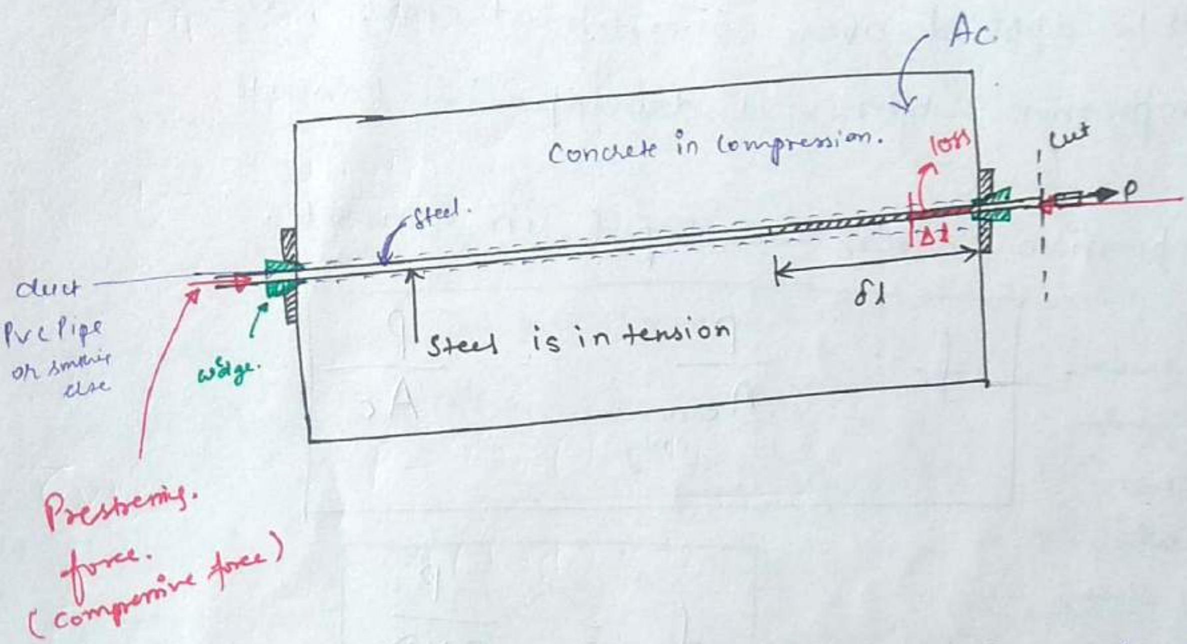
PRESTRESSED CONCRETE

Basic Concept :

Let us consider ex^am^ple of a Post tension member.



→ If a steel reinforcement Provided in a duct in tension-ed at ends and fixed after providing an elongation.

→ steel Reinforcement is in tension. Elongation provided

$= \delta l$

→ Stress, stain etc is due to δl.

→ Strain provided in steel $= \frac{\delta l}{l} = e$

→ Stress developed in steel.

$$\boxed{p = e \times E_s = \left(\frac{\delta l}{l} \times E_s\right)}$$

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→ Total force developed in steel

$$P_s = p_s \times A_s = \frac{\delta l}{l} \times E_s \times A_s = P$$

This force is called <u>prestressing force</u>. This force will be applied over concrete at ends, due to which compressive stress will develope in concrete.

→ Compressive stress developed in concrete.

$$\boxed{f_c = \frac{P}{A_{\text{concrete only}}} = \frac{P}{A_c}}$$

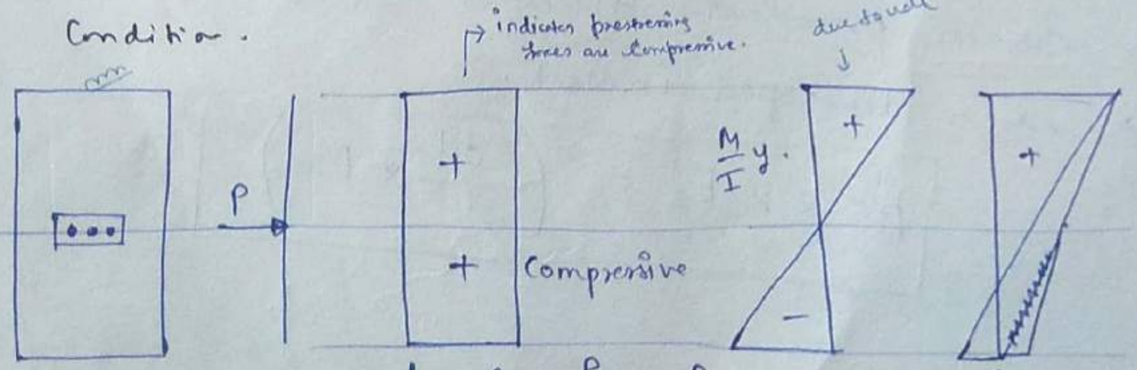
In prestress concrete. we do not consider. area of steel. in calculating compressive stress. cuz steel is in Tension ~~scoo~~ and it has developed Compressive force.

$$\boxed{f_c = \frac{P}{B \times D}}$$

Now To bear Compressive Force.

1st ~~we have to~~ Steel will go to '0' then it can bear Compressive stress.

In column equivalent area of conc. and steel was considered cuz both were initially in 0 stress condition.



$$p_0 = f_c = \frac{P}{A_c} = \frac{P}{B \cdot D}$$

Stress developed due to Moment

$$= \pm \frac{M}{I} y .$$

Total stress at any section.

at top = $\left(\frac{P}{A} + \frac{M}{I} y\right)$ Compressive (+ve)

at bottom = $\left(\frac{P}{A} - \frac{M}{I} y\right)$ $\begin{bmatrix} +ve \\ 0 \\ -ve \end{bmatrix}$

Stress due to moment are counter balanced by stresses developed due to prestressing force.

(II) If due to any reason, some part of the deformation provided is lost. say by "Δl". This will result in loss of stress in reinf^n. Due to which prestressing force will reduce.

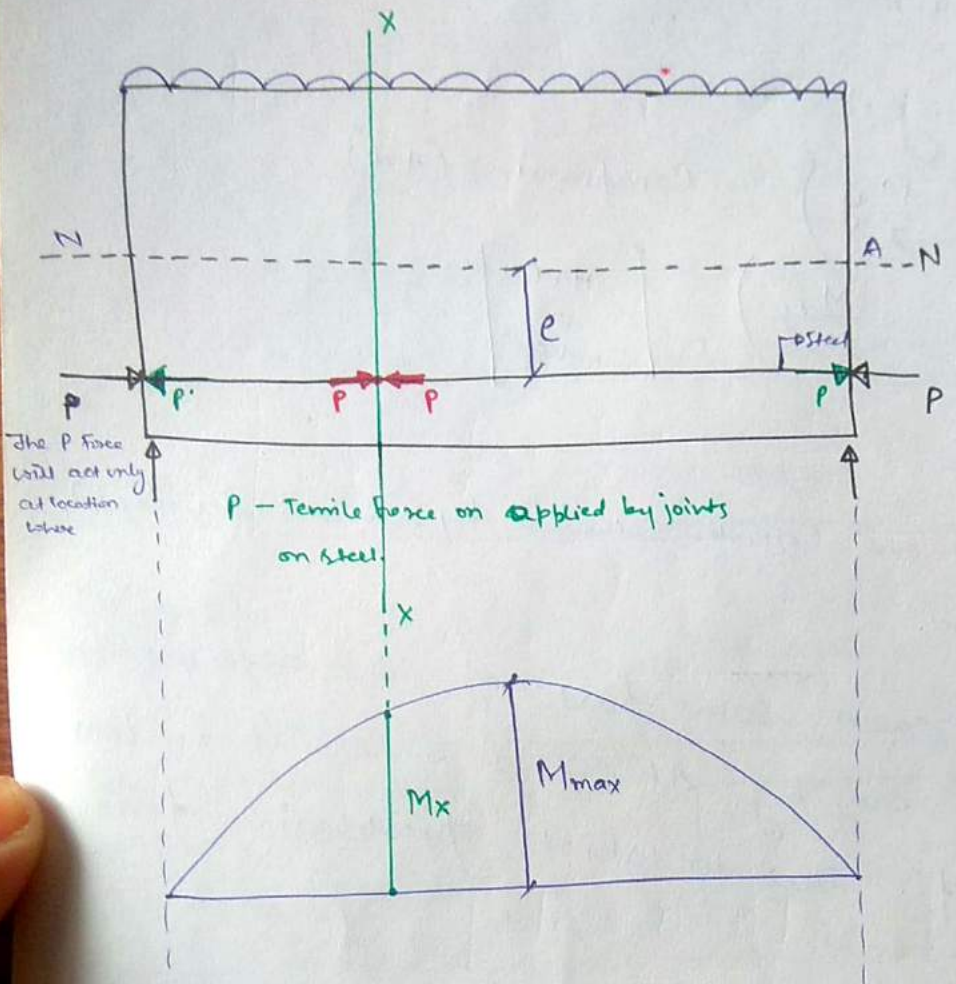
Loss of deformation $= \Delta l$

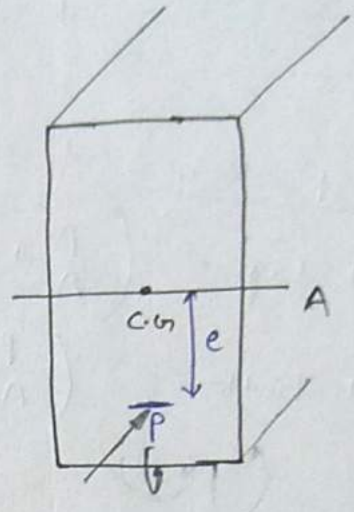
$$\boxed{\text{loss of stress} = \left(\frac{\Delta l}{l}\right) \times E_s}$$

Remaining stress is reinf^n.

$$\boxed{= \frac{(\delta l - \Delta l)}{l} \times E_s .}$$

Case 1 Reinf^{n}. Provided at an eccentricity.



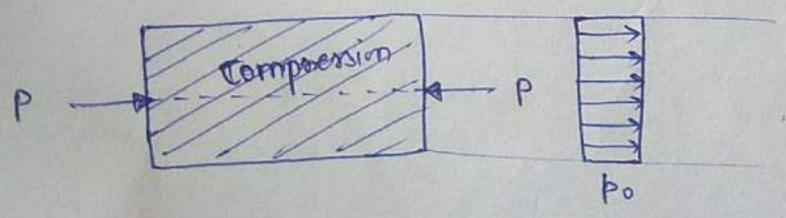


NA is CG of only Concrete area.

Stress Concept Method.

At any cross-section the beam is subjected to stresses due to three reasons.

① Direct stress due to P-Force.



$$p_o = (+)\ \frac{P}{A}$$

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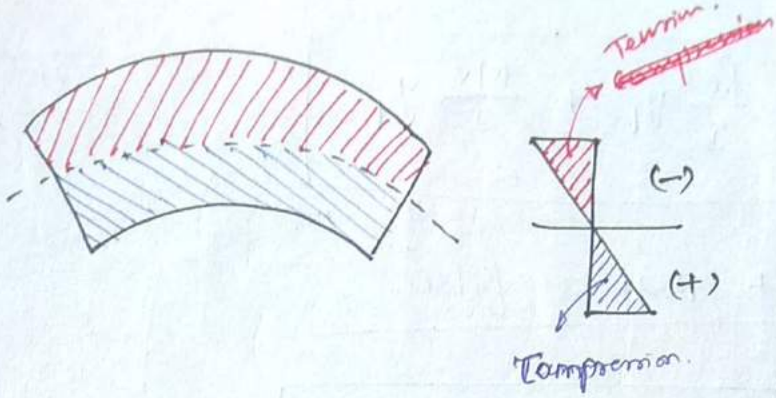
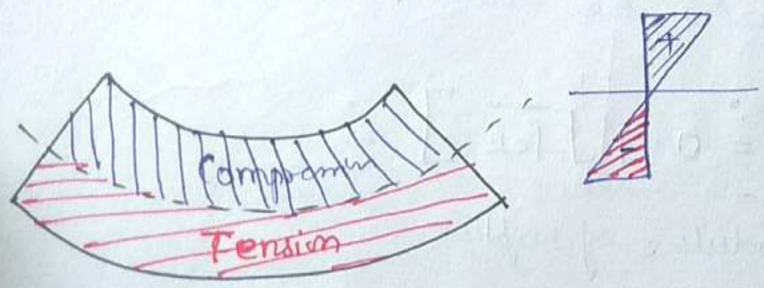
② Bending stress due to Prestressing force / eccentricity.



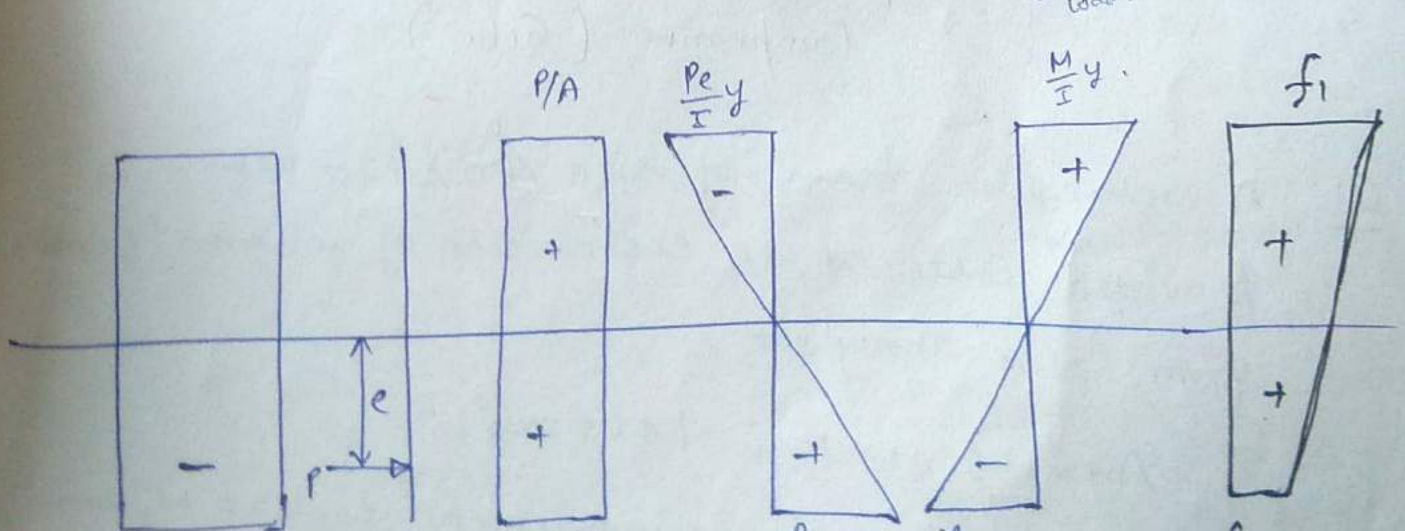
Stress

$$f_{b_1} = (\mp)\ \frac{P.e}{I}.y.$$

③ stress due to Load (Bending stresses)



Total stress



↓

is develope due to Prestressing force.

