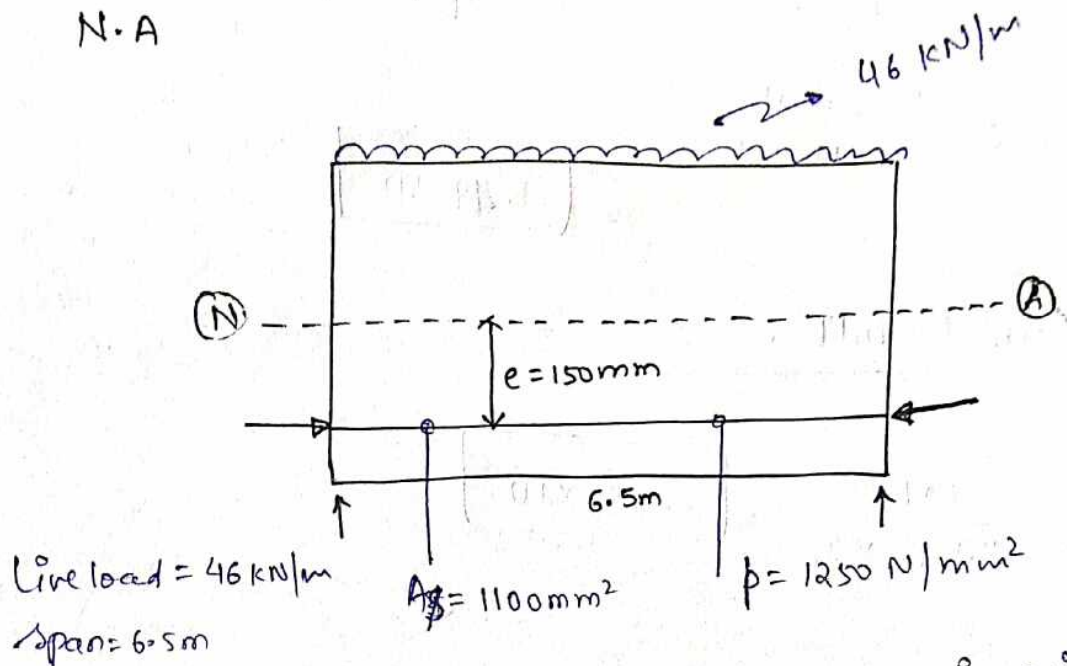
Net slope = $\theta = (\theta_2 - \theta_1)$ sagging.



Gain of stress in steel = $\frac{2e\theta}{l} \times E_s$

↓ assumption supp minor loss/gain

Q6. A ss beam of size 300 mm x 560 mm is provided with a straight cable at an eccentricity of 150 mm below N.A



Calculate Net increase/decrease of stress in reinforcement only due to the effect of P-force and loading.

$$E_s = 2.1 \times 10^5 \text{ N/mm}^2$$

$$E_c = 34000 \text{ N/mm}^2$$

Ans:

① prestressing force

$$P = A_s \times p_0$$

$$= 1100 \times 1250 \times \frac{1}{10^3} = 1375 \text{ kN}$$

$$e = 150 \text{ mm}$$

$$\text{② D.L} = 0.30 \times 0.56 \times 1 \times 25$$

$$= 4.20 \text{ kN}$$

$$\text{L.L} = 46 \text{ kN}$$

$$\omega = \underline{50.20 \text{ kN/m}}$$

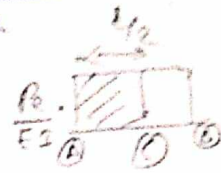
③ Slope due to P-Force.

diff b/w A & C

$$\theta_1 = \frac{P \cdot e \cdot L}{2 E_c \cdot I_c} = \frac{1375 \times 10^3 \times 6500}{2 \times 34000 \times \frac{300 \times 560^3}{12}}$$

keep units in N and mm.

$$= 4.49 \times 10^{-3}$$



④ Slope due to udl.



diff b/w A & C

$$\theta_2 = \frac{wL^3}{24 E_c \cdot I_c} = 3.85 \times 10^{-3}$$

So $\theta_1 > \theta_2$ $\rightarrow 3.85 \times 10^{-3}$

So net slope will be hogging.

$$\theta_1 - \theta_2 = 0.64 \times 10^{-3} \text{ (Hogging)}$$

⑤ Loss of stress in reinforcement

$$\text{Stress} = \frac{2e\theta}{l} \times E_s$$

$$= \frac{2 \times 150 \times 0.64 \times 10^{-3}}{6500} \times 2.1 \times 10^5$$

$$= 6.2 \text{ N/mm}^2$$

Ans

very less

hence

as per

assumption

loss of stress in

steel

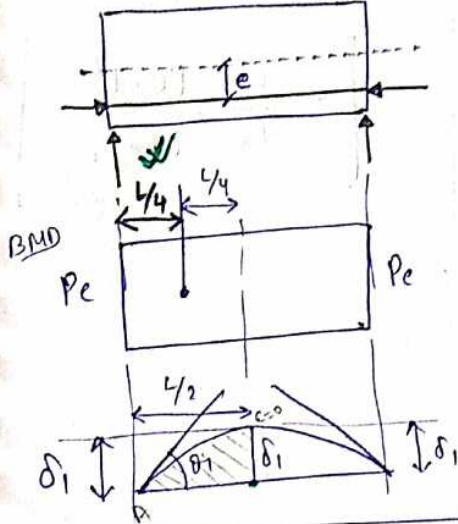
not considered.

Slope and deflection.

(A) Due to Prestressing.

Cases

① Straight Cable.



Slope

$$\theta_1 = \frac{Pe \cdot \frac{L}{2}}{Ec \cdot Ic}$$

$$= \frac{Pe \cdot L}{2 Ec \cdot Ic}$$

Deflection.