→ Prestressing force develops, stress [illegible] to stress [illegible]

Total stresses at top fibre.

$$\boxed{f_1 = \frac{P}{A} - \frac{Pe}{I} y_t + \frac{M_x}{I} y_t}$$

y_t - dist upto top fibre and Neutral axis

Total stress at Bottom fibre.

$$\boxed{f_2 = \frac{P}{A} + \frac{Pe}{I} y_B - \frac{M_x}{I} y_B}$$

M_x = Moment at the section.

(-ve) stress may also be allowed but value.

$$< \left[f_{cr} = 0.7 \sqrt{f_{ck}} \right]$$

modulus of rupture.

(+) value < permissible stress in concrete is Compressive (σ_{cbc})

Q1. A rectangular beam of size $\overset{B}{400} \times \overset{D}{720}$ mm is provided Cable at an eccentricity of 200mm (Const.) from N.A Throughout:

Area of steel is 1400 mm^2.

Initial stress in reinforcement = 1500 N/mm^2.

The beam is subjected to a live load of 60 KN/m over a simply supported effective span of 9m.

Calculate stresses in the beam —

(I) At ends.

(II) At mid span

(III) At 3.60 m from support

Neglect any loss of prestress in reinforcement.

$$f_1 = \frac{P}{A} - \frac{P.e.}{I} \cdot y_t + \frac{M_x}{I} y_t.$$

$$I = \frac{bd^3}{12}$$

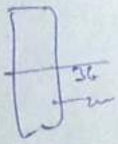
$$= \frac{400 \times 720^3}{12} = 1.24 \times 10^{10}$$

$$P = 21 \times 10^5 \text{ N}$$

$$= \frac{21 \times 10^5}{400 \times 720} - \frac{21 \times 10^5 \times 200}{1.24 \times 10^{10}} \times 480$$

720

$$\boxed{f_1 = -8.966}$$



$$f_2 = 15.42$$

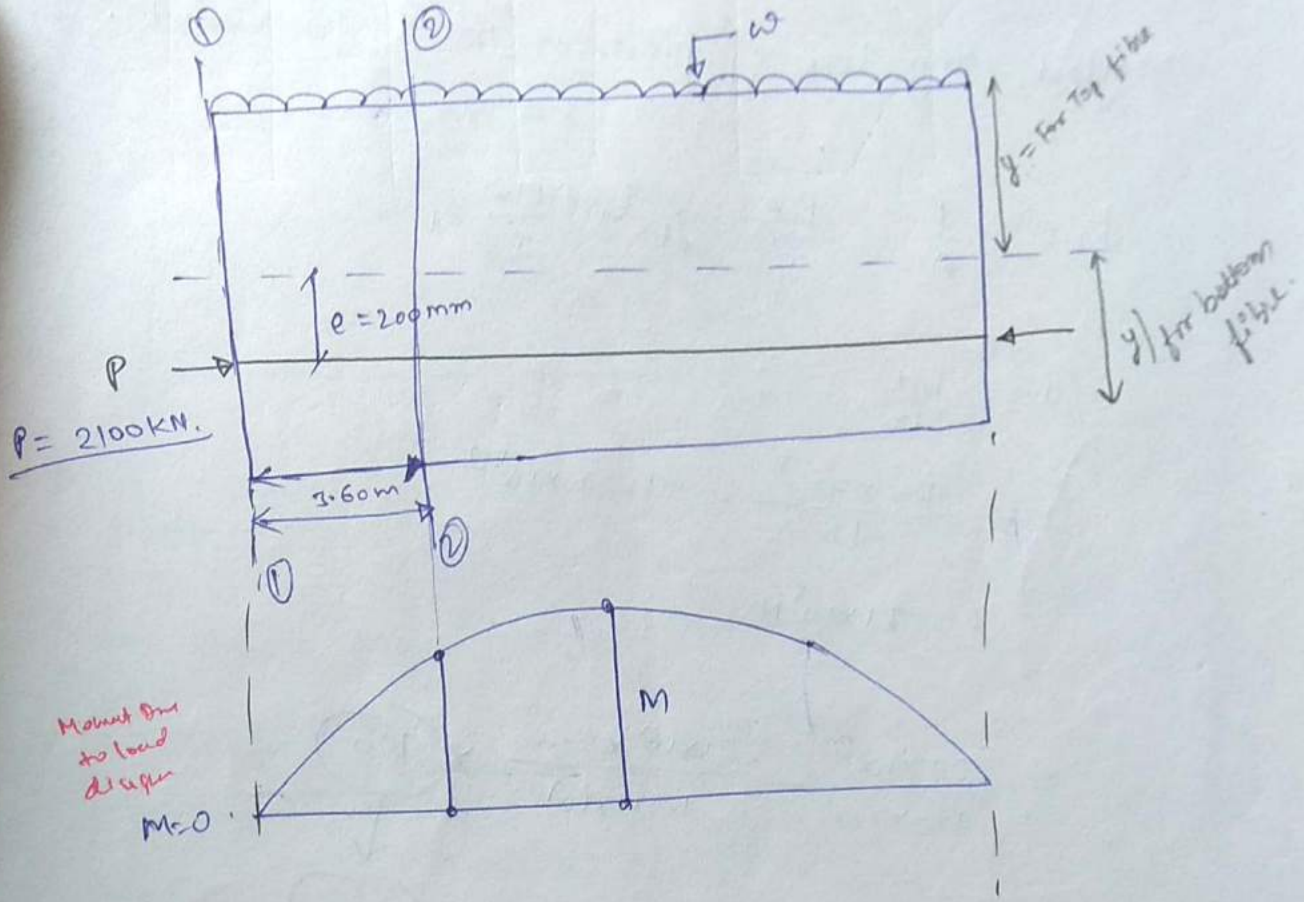
$\frac{11 \times 2^3}{12}$

Sol^n

① Prestressed forces.

$$P = A_s \cdot f_s = 1400 \times \frac{1500}{10^3} = \underline{2100\ kN}$$

$e = 200mm$



$\Rightarrow$ $A = 400 \times 720$

$I = 400 \times 720^3/12$

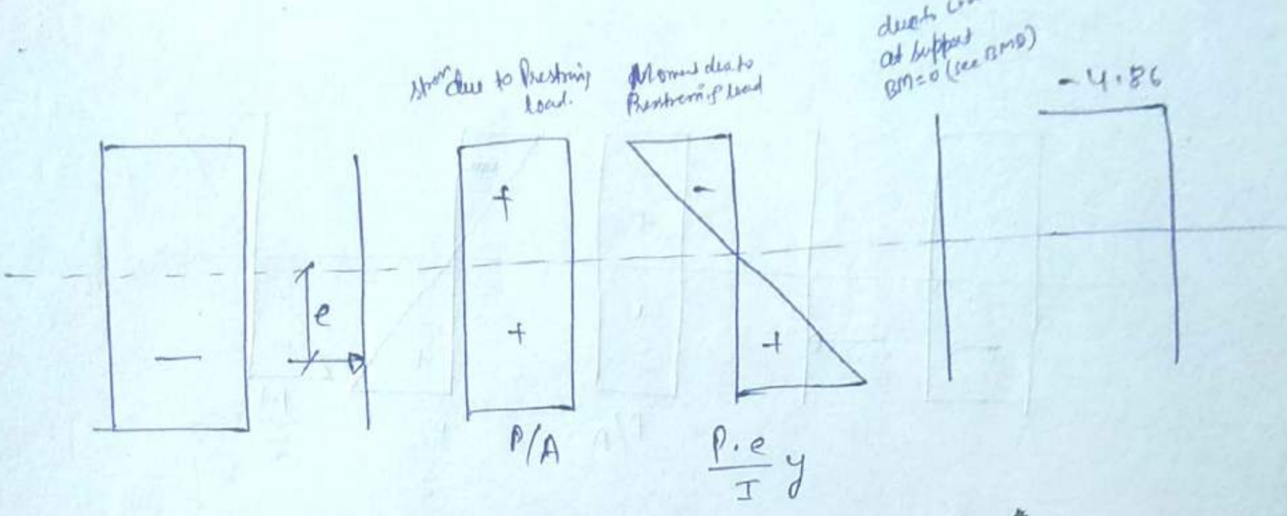
$Z = 400 \times 720^2/6$

$D.L = 0.40 \times 0.72 \times 1 \times 25 = 7.20\ kN/m$

$L.L = \qquad\qquad = 60.0\ kN/m$

$w = \underline{67.20\ kN/m}$

(i) At ends.



Stress at top.

$$\Rightarrow \frac{2100\times10^3}{400\times720} - \frac{2100\times10^3\times200}{400\times\frac{720^3}{12}}\times\left(\frac{720}{2}\right)$$

$$= \frac{P}{A} - \frac{Pe}{I}y.$$

$$= \frac{P}{A} - \frac{P.e}{Z} =$$

$$= \frac{2100\times10^3}{400\times720} - \frac{2100\times10^3\times200}{400\times720^2/6}$$

$\frac{I}{y} =$

$$= 7.29 - 12.15$$

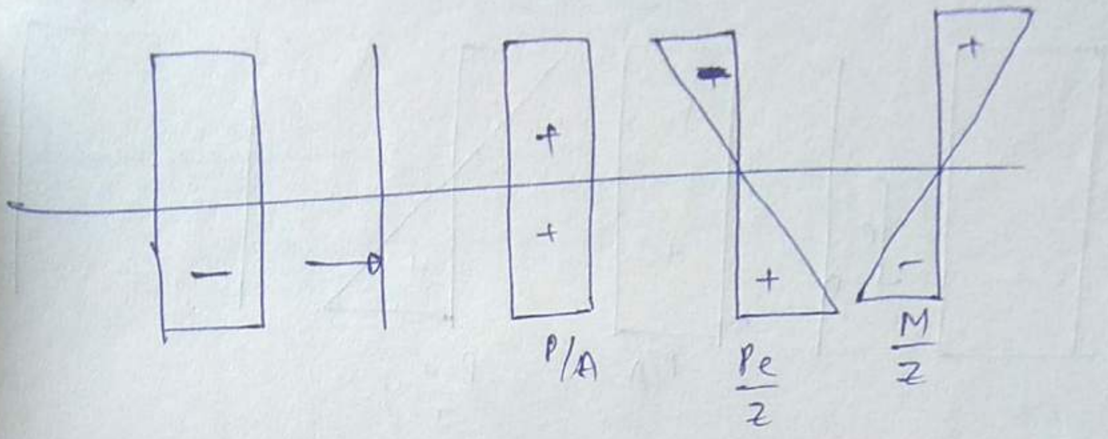
$$= -4.86$$

Stress at Bottom.

$$= \frac{P}{A} + \frac{Pe}{Z}$$

$$= 7.29 + 12.15 = 19.44 \ N/mm^2$$

② At mid span.



Moment at mid span

$$= \frac{Wl^2}{8} = 67.2 \times \frac{9^2}{8} = 680.40 \text{ kN-m.}$$

$$\frac{M}{Z} = \frac{680.4 \times 10^6}{400 \times 720^2/6} = \pm 19.69 \text{ N/mm}^2.$$

Stress at top.

$$= \frac{P}{A} - \frac{P.e}{Z} + \frac{M}{Z}$$

$$= 7.29 - 12.15 + 19.69 = 14.83 \text{ N/mm}^2$$

Stress at bottom.

$$= \frac{P}{A} + \frac{Pe}{Z} - \frac{M}{Z}$$

$$= 7.29 + 12.15 - 19.69 = -0.25 \text{ N/mm}^2.$$

③ At 3.6 m from support.

$$R_1 = R_2 = \frac{wl}{2} = 67.2\cancel{8} \times \frac{9}{2} = 302.4 \text{ kN}$$

$$\left\{ M_x = R_1 \cdot x - \frac{wx^2}{2} \right\}$$

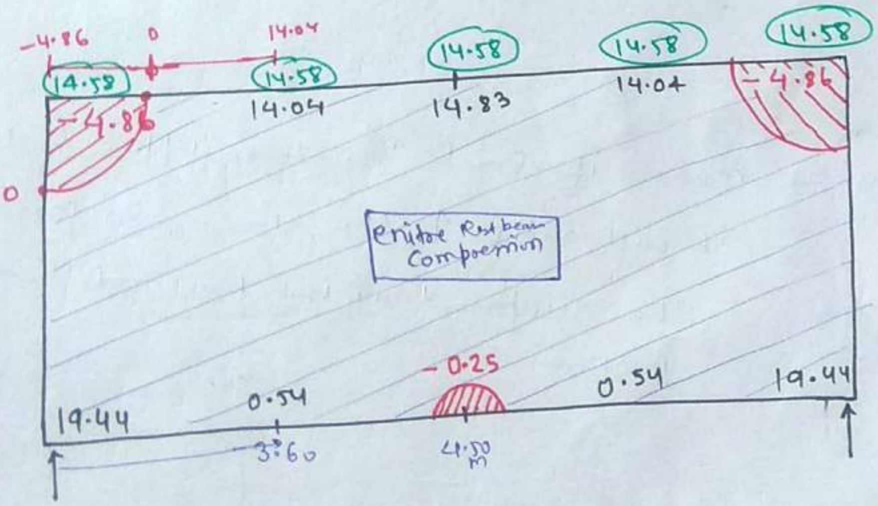
$$= 302.4 \times 3.60 - 67.2 \times \frac{3.60^2}{2}$$

$$= 653.18 \text{ kN-m}$$

$$\frac{M_x}{Z} = \frac{653.18 \times 10^6}{\frac{400 \times 720^2}{6}} = 18.90$$

Stress at top: $= 7.29 - 12.15 + 18.90 = 14.04 \text{ N/mm}^2$

at bottom $= 7.29 + 12.15 - 18.90 = 0.54 \text{ N/mm}^2$



$$\begin{array}{r} -4.86 \\ +\ 19.44 \\ \hline 14.58 \end{array}$$

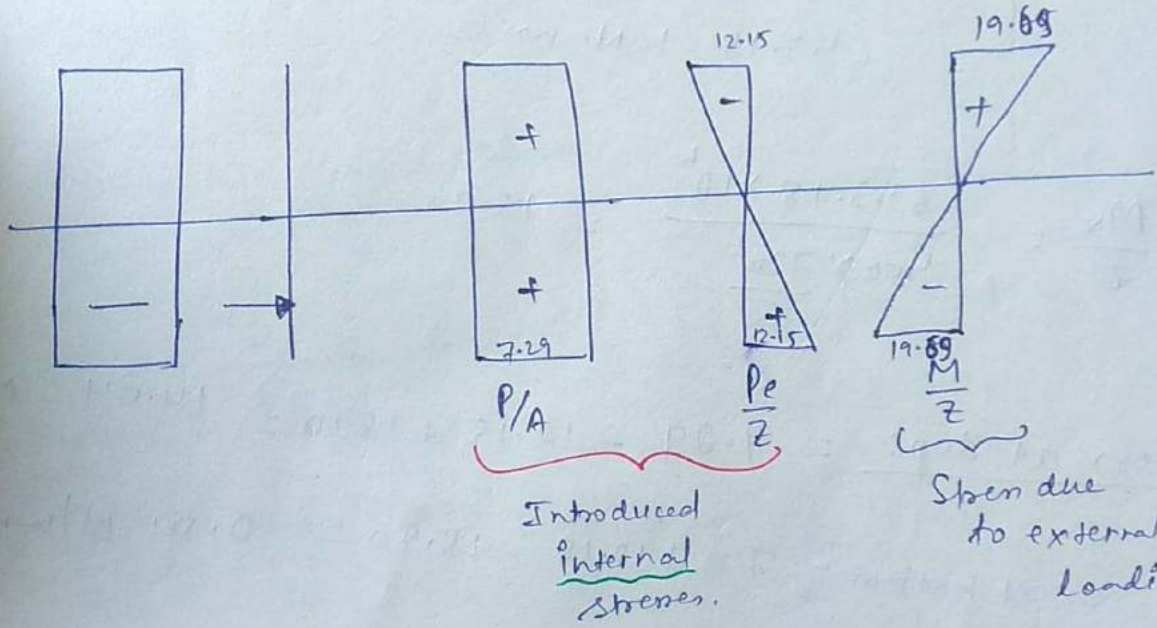
$$\begin{array}{r} 14.04 \\ 0.54 \\ \hline 14.58 \end{array}$$

$$\begin{array}{r} 14.83 \\ -0.25 \\ \hline 14.58 \end{array}$$

(7) Prestressed Concrete.

Def:

"Prestressed Concrete is the one, in which there have been introduced internal stresses of such magnitude and distribution so that the stresses resulting from External loading can be Counterbalanced upto a desired degree".



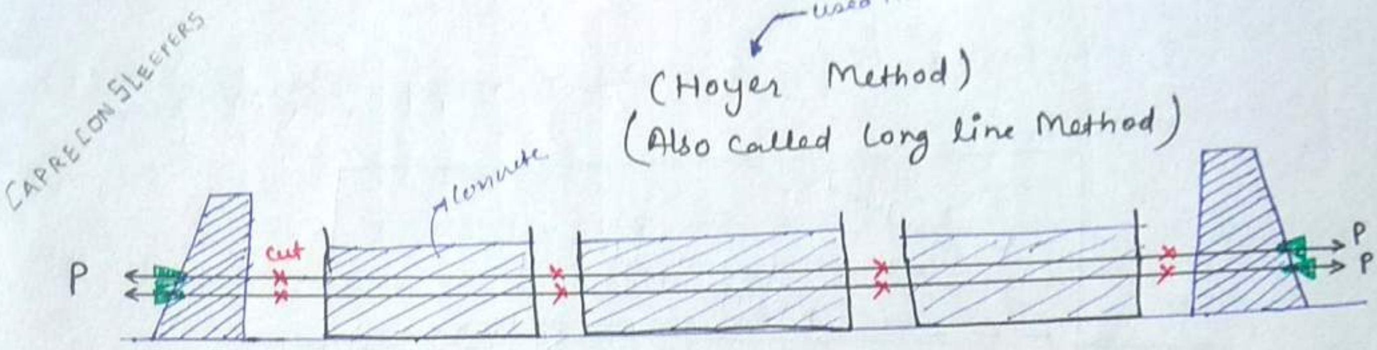
{distribution – means if Reinf Provide on upper side it will rather ↑ the stress. So provide the reinforcement distribution at lower portion.}

Type of Prestressing.

① Pretensioned Prestressed Concrete.

used in Railway sleepers

(Hoyer Method)
(Also called Long line Method)



① Tensioning :- Reinf^n tensioned and fixed.

② Casting :- Concrete is Casted. Reinf^n is kept in direct contact with concrete.

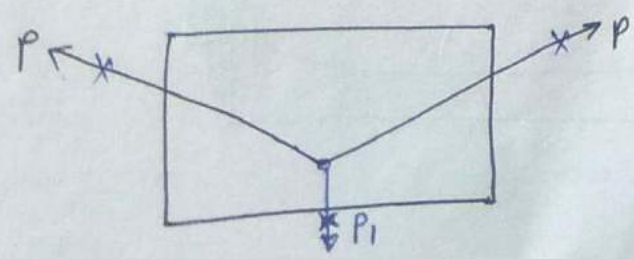
→ Concrete allowed to get proper strength.

③ Transfer of prestressing force. :- By cutting the wire.

Note: In this bond is developed which provides compression in Concrete.

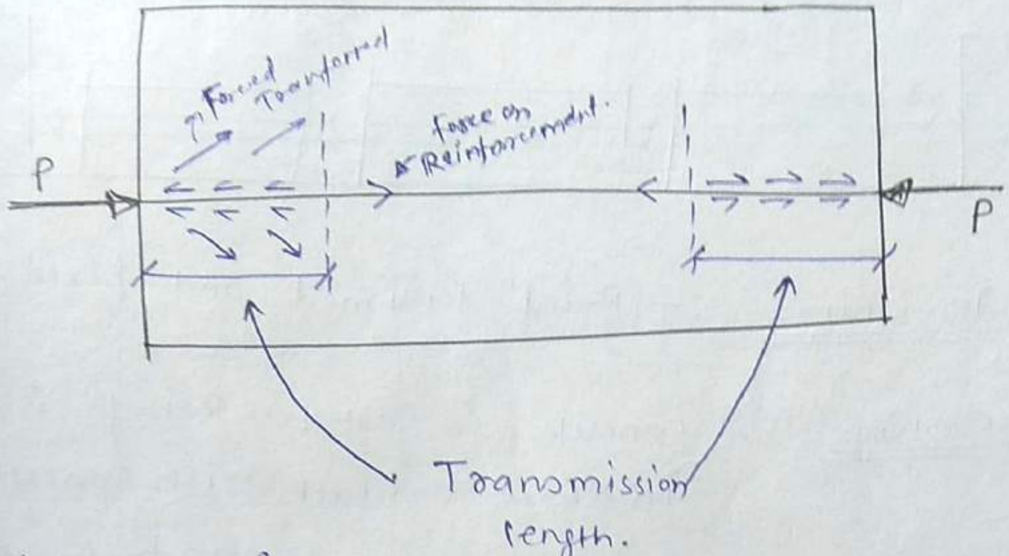
In pretension Member

Cables may be mostly - straight
→ may be bent also in some cases.

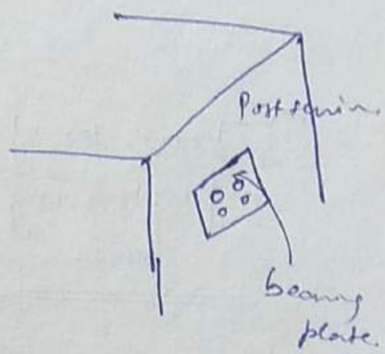


In Case of pretention.

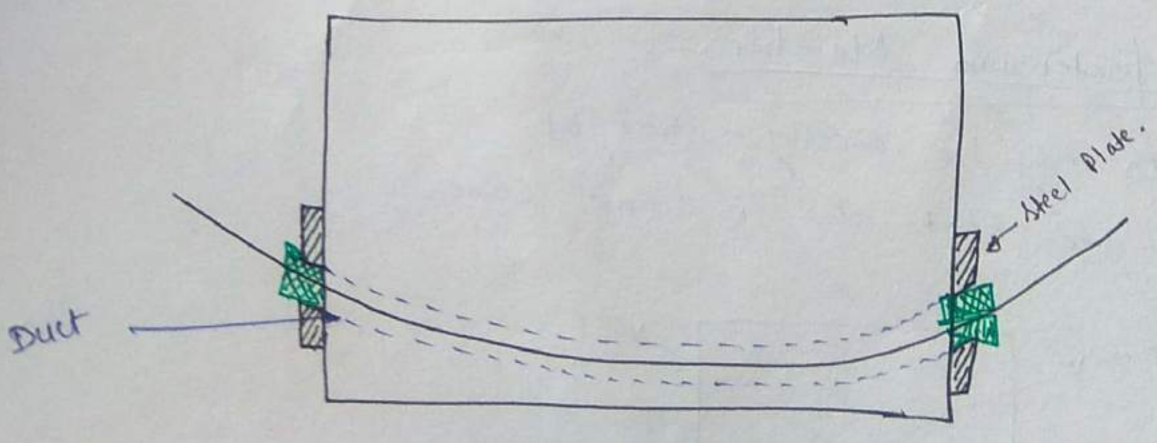
→ Transmission length is required at ends to transfer the prestressing force in Concrete by bond action.



* This method is used for repetitive type member for "factory production"



(2) Post Tensioned Members.



(1) <u>Casting</u>: Beam of required shape Casted by providing ducts at suitable location. Any shape is possible for Cable.

(2) <u>Tensioning</u>: After getting sufficient strength in concrete. reinforcement is provided in duct and tensioned at ends.

There are two methods:-

(1) Tension the Reinforcement from one end.
(2) Tensioning from both ends.

loss will be diff. in these two cases.

(3) <u>Anchoring</u>: After tensioning Reinforcement are anchored at ends.

There are many methods.

(I) Fressinet Method.
(II) Magnel Blatton Method.
(III) Le-mcall method.
(IV) Gifford udall method.

See fig. in book.

<u>This method</u>

→ used for cast in suit work.
→ large size members can be Casted at site.
→ stress transfer over Concrete is by bearing action.

Force is transferred by a steel plate fixed over Concrete at ends.

01/09/2015

(#) In Prestressed Concrete

Very high strength steel and Concrete are used :-
High tensile strength is generally achieve by slightly ↑ the % of Carbon Content in steel as compared to mild steel. generally - 0.6 - 0.85% of Carbon.

Steel → strength = 1000 to 2000 N/mm^2
i.e Higher grade steel used.

Concrete → Pre-tensioned = min^m grade = M40 (min^m)

Post tensioned — M30 (min^m)

Generally Concrete M50 to M100.

Why higher grade used:

Reason: ① Steel
In steel loss of stress is very high

Total loss of stress due to all reason } = 200 to 300 N/mm^2

i.e why we use higher grade.

if use 250 etc. then all will be lost our purpose will not will served

Total loss of stress $\not>$ 10 to 20 % of p_o

200 $\not>$ 10 % of p_o

$$p_o \not< \frac{200}{0.10} = 2000 \text{ N/mm}^2$$

200 $\not>$ 20 % of p_o

$$p_o \not< \frac{200}{0.2} = 1000 \text{ N/mm}^2.$$

② **Concrete:** Due to high stress value in steel, stresses in Concrete due to bond stress or bearing stress will also be very high.

Due to this reason concrete is also kept of very high strength.

⑪ Analysis of prestressed Concrete.

Assumptions.

① Hook's law is valid:

$$\boxed{p_s = \frac{\delta l}{l} \times E_s}$$

for Steel and Concrete both hook's law is applicable.

② At any section, plane section before bending remains plane after bending

meaning → Strain diagram is linear.

③ In steel any change in stresses due to prestressing force and external loading is to be neglected

⇓

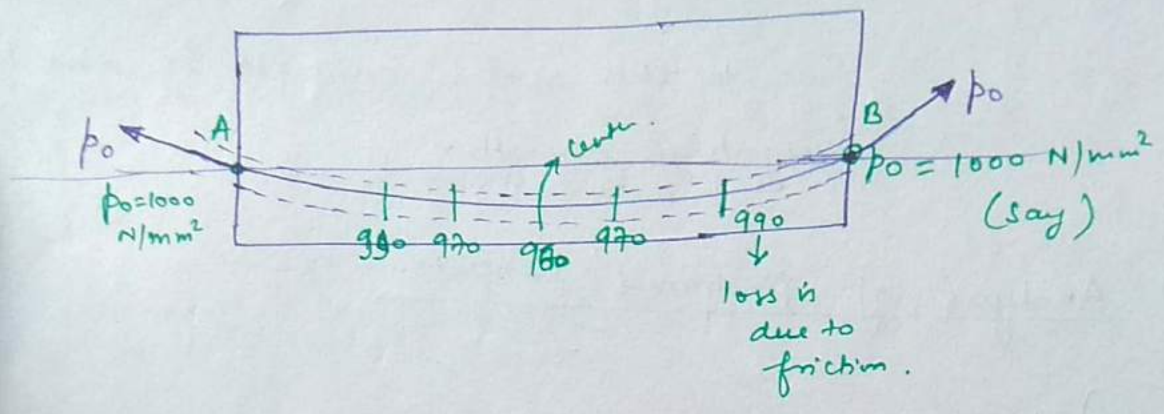
{ See Q6. of Copy. Result is very small Step⑤ }

At initial stage $= p_0$.

At final stage (Considering all losses)