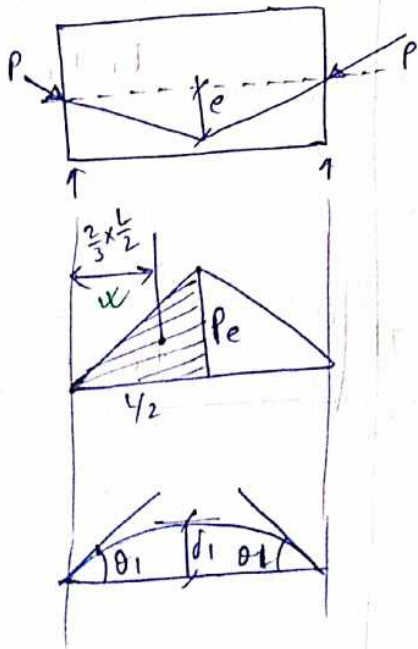
Normal of deflection

$$\delta_1 = \frac{Pe \cdot \frac{L}{2} \times \frac{L}{4}}{Ec \cdot Ic}$$

$$= \frac{Pe \cdot L^2}{8 Ec \cdot Ic}$$

Made for Deflection:
Calculating Moment of Area which axis is taken - moment of half area, is not we get tangent at C. It is to the line of beam.

② Bend Cable.



$$\theta_1 = \frac{\frac{1}{2} \cdot Pe \times \frac{L}{2}}{Ec \cdot Ic}$$

$$\theta_1 = \frac{Pe \cdot L}{4 Ec \cdot Ic}$$

$$\delta_1 = \frac{\frac{1}{2} \times Pe \cdot \frac{L}{2} \times \frac{2}{3} \times \frac{L}{2}}{Ec \cdot Ic}$$

$$\delta_1 = \frac{Pe L^2}{12 Ec \cdot Ic}$$

To find Deflection
(1) Condition
A fixed. \rightarrow Tangent at A
BMD \rightarrow \rightarrow
"X" is considered for cal. δ at C

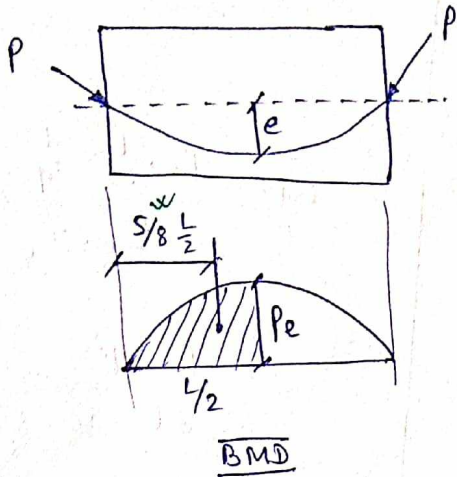
Condition
BMD. \rightarrow \rightarrow
Deflection δ at C
You can't get actual deflection at C if tangent at A taken
 \rightarrow Tangent. So consider C as fixed and calculate δ at A which is same as δ at C

Case

Slope

Deflection

③ Parabolic Cable



$$\theta_1 = \frac{\frac{2}{3} P \cdot e \times \frac{L}{2}}{E_c \cdot I_c}$$

$$\theta_1 = \frac{P_e \cdot L}{3 E_c \cdot I_c}$$

$$\delta_1 = \frac{\frac{2}{3} \cdot P_e \cdot \frac{1}{2} \times \frac{5}{8} \times \frac{L}{2}}{E_c \cdot I_c}$$

$$= \frac{5}{48} \frac{P_e \cdot L^2}{E_c \cdot I_c}$$

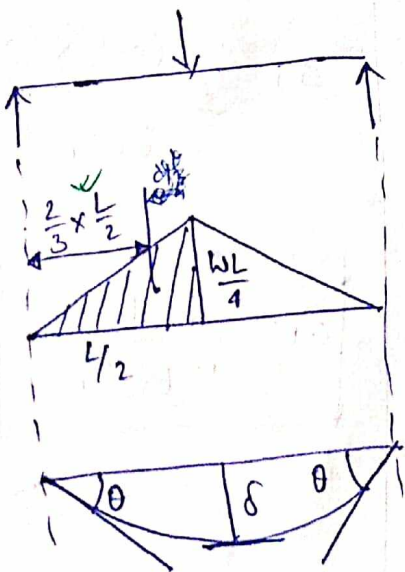
⑥ Due to loading.

Case

Slope

Deflection.

④ Point load



$$\theta = \frac{\overbrace{\frac{1}{2} \times \frac{WL}{4} \times \frac{L}{2}}^{\text{area}}}{E_c I_c}$$

$$\theta = \frac{WL^2}{16 E_c I_c}$$

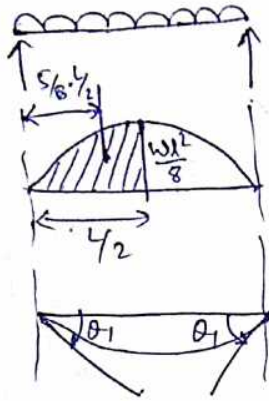
$$\delta = \frac{\left(\frac{1}{2} \times \frac{WL}{4} \times \frac{L}{2} \right) \times \left(\frac{2}{3} \times \frac{L}{2} \right)}{E_c I_c}$$

$$\delta = \frac{WL^3}{48 E_c I_c}$$

5

Case -

Udl



Slope

$$\theta = \frac{\frac{2}{3} \times \frac{wL^2}{8} \times \frac{L}{2}}{E_c I_c}$$

$$\theta = \frac{wL^3}{24 E_c I_c}$$

Deflection

$$\delta_c = \frac{\frac{2}{3} \times \frac{wL^2}{8} \times \frac{L}{2} \times \frac{5}{8} \times \frac{L}{2}}{E_c I_c}$$

$$\delta_c = \frac{5}{384} \frac{wL^4}{E_c I_c}$$

Q7. For Q6 Calculate ^{Net} deflection at Centre of Beam

due to prestressing force Cable

$$\delta_1 = \frac{P_e \cdot L^2}{8 E_c I_c} = \frac{1375 \times 10^3 \times 150 \times (6500)^2}{8 \times 34000 \times 350 \times \frac{860^3}{12}} = 7.30 \text{ mm } \uparrow$$

(due to hoggs)

due to udl.

$$\delta_2 = \frac{5}{384} \times \frac{wL^4}{E_c I_c} = 7.82 \text{ mm } \downarrow$$

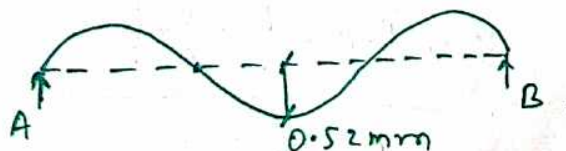
(due to hoggs)

$\therefore \delta \rightarrow$ net deflection at Centre.

$$\delta_2 - \delta_1 = 0.52 \text{ mm } \downarrow$$

Max

Note: Net moment was \ominus hogging but we got deflection \downarrow it is due to we got two Point of contraflexure.



Q88. A simply supported beam is subjected to a live load of 65 kN/m, over a span of 10m.

Size of beam = 400mm x 700mm.

Prestressing force = 1800 kN

Parabolic cable

eccentricity at ends = 0

at Centre = 180mm

Calculate ~~it~~ :-

(i) net deflection of beam at Centre.

(ii) net slope at ends

$$E_s = 2 \times 10^5 \text{ N/mm}^2$$

M60 Conc. used.

Ans:

$$\begin{aligned} E_c &= 5000 \sqrt{f_{ck}} \\ &= 5000 \times \sqrt{60} \\ &= 38729.83 \text{ N/mm}^2 \end{aligned}$$

Load Calculation:

$$D.L = 0.4 \times 0.7 \times 1 \times 25 = 7 \text{ kN/m}$$

$$L.L = 65 \text{ kN/m}$$

$$W = 72 \text{ kN/m}$$

(i) Net deflection due to prestressing

$$\delta = \frac{\frac{2}{3} \cdot P \cdot e \cdot \frac{L}{2} \cdot \frac{5}{8} \times \frac{L}{2}}{E_c I_c}$$

$$\delta = \frac{\frac{5}{32} \times 1800 \times 10^3 \times 180 \times \frac{10 \times 10^3}{2} \cdot \frac{5}{8} \times \frac{5000}{2}}{38729.93 \times \frac{100 \times 1000^3}{12}}$$

Calculator
error

$$\delta = \frac{5}{48} \times \frac{P \cdot e \cdot L^2}{E_c \cdot I_c}$$

$$= \frac{5}{48} \times \frac{1800 \times 10^3 \times 180 \times 10000^2}{5000 \sqrt{60} \times \frac{400 \times 1000^3}{12}}$$

$$= 7.62 \text{ mm } \uparrow$$

(ii) deflection due to load.

$$\delta_2 = \frac{5}{384} \frac{WL^4}{E_c I_c}$$

$$= \frac{5}{384} \frac{22 \times 10000^4 \times 12}{5000 \sqrt{60} \times 400 \times 1000^3} = 21.17 \text{ mm } \downarrow$$

Net deflection

$$= \delta_2 - \delta_1 = 21.17 - 7.62 = 13.55 \text{ mm } \downarrow$$

(2) slope.

(a) Due to Pe.

$$\theta_1 = \frac{P \cdot e \cdot L}{3 E_c \cdot I_c} = \frac{1800 \times 10^3 \times 180 \times 10000}{3 \times 5000 \sqrt{60} \times \frac{400 \times 1000^3}{12}}$$

$$= 2.44 \times 10^{-3}$$

(b) due to

$$\theta_2 = \frac{wl^3}{24 E_c I_c} = \frac{72 \times 10000^3}{24 \times 50000 \sqrt{60} \times \frac{400 \times 700^3}{12}}$$
$$= \underline{6.77 \times 10^{-3}}$$

Net slope,

$$\theta = \theta_2 - \theta_1$$

$$= 6.77 \times 10^{-3} - 2.44 \times 10^{-3}$$

$$= \underline{4.33 \times 10^{-3}}$$

2nd Method

(1) Balancing U.d.l.

$$w_p = \frac{8Ph}{l^2} = \frac{8 \times 1800 \times 0.18}{10^2}$$

$$= \underline{25.92 \text{ KN/m} \uparrow}$$

$$\underline{w = 72 \text{ KN/m}}$$

Net load,

$$w_{\text{net}} = 72 - 25.92$$
$$= 46.08 \text{ KN/m.}$$

(i) deflection

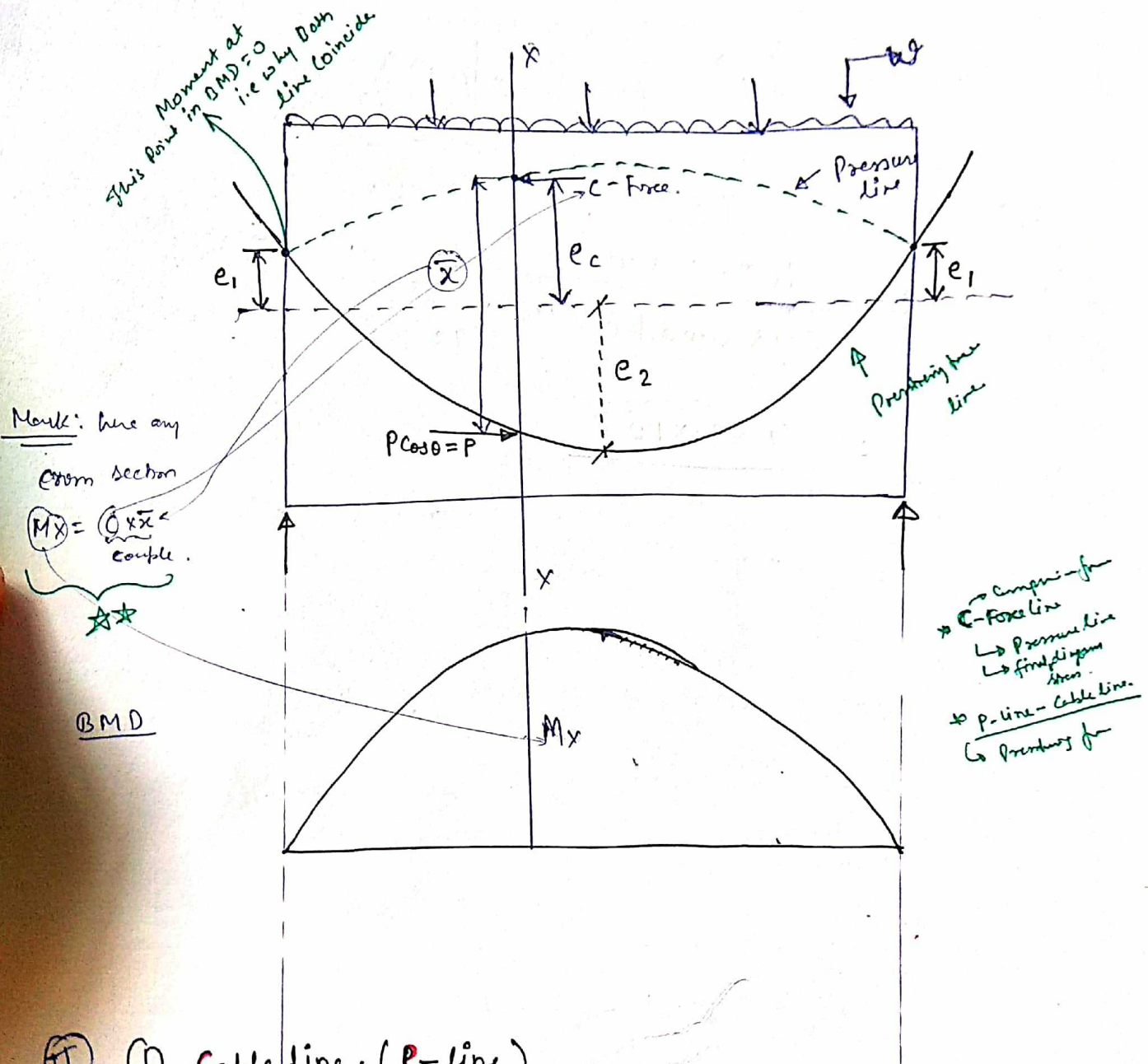
$$\delta_{\text{net}} = \frac{5}{384} \times \frac{46.08 \times 10000^4}{5000 \sqrt{60} \times 400 \times \frac{700^3}{12}} = 13.55 \text{ mm.}$$