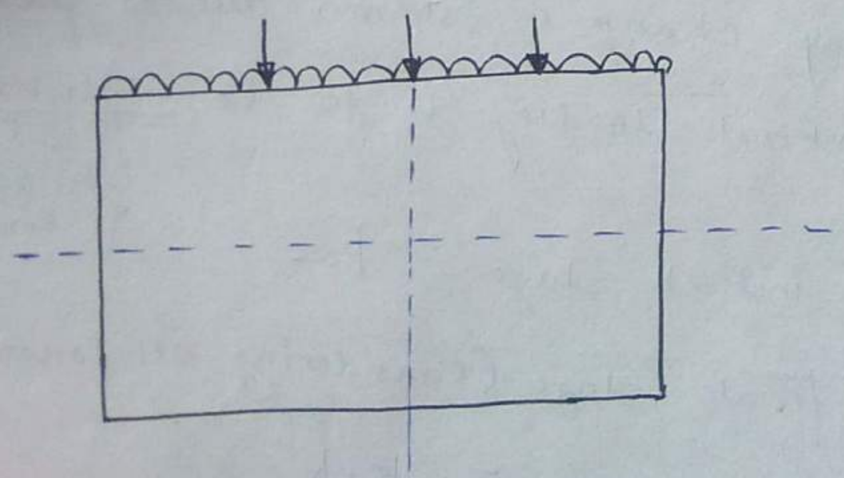
$= K \cdot p_0$

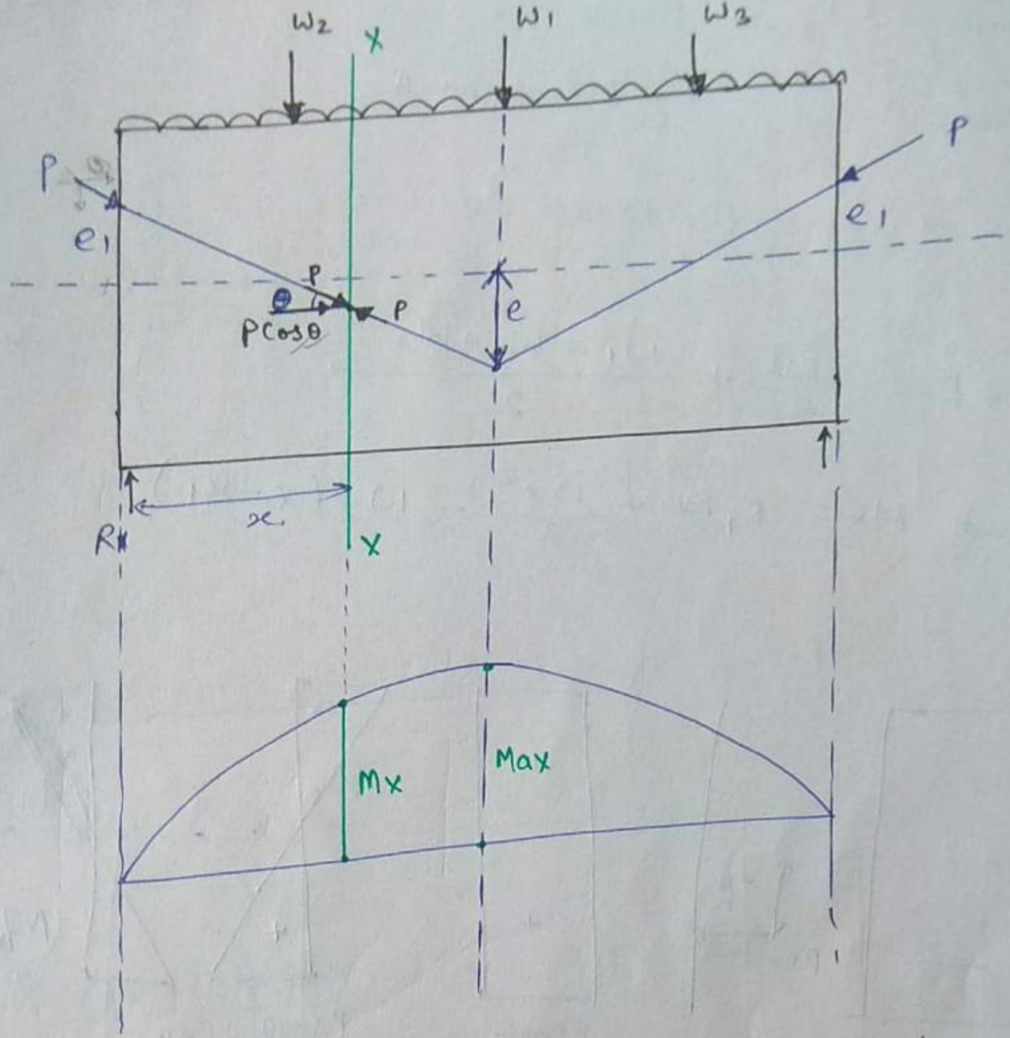
(4) Any change in stress in reinf^n along its length is also to be neglected.



<u>Note</u>: In case of if fix, reinf^n at 'A' and pulled or force Po applied only at B then more losses occur, cuz Po (=1000 say) then at Centre it will be 960, but further toward A ~~it~~ ~~...~~ losses will increase. ~~and~~ like 960, 950, 940, 930 ... and finally at A we get more losses.

Case 2 Pre-stressed Conc. Beam with bent Cable.





Stresses in beam at any section can be found by:

(I) stress concept Method.

(II) Load balancing Concept.

(III) strength Concept. (C-Line)

(I) stress Concept Method.

→ NO change

→ Stresses are found due to actual prestressing force acting at the location of Cable and due to actual moment at the section (without any load balancing). due to all loads.

means the location where you cut section and the location where you get cable line

① At – x–x. {i.e at a particular section}

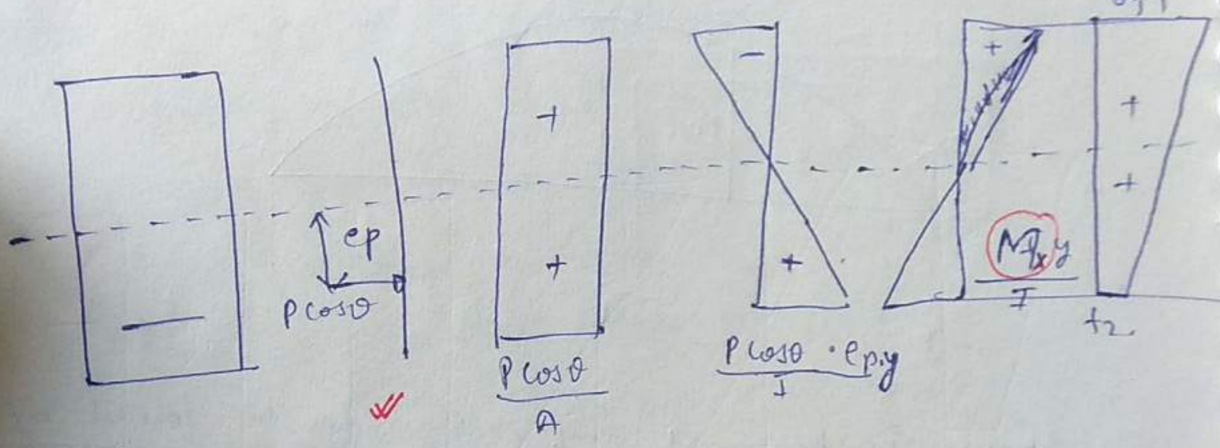
① Prestressing force = $P\cos\theta$.

② Location = e_q below N.A

③ Moment at x–x due to all loads.

$$R_1 = R_2 = \frac{wl}{2} + \frac{W_1 + W_2 + W_3}{2}$$

Moment $M_x = R_1 \cdot x - \frac{wx^2}{2} - W_2(x - x_1)$



Stress at top.

$$= \frac{P\cos\theta}{A} - \frac{P\cos\theta \cdot e_p}{I} \cdot y + \frac{M_x}{I} \cdot y = f_1$$

Stresses at Bottom.

$$= \frac{P\cos\theta}{A} + \frac{P\cos\theta \cdot Pe}{I} \cdot y - \frac{M_x}{I} y = f_2$$

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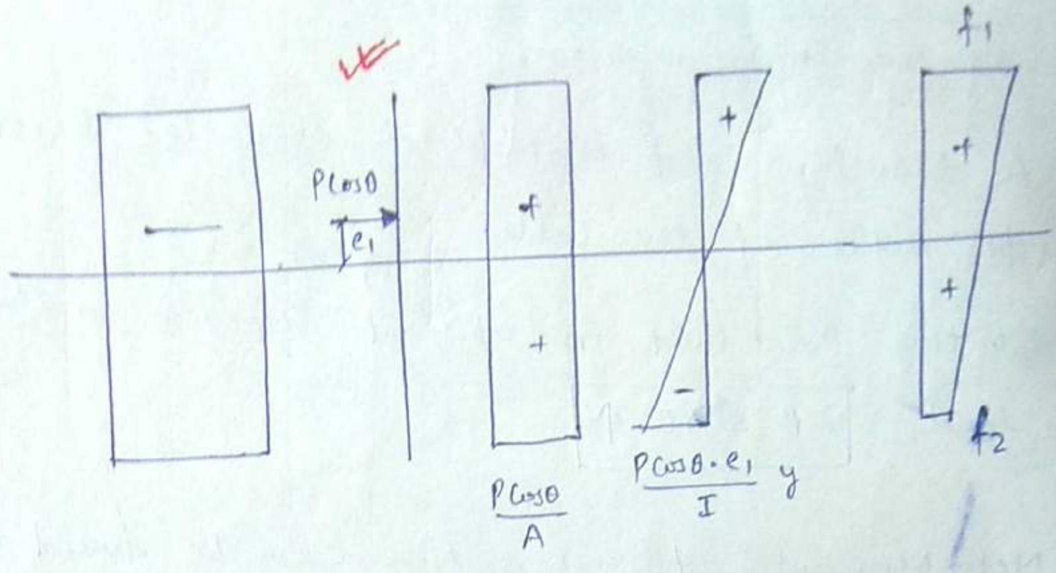
② At ends.



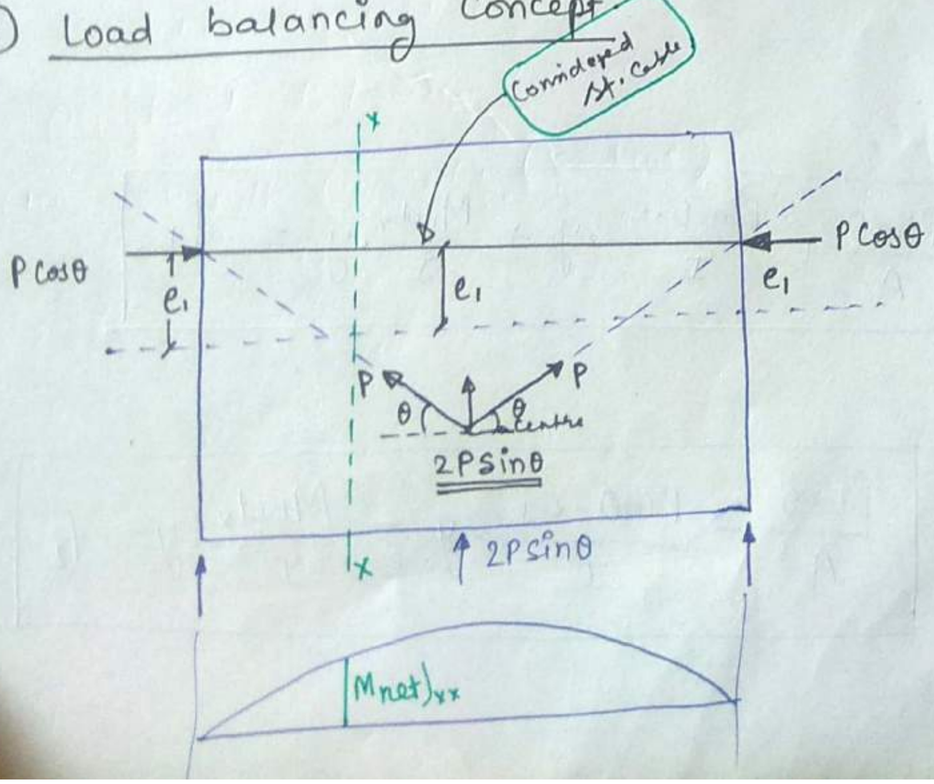
Stress at top.

$$= \frac{P\cos\theta}{A} + \frac{P\cos\theta \cdot e_1}{I} \cdot y.$$

Stress at Bottom

$$= \frac{P\cos\theta}{A} - \frac{P\cos e_1}{I} \cdot y$$

② Load balancing Concept.



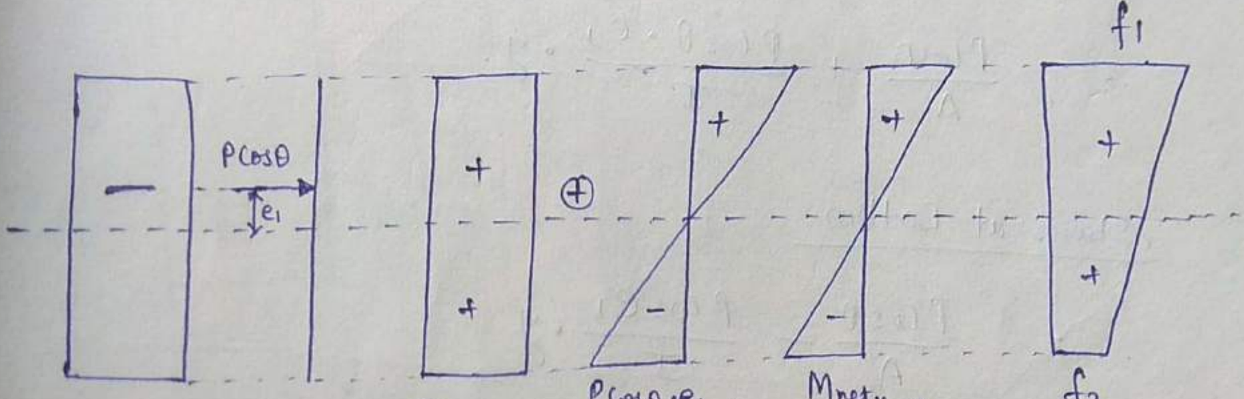
Load Blancing Concept.

(I) Cable profile is Considered straight from the points where ever it is at ends.

(II) A blancing load is applied over the beam as per profile of the Cable.

In this Point load in upward direction as balancing load = $\boxed{2P \sin\theta \uparrow}$

(III) Net Moment at any section can be found due to all loads i.e balancing load.



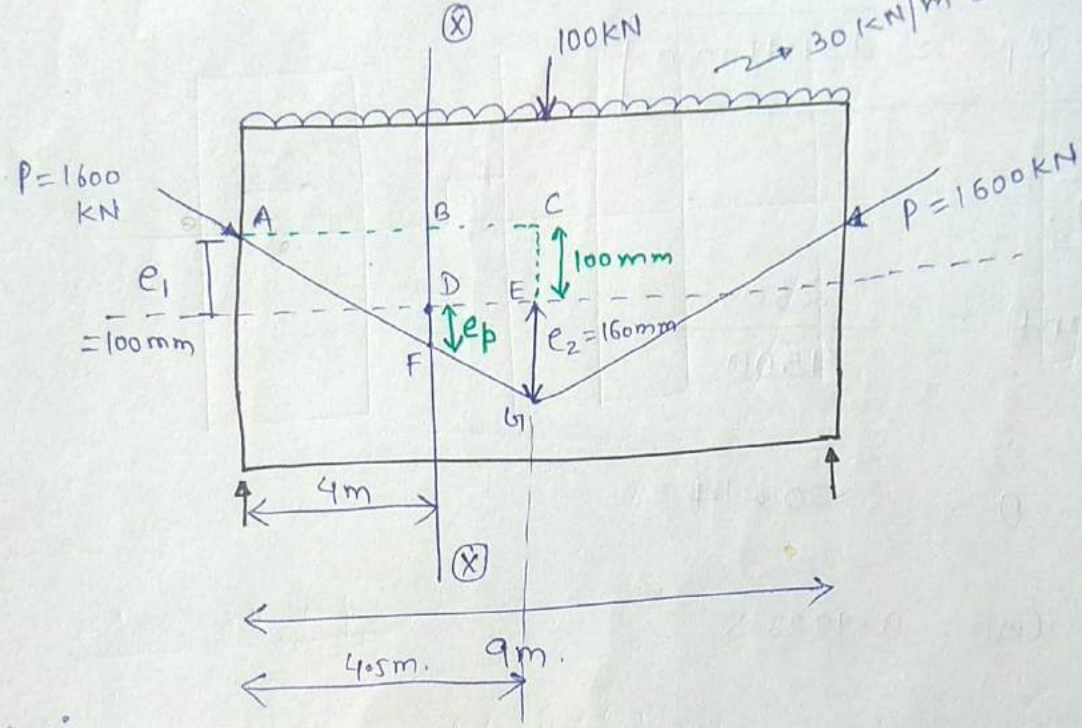
$\frac{P\cos\theta}{A}$ $\quad \frac{P\cos\theta \cdot e_1}{I} \cdot y$ $\quad \frac{M_{net_x}}{I} \cdot y$

Stresses:

at top: $\boxed{\frac{P\cos\theta}{A} + \frac{P\cos\theta \cdot e_1}{I} \cdot y + \frac{M_{net_x}}{I} \cdot y = f_1}$

at Bottom $\boxed{\frac{P\cos\theta}{A} - \frac{P\cos\theta \cdot e_1}{I} \cdot y - \frac{M_{net_x}}{I} \cdot y = f_2}$

Q.2 A Rectangular beam of size 350 mm X 650 mm is provided as shown in figure.