(ii) slope =

$$\theta = \frac{46.08 \times 10000^3}{24 \times 5000 \sqrt{60} \times 400 \times \frac{700^3}{12}}$$

$$= \frac{4.33 \times 10^{-3}}{1}$$

Pressure line and Cable line.



(I) Cable line (P-line)

Prestressing force line.

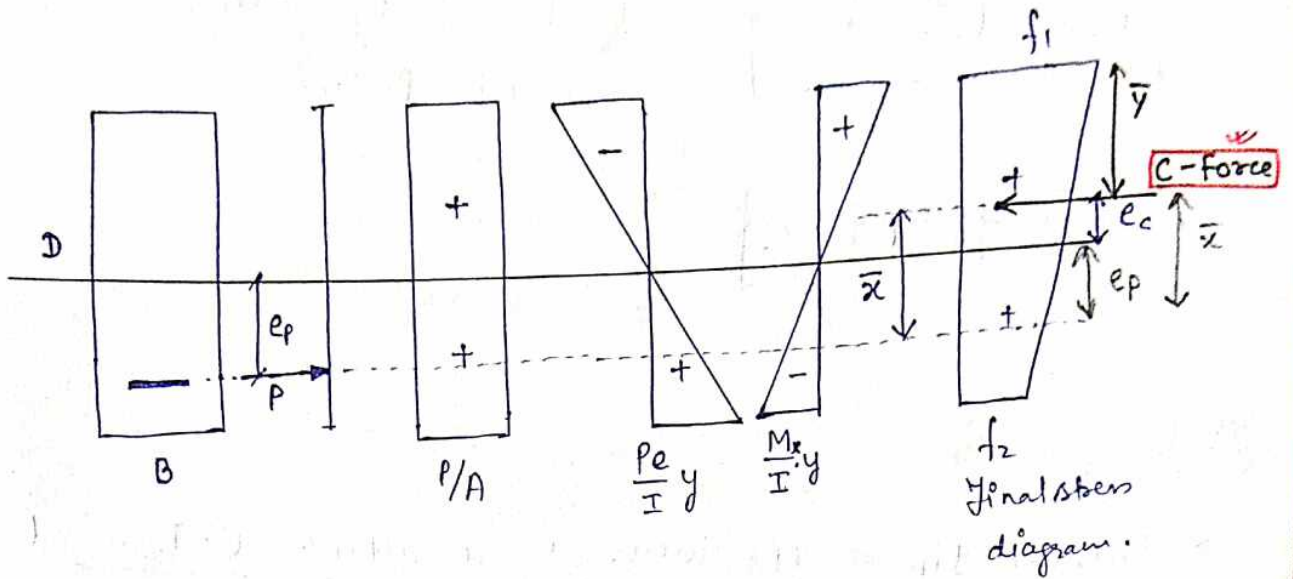
~~Steen~~ Steen Concept method is based on this force (P-force)

-> Cable line (P-line) is the location of prestressing force in the beam at different cross section.

(II) Make a Note: that lever arm changes for every location x -> but force is constant.

At x-x

By stress Concept Method.



Stress at top:

$$f_1 = \frac{P}{A} - \frac{P \cdot e}{I} \cdot y + \frac{M_x}{I} \cdot y$$

Stress at bottom:

$$f_2 = \frac{P}{A} + \frac{P \cdot e}{I} \cdot y - \frac{M_x}{I} \cdot y$$

C-Force: "Total Net Compressive force acting over the section due to final stress diagram."

$$\left. \begin{aligned} \text{C Force} &= B \times D \times \left(\frac{f_1 + f_2}{2} \right) \\ &= \text{P-Force} \end{aligned} \right\} \text{at all cross section}$$

Location of C-force.

$$\bar{y} = \left(\frac{f_1 + 2f_2}{f_1 + f_2} \right) \times \frac{D}{3}$$

Eccentricity of C-force.

$$e_c = \left(\frac{D}{2} - \bar{y} \right)$$

⇒ Total shift of C-line
w.r.t P-line

$$\bar{x} = e_p + e_c$$

② Pressure line

- Pressure line is the locus of resultant C-Force at different cross-section of beam.
- Pressure line will show the location of Net compressive force in the beam at different cross-section.

Important properties.

① $C\text{-Force} = P\text{ Force}$ at all cross-section of beam.

or

$$C = P \cos \theta$$

② The shift of C-line is proportional to the moment in beam at the particular C/S.

i.e. at any cross section

$$M_x = P \times \bar{x} = C \times \bar{x} \quad \text{see fig}$$

$\bar{x} = \text{shift of C-line w.r.t P-line}$

$$\therefore \bar{x} = \frac{M_x}{P} = \frac{M_x}{C}$$

→ Shift of C-line w.r.t P-line.

→ eccentricity of C-line

$$e_c = \bar{x} - e_p$$

③ At ends

moment = 0

Shift of C line = 0

→ C-line and P line shall be at same location at cross section where $M=0$ ✓

Strength Concept Method.

① Calculate Moment due to all given loads at the section (without any load balancing)

$$= M_x$$

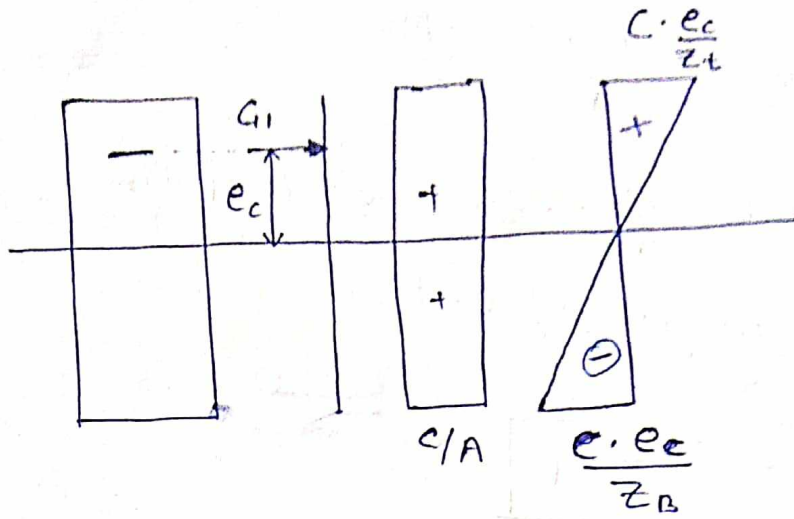
② Shift of C-line w.r.t P-line

$$\bar{x} = \frac{M_x}{P}$$

③ Eccentricity of C-line.

$$e_c = (\bar{x} - e_p)$$

④

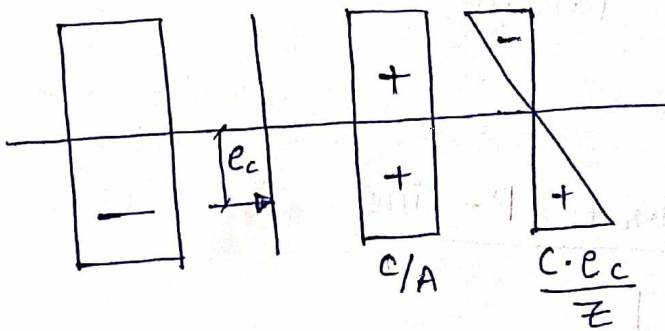


Case 1: if \$e_c\$ is above NA.

$\underline{\text{stress at top}} = \frac{C}{A} + \frac{C \cdot e_c}{Z_t}$
$\underline{\text{stress at bottom}} = \frac{C}{A} - \frac{C \cdot e_c}{Z_B}$

No effect of Moment ^{due to lead} is to be added here.

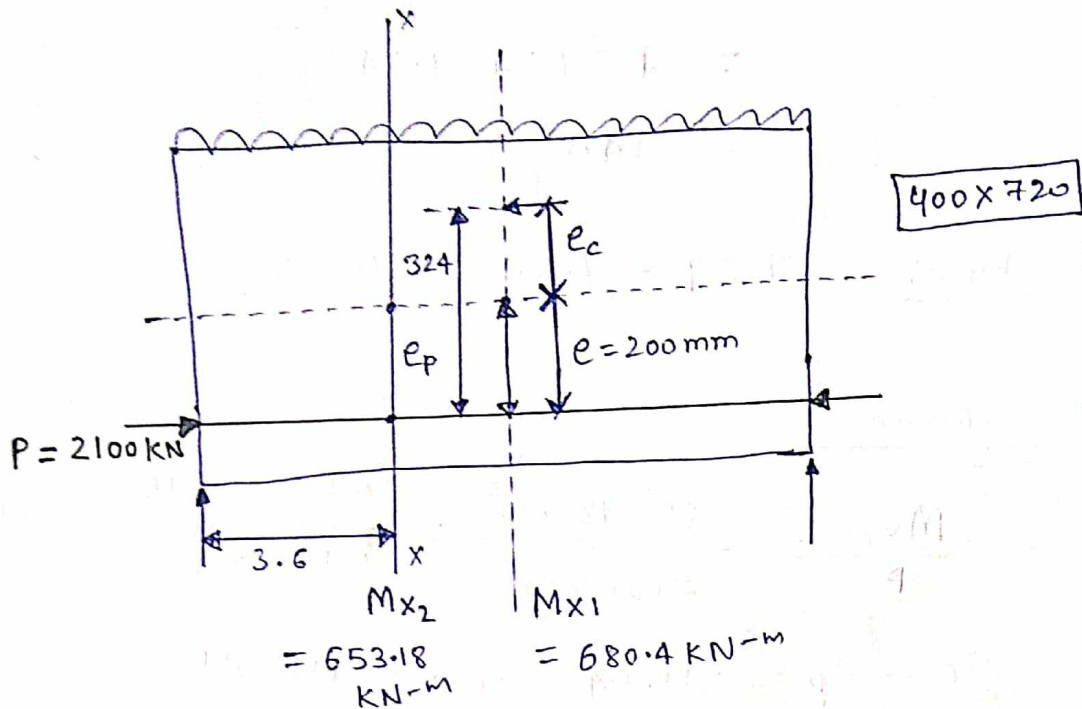
Case 2: if \$e_c\$ is below NA.



$\text{stress at top} = \frac{C}{A} - \frac{C \cdot e_c}{Z}$
$\text{at bottom} = \frac{C}{A} + \frac{C \cdot e_c}{Z}$

Q1. Solve by strength Concept.

- (i) at mid span.
- (ii) at 3.6 m from support.



(i) at mid span.