Solution:

Calculation of load.

D.L = 0.35 X 0.65 X 1m X 25 = 5.69 KN/m.

L.L = = 30 KN/m.

Total : 35.69 KN/m.

Reaction : $\frac{wL}{2}$ = 160.59 KN + 50 = 210.59 KN. due to Point load.

$M)_{xx}$ = ~~160.59~~ 210.59 X $\frac{4^2}{2}$ = 1684.72 KN-m.
$-35.69 \times \frac{4^2}{2}$ — 1127.82 KN-m.
556.9

* from Stress Concept Method.

from similar Δ

(a) $\boxed{\frac{BF}{AB} = \frac{CG}{AC}}$

BF = 4m X $\frac{260}{4.50}$

= 231.11

$\therefore \quad FD = BF - BD$

$= 231.11 - 100$

$$\boxed{e_p = 131.11\ mm}$$

(b) <u>P cosθ.</u>

$$\tan\theta = \frac{260}{4500}$$

$$\theta = 3.30674^\circ$$

$$\cos\theta = 0.998335$$

$\therefore \quad P\cos\theta = 1600 \times 0.998335$

$= 1597.34\ kN.$

(c) <u>Moment.</u>

$DL = 0.35 \times 0.65 \times 1.0 \times 25 = 5.69\ kN/m$

$LL = \quad 30.$

$w = 35.69\ kNm$

$$R_1 = R_2 = \frac{wl}{2} + \frac{w}{2} = \frac{35.69 \times 9}{2} + \frac{100}{2}$$

$= 210.605\ kN.$

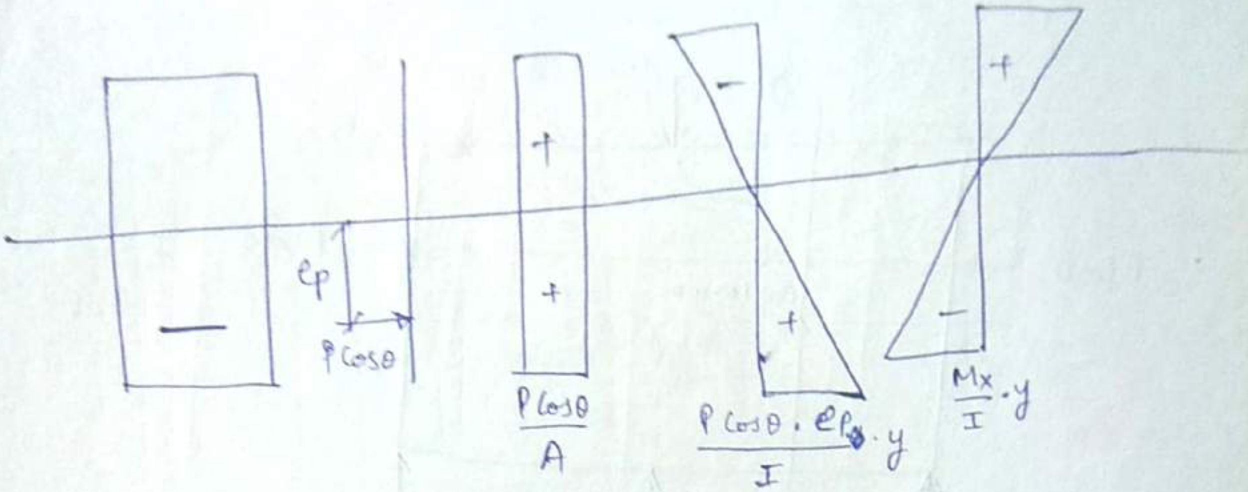
Marks ✓

$$M_x = R_1 \cdot x - \frac{wx^2}{2}$$

$= 210.605 \cdot 4 - \frac{35.69 \times 4^2}{2}$

$= \underline{556.9\ kN\text{-}m}$

Stress Concept Mtd.



$$\frac{P\cos\theta}{A} = \frac{1597.34 \times 10^3}{350 \times 650} = 7.02 \ N/mm^2$$

$$\frac{P\cos\theta \cdot e_p}{Z} = \frac{1597.34 \times 10^3 \times 131.11}{350 \times 650^2/6} = 8.50 \ N/mm^2.$$

$$\frac{M_x}{Z} = \frac{556.9 \times 10^6}{350 \times 650^2/6} = 22.60 \ N/mm^2.$$

$\therefore$ Stress at top $= 7.02 - 8.50 + 22.60$

$= 21.12 \ N/mm^2.$

Stress at Bottom $= 7.02 + 8.50 - 22.60$

$= -7.08 \ N/mm^2$

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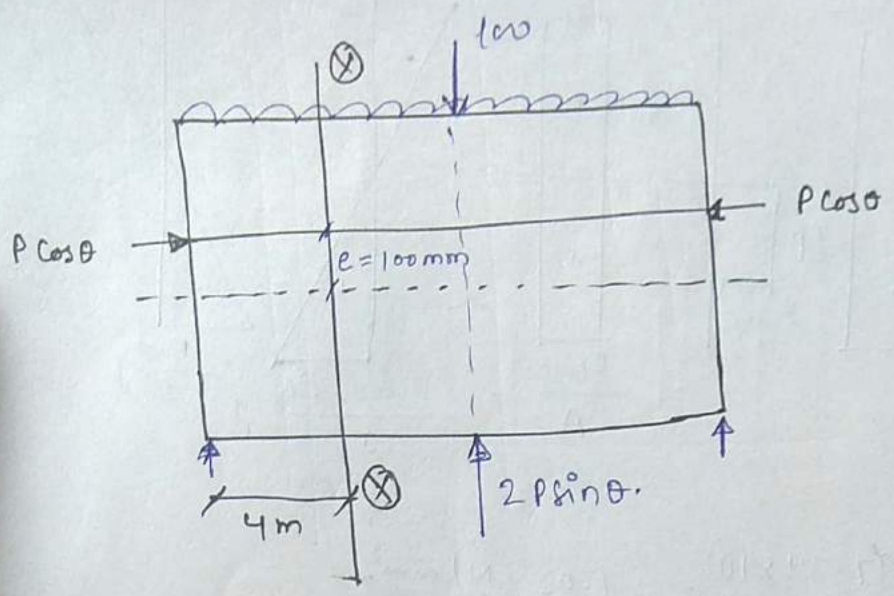
Method II.

Now from load balencing method.



(a) e = 100 mm above N.A.

(b) P cos θ

From similar △ initial fig. find tan θ → θ

then cos θ → $\boxed{P\cos\theta = 1597.34\ kN}$

$\tan\theta = e/L$

(c) upward balancing Load:

$= 2P\sin\theta$

$= 2 \times 1600 \times \sin 3^\circ.30675$

OR $2 \times 1600 \times \frac{260}{\sqrt{4500^2+260^2}}$

$= 184.581$

$= 184.581$ KN.

(d) Moment

$w = 35.69$ kN/m (Calculated Ealier)

$W = 100$ kN ↓

$2P\sin\theta = 184.581$ kN ↑

$$R_1 = R_2 = \frac{wl}{2} + \frac{w}{2} - \frac{2P\sin\theta}{2}$$

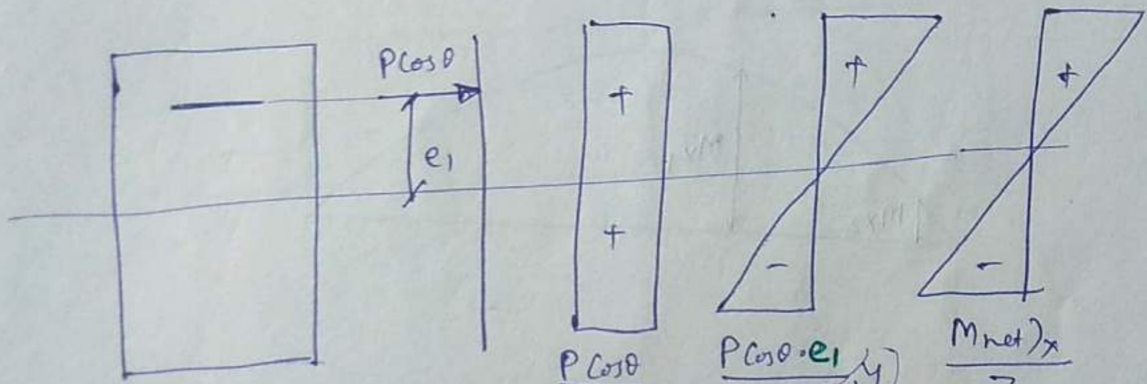
$$= \frac{35.69 \times 9}{2} + \frac{100}{2} - \frac{184.581}{2}$$

$$= 118.31 \text{ KN}.$$

$$M_{x\,net} = R_1 \cdot x - \frac{wx^2}{2}$$

$$= 118.31 \times 4 - \frac{35.69 \times 4^2}{2}$$

$$= 187.738 \text{ KN-m}.$$



$$\frac{P\cos\theta}{A} \qquad \frac{P\cos\theta \cdot e_1}{I} \cdot y \quad \frac{M_{net})_x}{Z}.$$

$$Z.$$

$$\frac{P\cos\theta}{A} = \frac{1597.34 \times 10^3}{350 \times 650} = +7.02 \text{ N/mm}^2$$

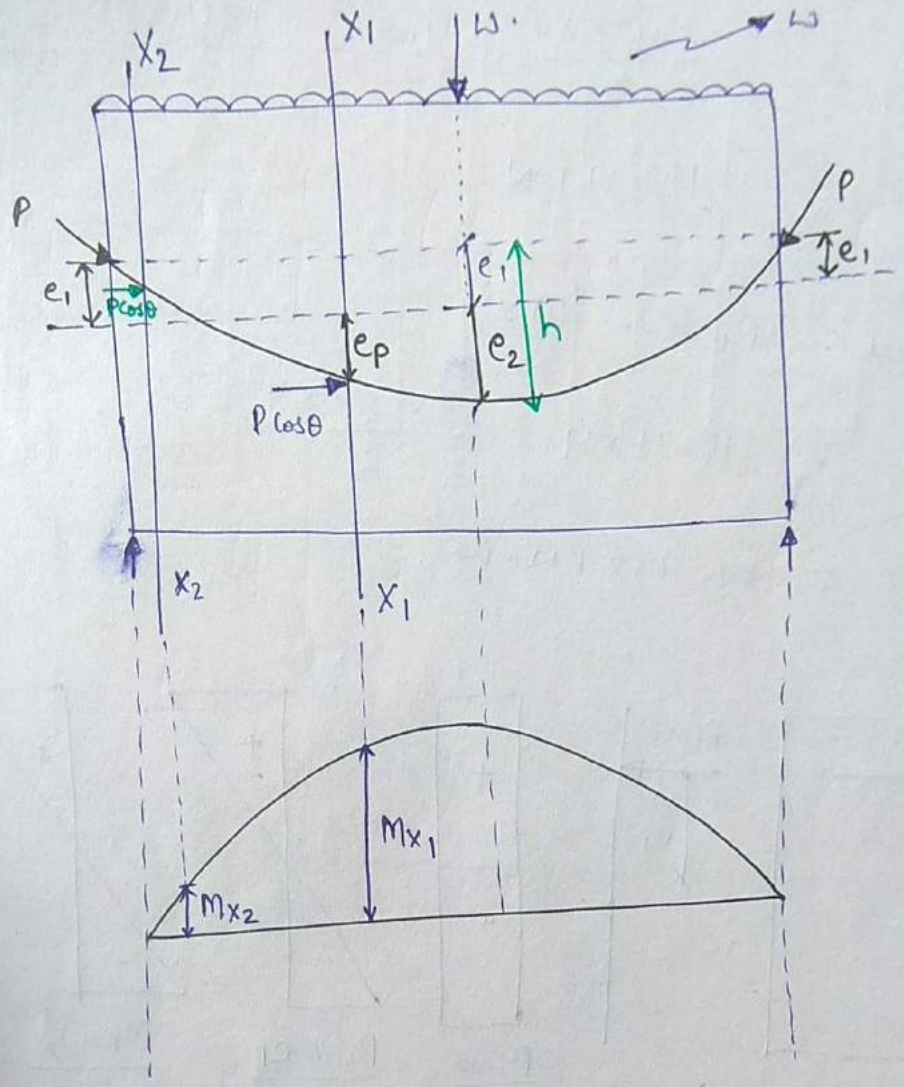
$$\frac{P\cos\theta \cdot e_1}{Z} = \frac{1597.34 \times 10^3 \times 100}{350 \times 650^2/6} = \pm 6.48 \text{ N/mm}^2.$$

$$\frac{M_{x\,net}}{Z} = \frac{187.738 \times 10^6}{350 \times 650^2/6} = \pm 7.62 \text{ N/mm}^2$$

Stress at top $= 7.02 + 6.48 + 7.62 = \underline{21.12 \text{ N/mm}^2}$

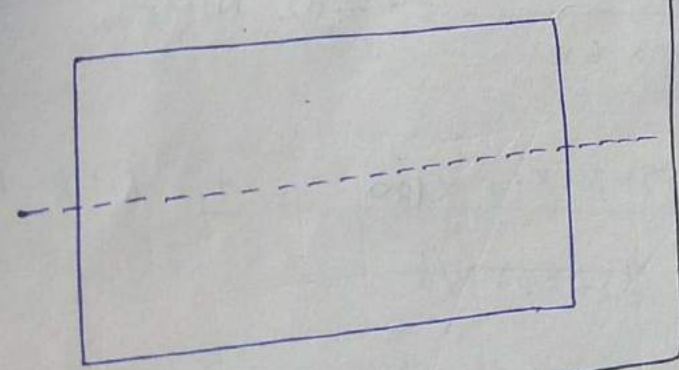
Stress at bottom $= 7.02 - 6.48 - 7.62 = \underline{-7.08 \text{ N/mm}^2}$

Case 3 Beam with parabolic profile of Cable.