- (i) $M_x = 680.40 \text{ kN-m}$
- (ii) shift of C-line w.r.t P-line

$$\bar{x} = \frac{M_x}{P} = \frac{680.40 \times 10^6}{2100 \times 10^3} = 324 \text{ mm}$$

(iii) Eccentricity of C-force.

$$e_c = \bar{x} - e_p = 324 - 200 = 124 \text{ mm} \quad \underline{\text{above NA}}$$

(IV) Steps

$$\text{at top: } \frac{C}{A} + \frac{C \cdot e_c}{Z} = \frac{2100 \times 10^3}{400 \times 720} + \frac{2100 \times 10^3 \times 124}{400 \times \frac{720^2}{6}}$$

$$= 7.29 + 7.54$$

$$= \underline{14.83}$$

$$\text{at bottom: } = 7.29 - 7.54 = \underline{-0.25}$$

(V) at 3.60 m

$$(i) \bar{x} = \frac{M \times 2}{P} = \frac{653.18 \times 10^6}{2100 \times 10^3} = \frac{653.18 \times 10^6}{2100 \times 10^3} = 311.04 \text{ mm}$$

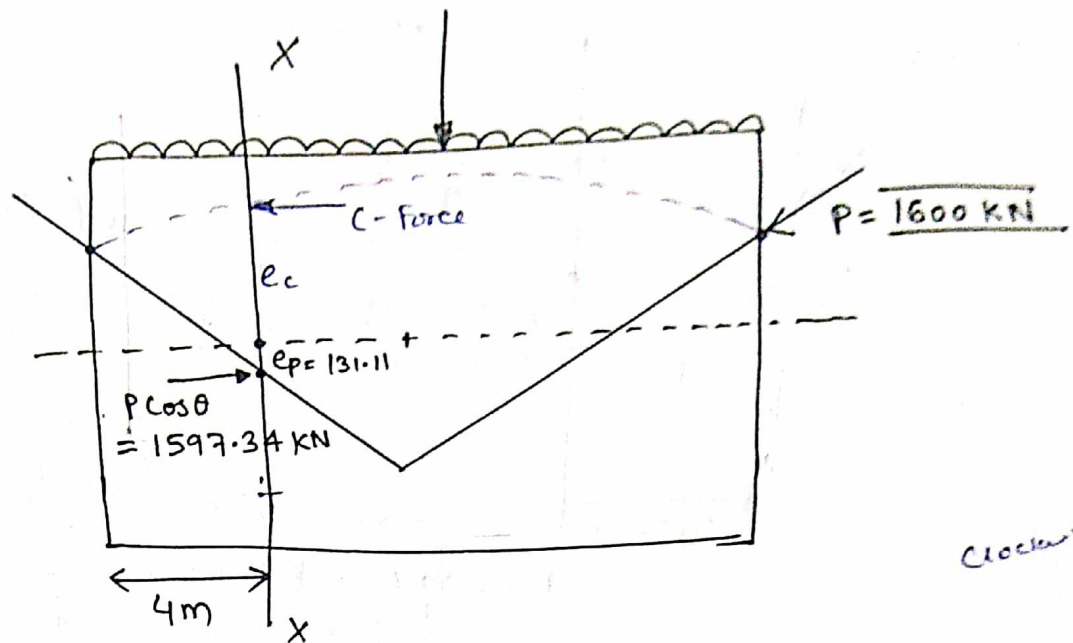
$$(ii) e_c = \bar{x} - e_p = 311.04 - 200 = 111.04$$

$$(iii) \text{at top: } = \frac{2100 \times 10^3}{400 \times 720} + \frac{2100 \times 10^3 \times 111.04}{400 \times \frac{720^2}{6}}$$

$$= 7.29 + 6.75 = 14.04$$

$$\text{at bottom} = 7.29 - 6.75 = 0.54.$$

Q2.



$$M_x = 556.90 \text{ kN-m}$$

Strength Concept:

$$\begin{aligned} \textcircled{1} \quad \bar{x} &= \frac{M_x}{P \cos \theta} = \frac{556.90 \times 10^6}{1597.34 \times 10^3} \\ &= \underline{348.642 \text{ mm}} \end{aligned}$$

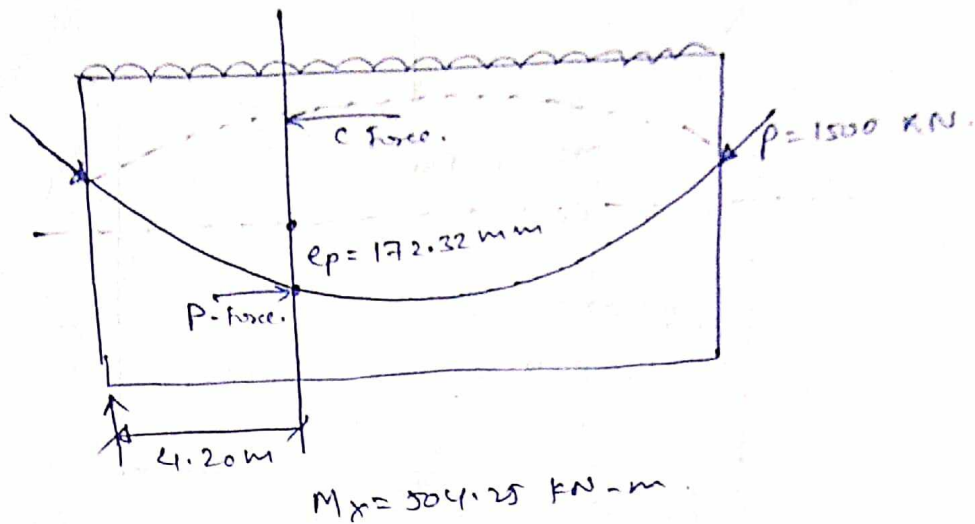
$$\begin{aligned} \textcircled{2} \quad e_c &= \bar{x} - e_p \\ &= 348.642 - 131.11 \\ &= \underline{217.532 \text{ mm}} \quad \underline{\text{above NA}} \end{aligned}$$