Load Balancing:

Left



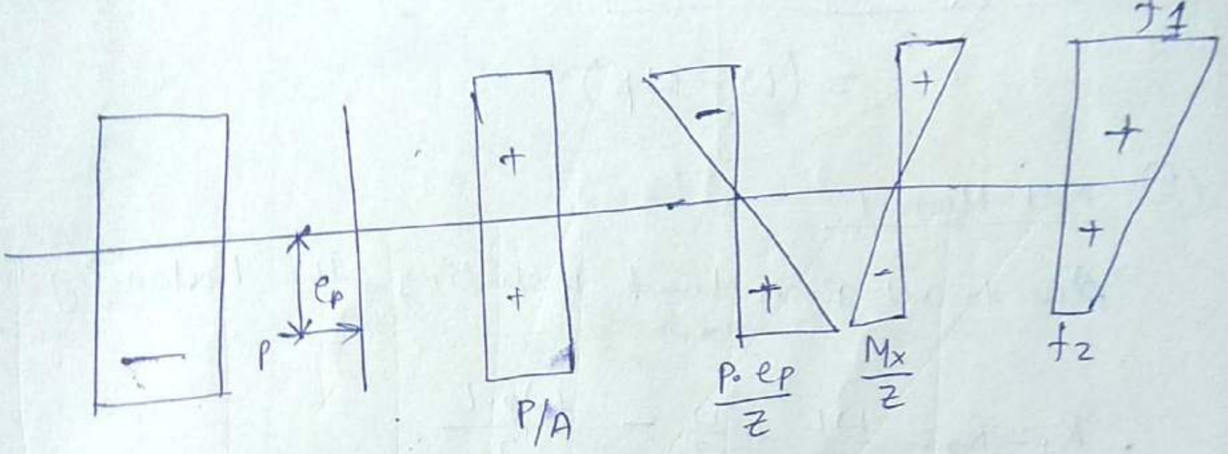
(1) Stress concept at any section

(1) Consider P force = $\boxed{P \cos\theta \simeq P}$

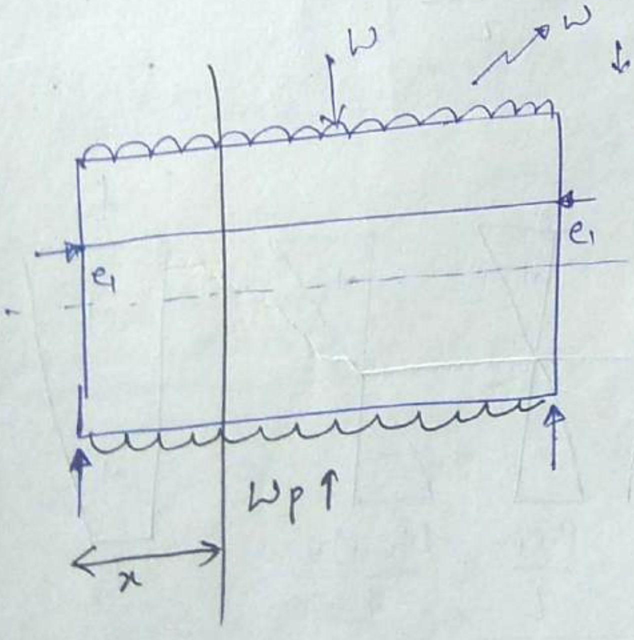
(2) eccentricity of P-force = e_p

(3) Moment at section = M_x

say at $x_1 - x_1$



(II) Load balancing.



① Cable profile made straight.

② Balancing load is a U.d.L

$$\boxed{w_p = \frac{8P\cdot h}{l^2}} \qquad \left(\text{here } \boxed{h = e_1 + e_2}\right)$$

h is like Central height.

Net U.d.L over beam.

$$= (w - w_p)$$

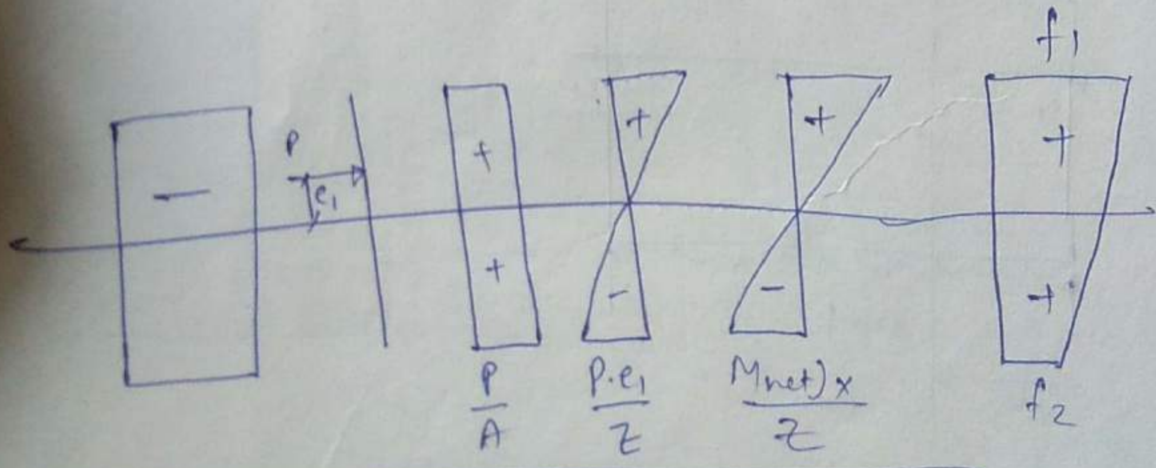
③ Net Moment

due to all given Load including the balancing load.

$$R_1 = R_2 = \frac{wL}{2} + \frac{W}{2} - \frac{w_p \cdot L}{2}$$

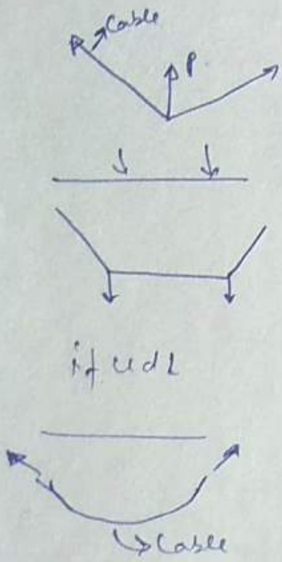
Moment

$$M_x = R_1 \cdot x - \frac{wx^2}{2} + \frac{w_p \cdot x^2}{2}$$

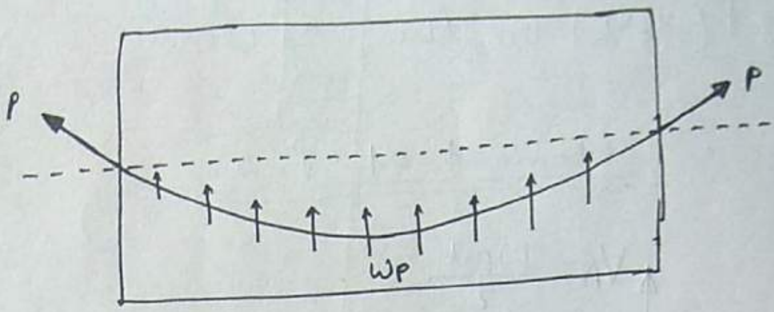


$$\boxed{\text{Stresses: } \frac{P}{A} \pm \frac{Pe_1}{Z} \pm \frac{M_{net})x}{Z}}$$

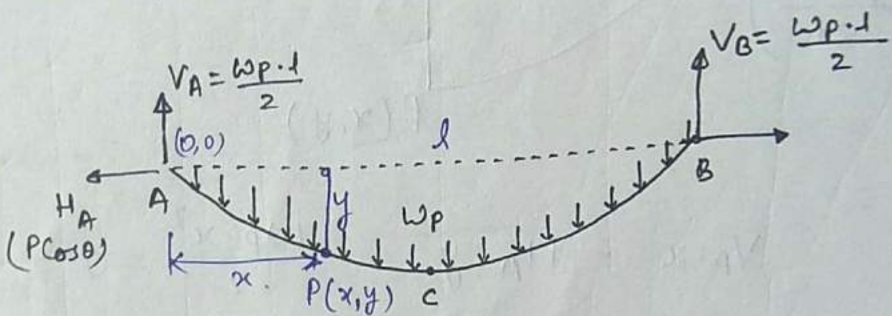
(#) Properties of Parabolic profile of Cable.



apply
Point load
at each
Point
Called udl



~~Reactions~~



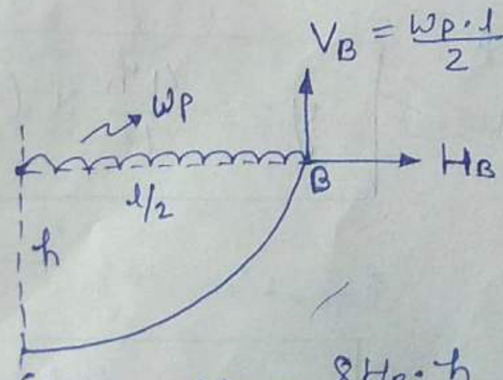
Property of Cable

(#) Important property of a Cable:

It cannot take any Moment.

* Moment at each and every point of Cable = 0

① $\boxed{\text{Moment at } C = 0}$



$$V_B \cdot \frac{l}{2} - H_B \cdot h - Wp \cdot \frac{l}{2} \cdot \frac{l}{4} = 0$$

$$\frac{Wp \cdot l}{2} \cdot \frac{l}{2} - H_B \cdot h = Wp \cdot \frac{l^2}{8} = 0$$

$$H_B \cdot h = \frac{Wp \cdot l^2}{4} - \frac{Wp \cdot l^2}{8}$$

$$\boxed{H_B \cdot h = \frac{Wp \cdot l^2}{8}}$$

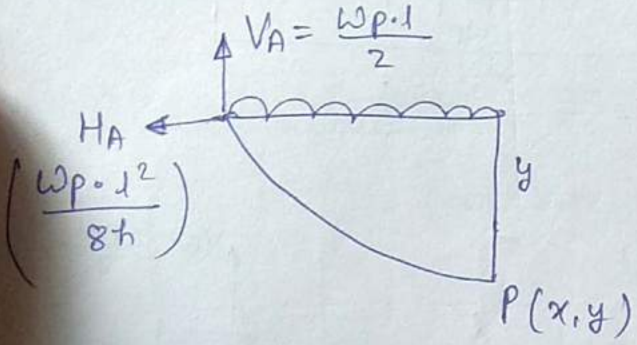
$\rightarrow$ $Wp = \frac{8 H_B \cdot h}{l^2}$

$$= \frac{8 \cdot P\cos\theta \cdot h}{l^2} = \frac{8Ph}{l^2}$$

Equation of parabolic Cable.

Consider A Point as origin consider any point $P(x, y)$ on the Cable.

Moment at $P = 0$.



$$V_A \cdot x - H_A \cdot y - \frac{w_p \cdot x^2}{2} = 0$$

$$\frac{w_p \cdot l}{2} \cdot x - \frac{w_p \cdot l^2}{8h} \cdot y - \frac{w_p \cdot x^2}{2} = 0$$

$$\boxed{l \cdot x - \frac{l^2}{4h} \cdot y - x^2 = 0}$$

$$\frac{l^2}{4h} \cdot y = x(l - x)$$

$$\boxed{y = \frac{4h}{l^2} x (l - x)} \rightarrow (A)$$

Equation of parabolic cable.

$$\boxed{\frac{dy}{dx} = \frac{4h}{l^2}(l - 2x)} \rightarrow \text{slope equation.}$$

$$= \theta = \tan\theta$$

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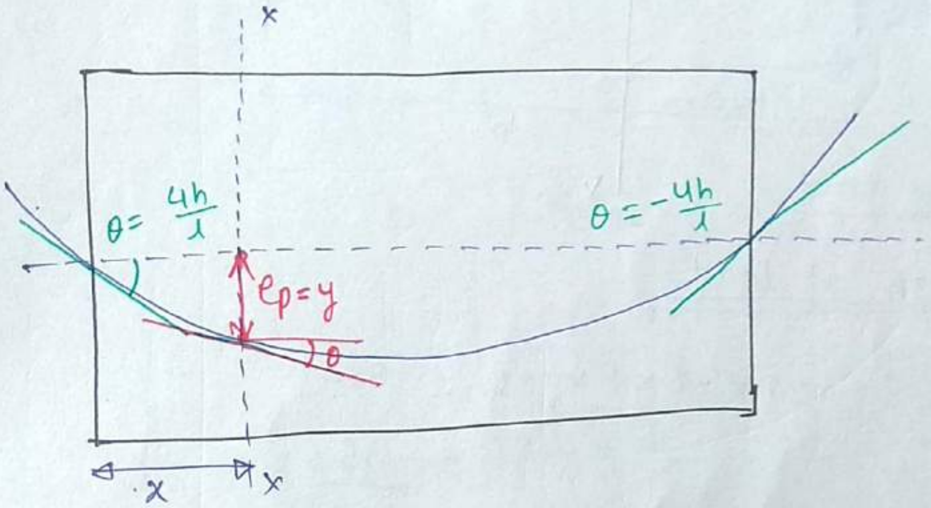
Slope at ends [$x=0$ and l]

$\boxed{x=0}$ $\quad \theta = \tan\theta = \frac{dy}{dx} = \frac{4h}{l}$

$\boxed{x=l}$ $\quad \theta = \tan\theta = \frac{dy}{dx} = -\frac{4h}{l}$



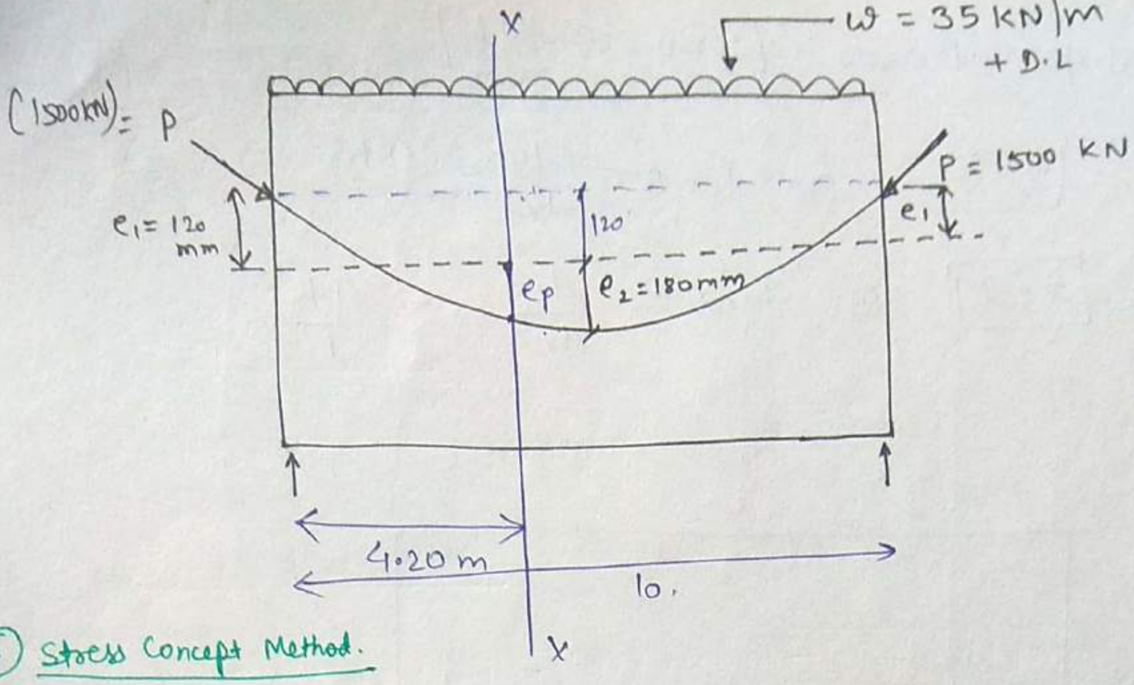
Q3. A prestressed Concrete beam of size 320mm x 800mm is used for a span of 10 m. Prestressing Cable of parabolic profile is used with P = 1500 kN

eccentricity at ends = 120 mm above NA

eccentricity at mid span = 180 mm below N.A

The beam is subject to a live load of 35 kN/m.

Calculate stresses in the Beam at a distance 4.20m from support.



(I) Stress Concept Method.

(1) Calculation of load.

$$D.L = .32 \times .8 \times 1 \times 25 = 6.4$$

$$L.L = = 35$$

$$41.4 \text{ KN/m}.$$

deflection at X-X

$$y = \frac{4h}{l^2} x(l-x)$$

$$= \frac{4 \times (.120 + .180)}{10^2} 4.2 \, (10 - 4.2)$$

$$= \frac{4 \times .3}{10^2} \times 4.2 \times (5.8)$$

$$= 0.29232 \text{ m}$$

$$= 292.32 \text{ mm}$$

$$\therefore e_p = 292.32 - 120 = 172.32 \text{ mm}.$$

② $P = 1500\ KN$

$\boxed{\text{(Neglec Cos}\theta)}$

③ DL - $= 6.40$

L.L $= 35$

$\omega = \underline{41.4\ KN/m}$

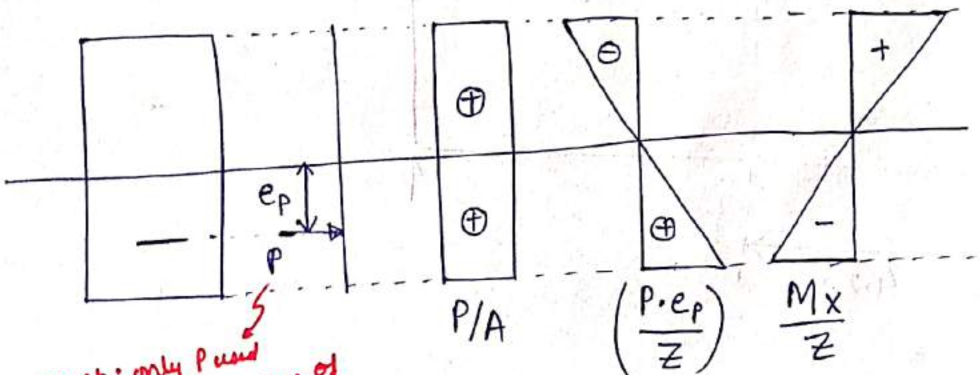
$$R_1 = R_2 = \frac{\omega l}{2} = \frac{41.4 \times 10}{2}$$

$$= \underline{207\ KN}$$

$$M_x = R_1 \cdot 4.2 - \omega \cdot \frac{4.2^2}{2}$$

$$= 207 \times 4.2 - 41.4 \times \frac{4.2^2}{2}$$

$$= \underline{504.252\ KN\text{-}m}$$



Mark: only P used no PCosθ in case of parabolic Profile

$$\frac{P}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$$

$$\frac{P \cdot e_p}{Z} = \frac{1500 \times 10^3 \times 172.32}{320 \times \frac{800^2}{6}} = \mp 7.57$$

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$$\frac{M_x}{Z} = \frac{504.252 \times 10^6}{320 \times \frac{800^2}{6}} = \pm 14.77$$

Stress at top.

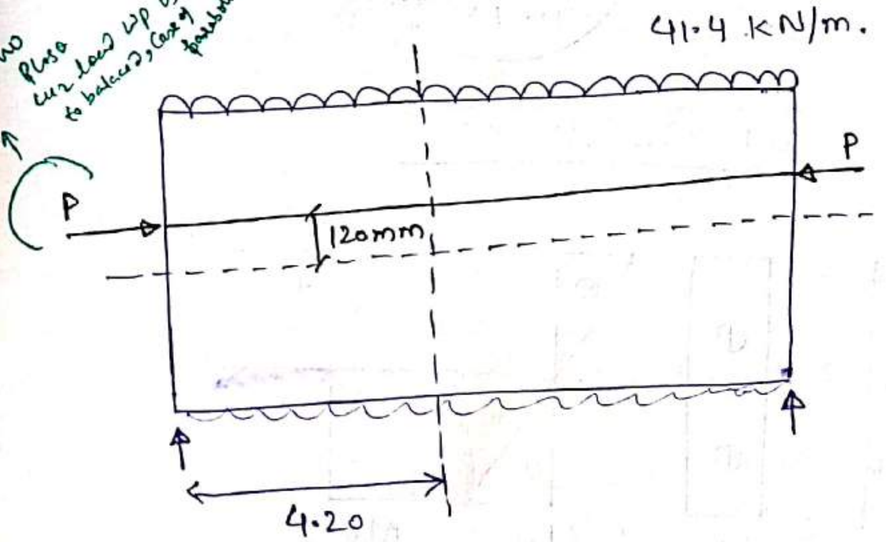
$$= 5.86 - 7.57 + 14.77 = \underline{\overline{13.06}}\ N/mm^2$$

Stress at bottom.

$$= 5.86 + 7.57 - 14.77 = \underline{\overline{-1.34}}\ N/mm^2$$

② Load Balancing Concept :

Remark: wo Ploss the load wp is used to balanced, Case of parabolic



Bal. U.d.L

$$w_p = \frac{8Ph}{l^2} = \frac{8 \times 1500 \times 0.30}{10^2}$$

$$= 36\ kN/m$$

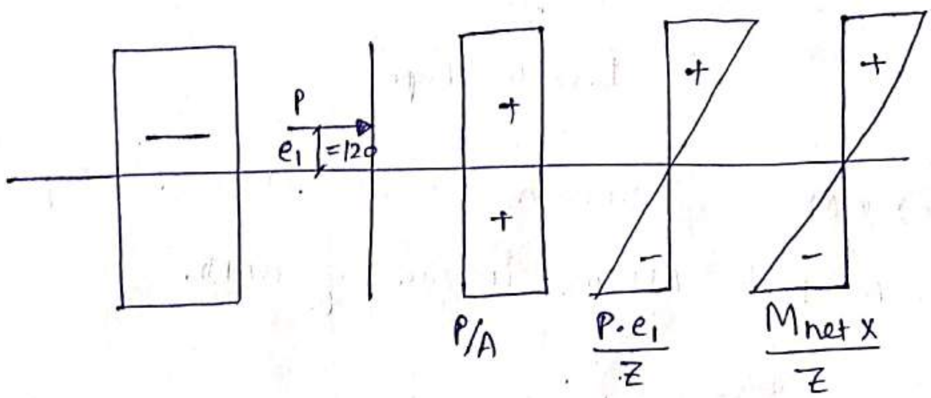
Net udl

Net udl.

$$(w - w_p) = 41.4 - 36$$
$$= 5.4 \text{ kN/m}.$$

$$R_1 = R_2 = \frac{w_{net} \cdot l}{2} = \frac{5.4 \times 10}{2} = 27 \text{ kN}$$

$$M_{net\ x-x} = R_1 \cdot x - \frac{w_{net} \cdot x^2}{2}$$
$$= 27 \times 4.2 - \frac{5.4 \times 4.2^2}{2}$$
$$= 65.772 \text{ kN-m}$$



$$\frac{P}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$$

$$\frac{P \cdot e_p}{Z} = \frac{1500 \times 10^3 \times 120}{320 \times 800^2/6} = \pm 5.27$$

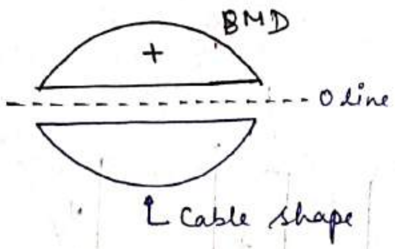
$$\frac{M_{xnet}}{Z} = \frac{65.772 \times 10^6}{320 \times \frac{800^2}{6}} = \pm 1.93$$

Stresses at top = 5.86 + 5.27 + 1.93 = $\underline{13.06}$ N/mm².

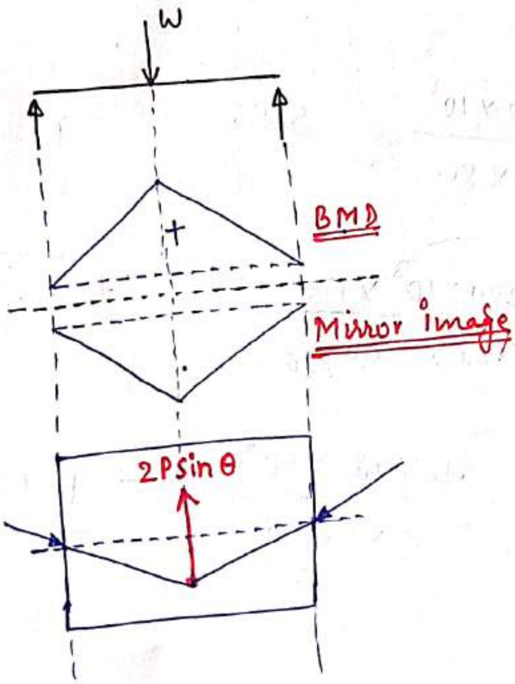
Stresses at bottom = 5.86 − 5.27 − 1.93 = $\underline{-1.34}$ N/mm²

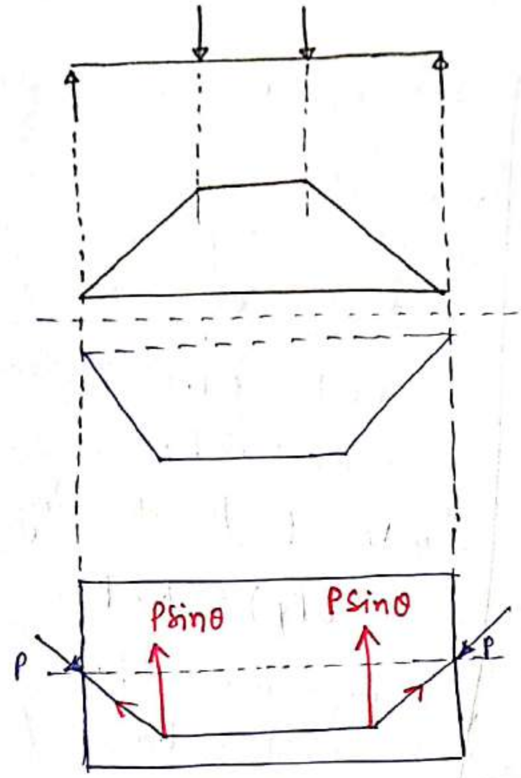
Load Balancing Profile of Cable for different Loading

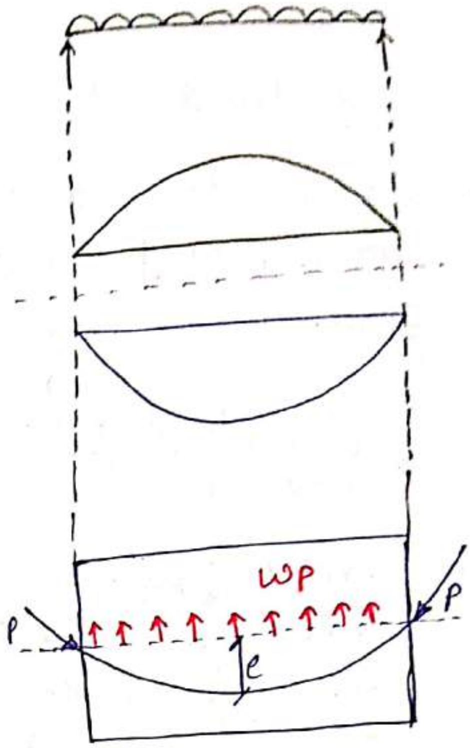
→ The shape of Cable for balancing a particular load is simillar as the BMD due to that load as per type of beam.

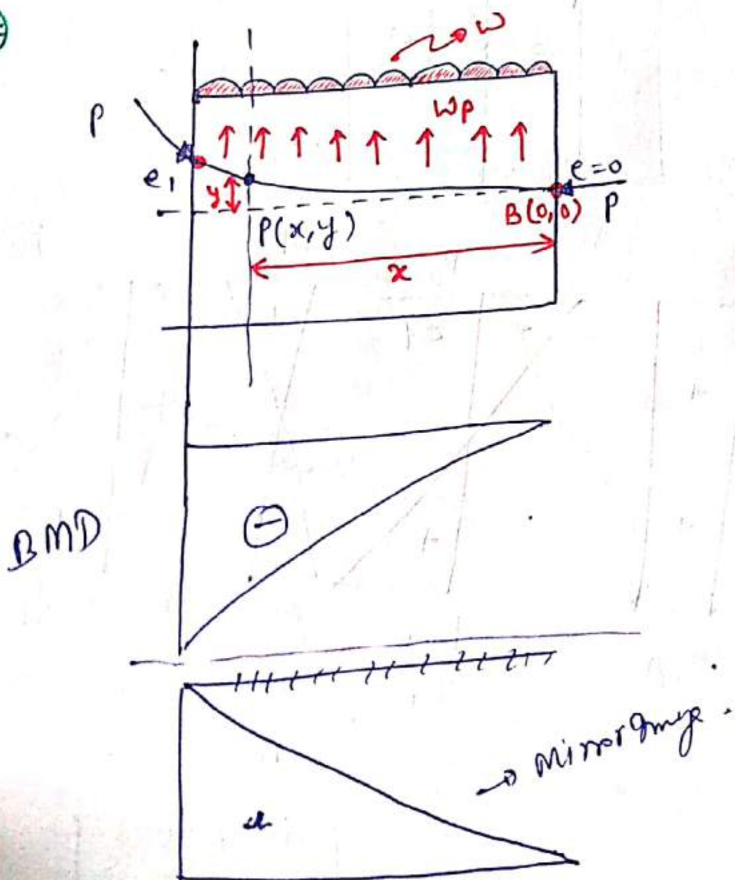


if (+ve) BM are shown above 0 line. shape of Cable shall be Mirror image of BMD.









$$P e_1 = w \cdot l \cdot \frac{l}{2}$$

$$\boxed{e_1 = \frac{wl^2}{2P}}$$

Moment abt P

$$P \cdot y = \frac{w \cdot x^2}{2}$$

* $$\boxed{y = \frac{wx^2}{2P}}$$

Equation of Cable.

(#) Stresses at different stage of Loading.

(1) At transfer.

→ Stress calculated just after transfer of prestressing force in the member.

Only dead load (DL) may be considered

→ Initial prestressing force is considered (i.e P_o value)

→ No losses are considered.
(Elastic shortning losses neglected)

(2) Final stage of loading

→ Stress calculated at final stage, when all loads are working over members.