③ Stress

$$\begin{aligned} \text{at top} &= \frac{C}{A} + \frac{C \cdot e}{Z} = \frac{1597.34 \times 10^3}{350 \times 600} + \frac{1597.34 \times 10^3 \times 217.532}{350 \times \frac{650^2}{6}} \\ &= 7.02 + 14.10 = \underline{21.12 \text{ N/mm}^2} \end{aligned}$$

$$\text{at bottom} = 7.02 - 14.10 = -7.08 \text{ N/mm}^2$$

Q.3



$$\textcircled{1} \quad \bar{x} = \frac{M_x}{P} = \frac{504.25 \times 10^6}{1500 \times 10^3}$$
$$= 336.167 \text{ mm.}$$

$$\textcircled{2} \quad e_c = \bar{x} - e_p$$
$$= 336.167 - 172.32$$
$$= 163.85 \text{ mm.}$$

$$\frac{C}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$$

$$\frac{C \cdot e_c}{I} = \frac{1500 \times 10^3 \times 163.85}{320 \times 800^2 / 6} = 7.20$$

$$\text{at top} = 5.86 + 7.20 = 13.06 \text{ N/mm}^2$$

$$\text{at bottom} = 5.86 - 7.20 = -1.34 \text{ N/mm}^2$$

Q4 at midspan (T-beam)

$A = 330000 \text{ mm}^2$

$I = 2.45 \times 10^{10} \text{ mm}^4$

$\bar{y}_t = 386.36 \text{ mm}$

$y_D = 513.64 \text{ mm}$

$M_d = 148.50 \text{ kN-m}$

$M_x = 720 \text{ kN-m}$

$P = 2240 \text{ kN}$

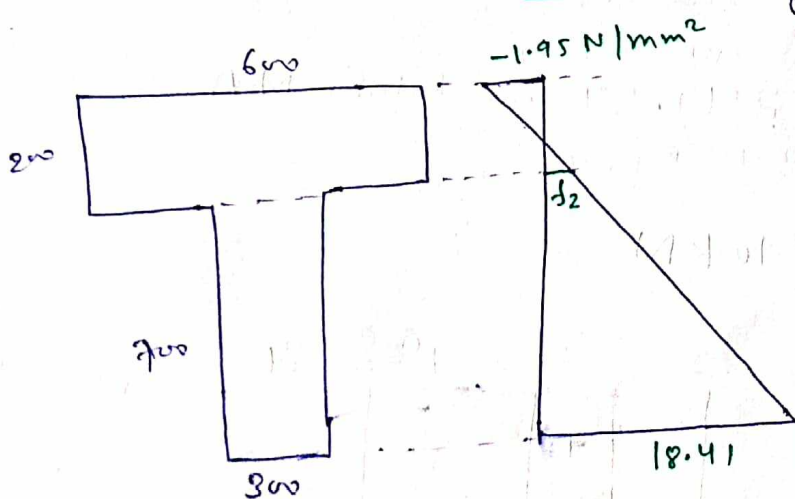
$k = 100 = 20 \%$

$P_f = 2240 \times 8$

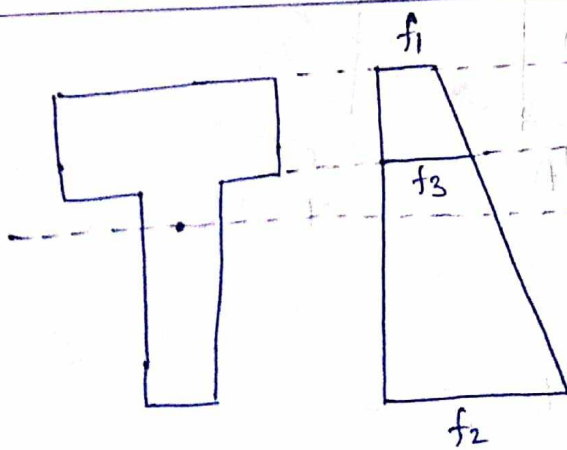
$= 1792 \text{ kN}$

$e = 313.64 \text{ mm}$

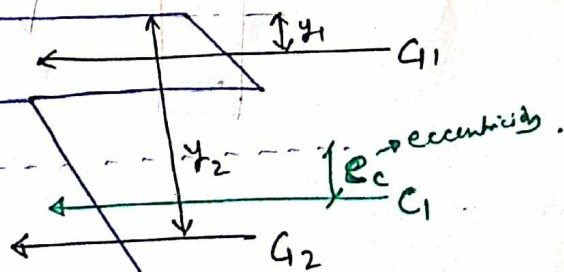
at T-center:



Very complex



Force diagram



(#) (i) At Transfer

$$M_x = 148.50 \text{ kN-m}$$

(ii) Shift of c-line w.r.t p-line

$$\bar{x} = \frac{M_x}{P} = \frac{148.50 \times 10^6}{2240 \times 10^3} = 66.295 \text{ mm.}$$

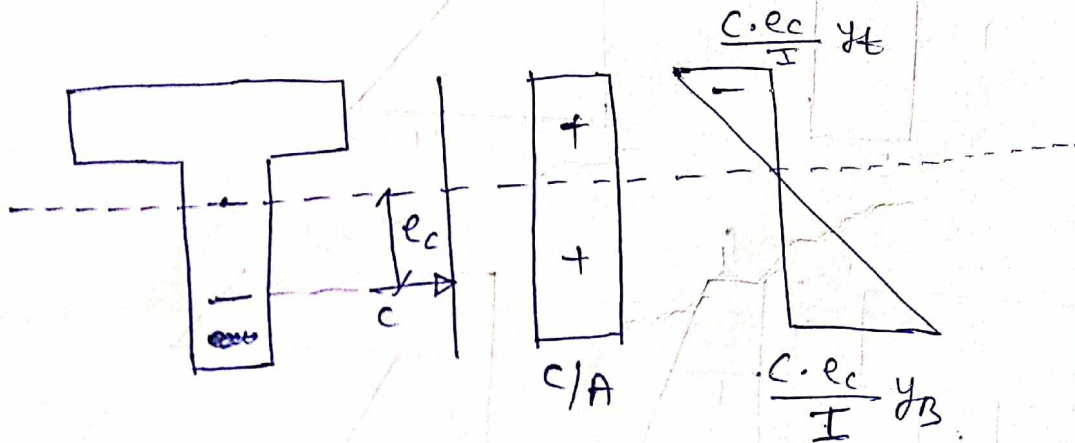
(iii) Eccentricity of c-force.

$$\begin{aligned} e_c &= \bar{x} - e_p \\ &= 66.295 - 313.64 \text{ mm} \\ &= -247.345 \text{ mm.} \end{aligned}$$

i.e. = 247.345 mm below N.A

$$C = P = 2240 \text{ kN}$$

(iv)



Stress at top.

$$\begin{aligned} &= \frac{C}{A} - \frac{C \cdot e_c}{I} y_t \\ &= \frac{2240 \times 10^3}{33000} - \frac{2240 \times 10^3 \times 247.345}{2.45 \times 10^6} \times 386.36 \\ &= \end{aligned}$$

$$= 6.79 - 8.74 = -1.95 \text{ N/mm}^2.$$

Stress at Bottom.

$$= \frac{2240 \times 10^3}{330000} + \frac{2240 \times 10^3 \times 240.345}{2.45 \times 10^{10}} \times 513.64$$

$$= 6.79 + 11.62$$

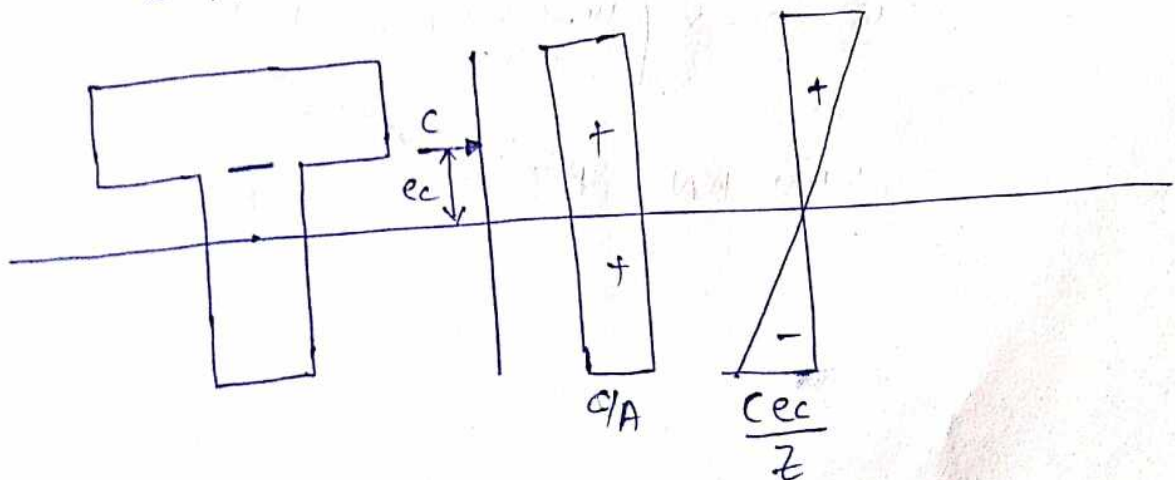
$$= \underline{18.41 \text{ N/mm}^2}$$

② At final stage.

① Total Moment = $148.5 + 720$
 $= 868.50 \text{ kN-m.}$

② $\bar{x} = \frac{868.50 \times 10^6}{1792 \times 10^3} = 484.654 \text{ mm}$

③ $e_c = \bar{x} - e_p$
 $= 484.654 - 313.64$
 $= 171.014 \text{ (above NA)}$



Stresses at top.

$$= \frac{C}{A} + \frac{C \cdot e_c}{I} \cdot y_t$$

$$= \frac{1792 \times 10^3}{330000} + \frac{1792 \times 10^3 \times 171.014}{2.45 \times 10^{10}} \times 386.36$$

$$= 5.43 + 4.83 = 10.26$$

Stress at Bottom -

$$= \frac{1792 \times 10^3}{330000} - \frac{1792 \times 10^3 \times 171.014}{2.45 \times 10^{10}} \times 513.64$$

$$= 5.43 - 6.42$$

$$= -0.99$$

Proof C Force = P Force.

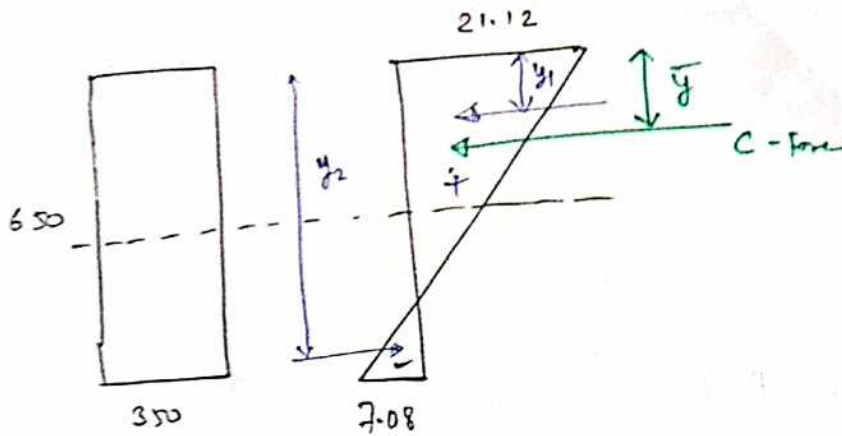
Ans 1

$$C = B \times D \times \left(\frac{f_1 + f_2}{2} \right)$$

$$= 400 \times 720 \times \left(\frac{14.825 + (-0.25)}{2} \right)$$

$$= 2100 \text{ kN} = P \text{ Force.}$$

Q2.



$$C\text{-force} = 350 \times 650 \times \left(\frac{21.12 - 7.08}{2} \right) \times \frac{1}{10^3}$$

$$= 1597 \text{ kN} = P_{\text{force}}$$

$$\bar{y} = \frac{f_1 + 2f_2}{f_1 + f_2} \times \frac{D}{3}$$

$$= \frac{21.12 + 2 \times (-7.08)}{21.12 + (-7.08)} \times \frac{650}{3}$$

$$= \underline{107.407 \text{ mm}}$$

$$e_c = \frac{D}{2} - \bar{y} = 325 - 107.407$$
$$= \underline{217.593}$$

\bar{y} can be directly calculated from this formula. Consider it a trapezoid. But put compressive tensile stress (ve) magnitude