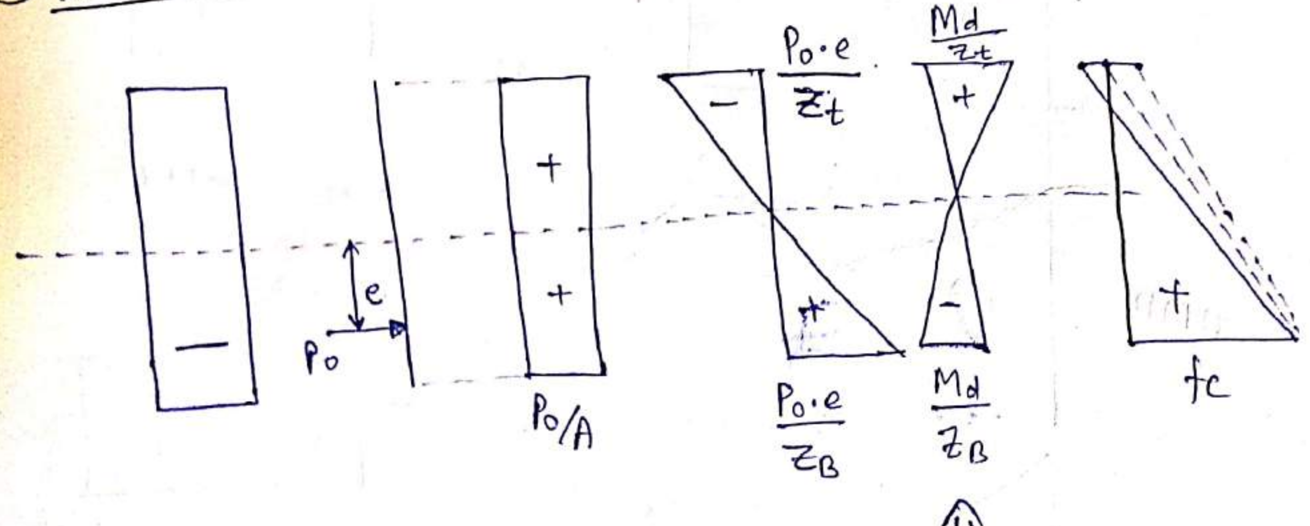
→ DL and L.L all loads are considered.

→ Final prestressing force $(P_f) = K \cdot P_o$ is considered.
K = Loss factor.

→ All losses are considered.

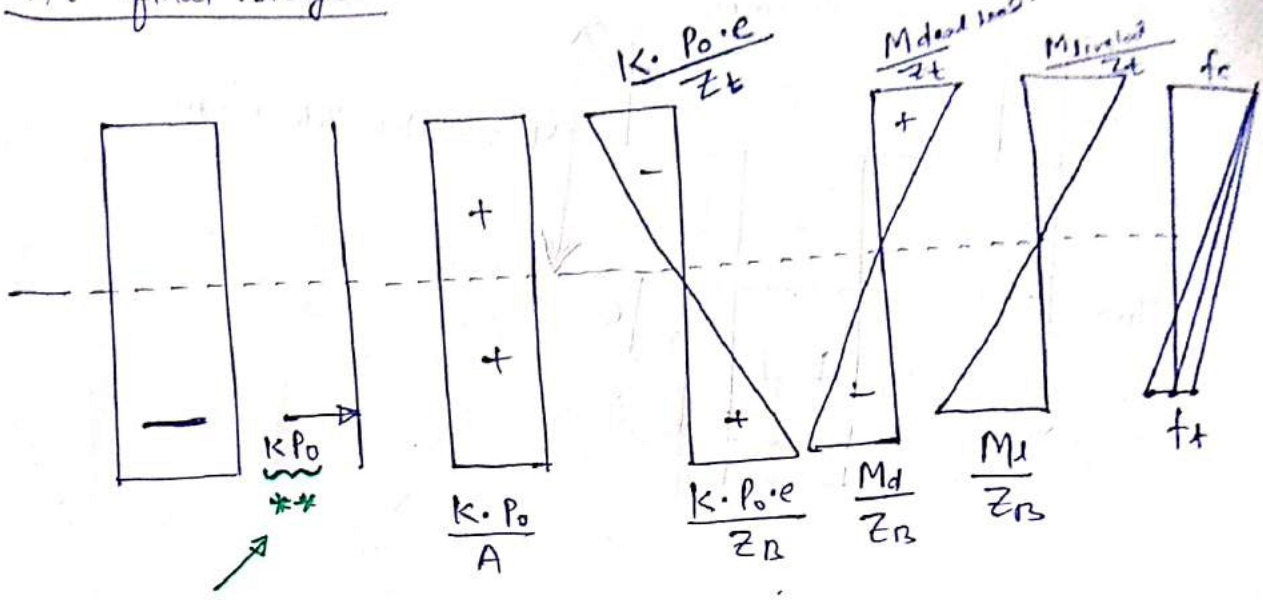
Consider a post tensioned beam.

(1) Stress at Transfer.



(due to dead load only)

(ii) At final stage.



k signifies losses considered

Mark: $k = 0.8$ means loss is 20% of P

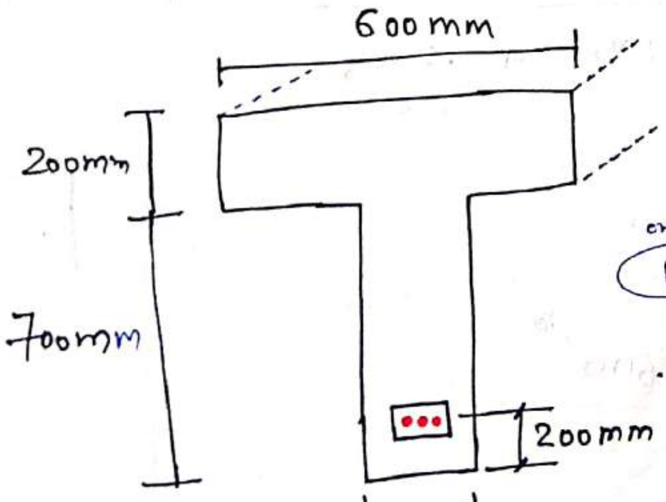
$M_l \rightarrow$ Moment due to live load.

$M_d \rightarrow$ " " " Dead "

$Z(t) \rightarrow$ at top

$Z(B) \rightarrow$ at Bottom

Q4. A Post tensioned Prestressed Concrete beam as shown in fig:



Live load = 40 kN/m

span of beam = 12 m

cross for steel

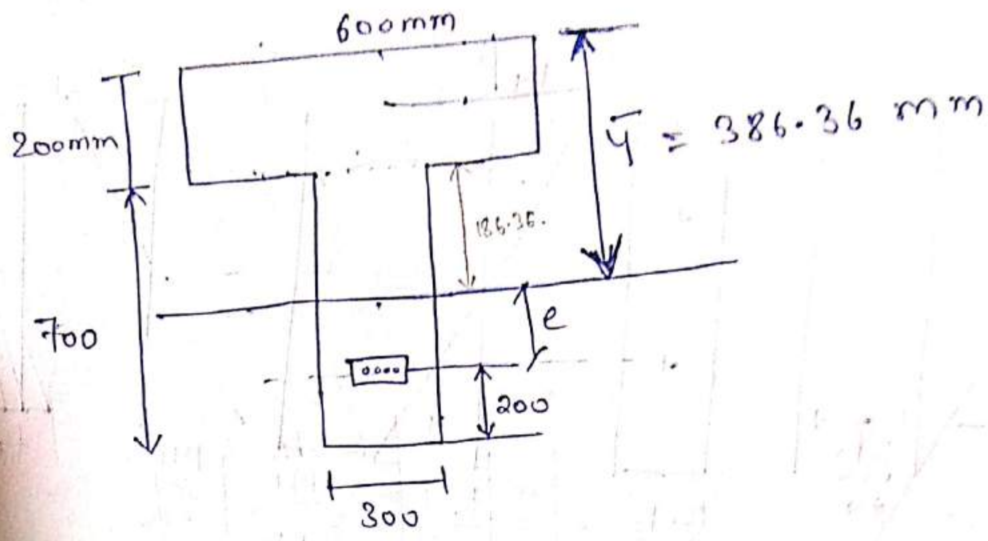
Area = 1600 mm^2

stress (Initial) = 1400 N/mm^2

loss of stress = 20 %

$\therefore K = 0.8$

Calculate max^m stresses in beam at mid span at two stage of loading.



A_1Y_1

sol^n: ① Dead load $= .6 \times .2 \times 25 + .7 \times .3 \times 25$

$= 8.25$

$L.L = 40$

$\omega = 48.25$

② $\bar{y} = \dfrac{A_1Y_1 + A_2Y_2}{A_1 + A_2}$ (Varignon's Th.)

$= \dfrac{.600 \times .200 \times .100 + .700 \times .300 \times (.350 + .2)}{.6 \times .2 + .7 \times .3}$

$\bar{y} = 386.36$ mm

③ ~~$I_{NA} = \dfrac{(.200 \times .600^3)}{12}$~~

Ar^2

(3) $$I_{NA} = \frac{600 \times 200^3}{12} + 600 \times 200 \times (386.36 - 100)^2$$

$$+ \frac{300 \times 700^3}{12} + 300 * 700\,(550 - 386.36)^2$$

$(350+200)$

or

$300 \times 700\,(350 - 186.36)^2$

$$= \cancel{4.58}\ 2.45 \times 10^{10}\ mm^4$$

(4) $P_o = A_s \times p_o = 1600 \times \frac{1400}{10^3} = \overline{\underline{2240\ KN}}$

$e = y_B - 200 = (900 - 386.36) - 200$

$e = \overline{\underline{313.64\ mm}}$

$y_t = 386.36\,mm$ (Dist. of top fibre from NA)

$y_B = 513.64$ (Dist of bottom fibre from NA)

(5) $y_B = 900 - 386.36$

$= 513.64\ mm$

(6) $DL = (0.6 \times 0.2 + 0.3 \times 0.7) \times 1 \times 25$

$w_d = \overline{\underline{8.25\ KN/m}}$

$M_d = \frac{w_d \cdot l^2}{8}$

$= 8.25 \times \frac{12^2}{8} = 148.5$

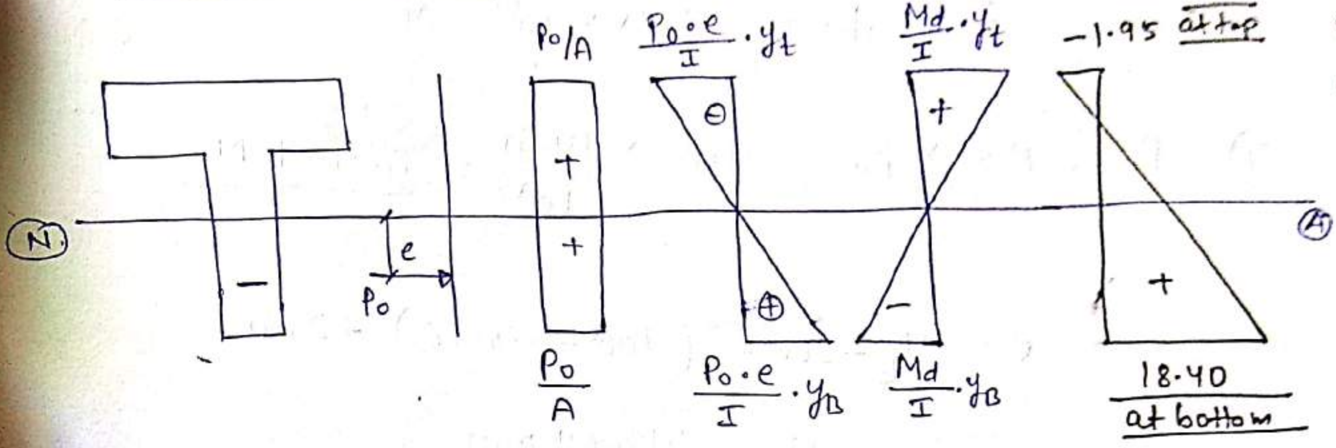
$M_L = 40 \times \frac{12^2}{8} = 720\ KN\text{-}m$

Calculate $M_{dead\,load}$ & M_{LL} separately

(7) $K = 0.80$

cu2 loss of stress = 20%.

(7) <u>At transfer.</u> (only dead load considered)



<u>Stress value at top:</u>

$$= \frac{P_o}{A} - \frac{P_o \cdot e}{I} y_t + \frac{M_d}{I} \cdot y_t$$

$$= \frac{2240 \times 10^3}{330000} - \frac{2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 386.36 + \frac{148.5 \times 10^6}{2.45 \times 10^{10}} \times 386.36$$

$$= 6.79 - 11.08 + 2.34 = -1.95 \ N/mm^2$$

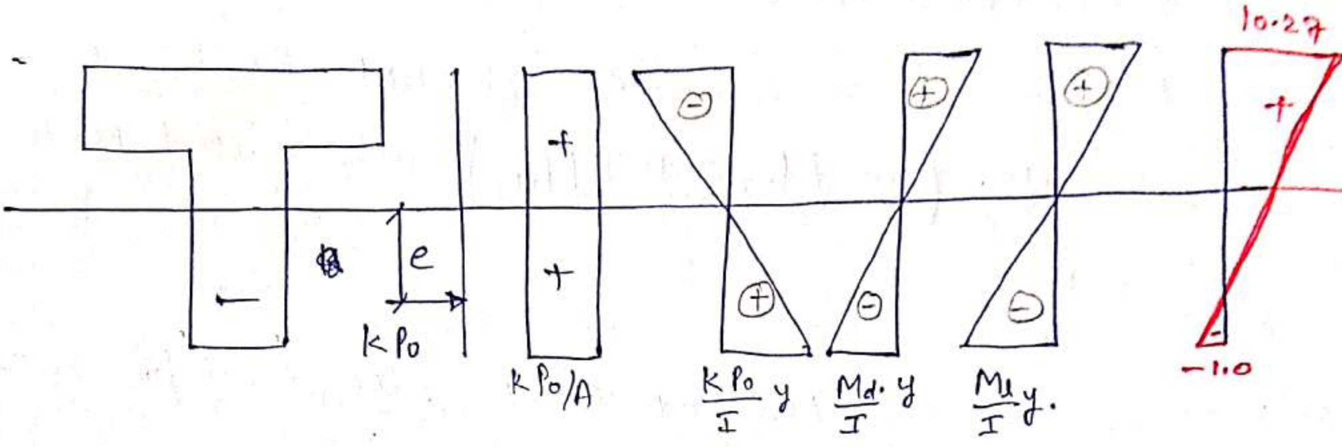
<u>Stress value at Bottom.</u>

$$= \frac{P_o}{A} + \frac{P_o \cdot e}{I} \cdot y_B - \frac{M_d}{I} y_B$$

$$= \frac{2240 \times 10^3}{330000} + \frac{2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 513.64 - \frac{148.50 \times 10^6}{2.45 \times 10^{10}} \times 513.64$$

$= 6.79 + 14.72 - 3.11 = \underline{18.40}$

At final Stage.



at top.

$$= K \cdot \frac{P_0}{A} - K \frac{P_0 \cdot e}{I} y_t + \frac{M_d + M_l}{I} y_t$$

$$= \frac{0.80 \times 2240 \times 10^3}{330000} - \frac{0.80 \times 2240 \times 10^3 \times 313.64}{2.45 \times 10^{10}} \times 386.36 + \frac{\overset{868.50}{\cancel{1418.50}} \times 10^6}{2.45 \times 10^{10}} \times 386.36$$

$$= 5.43 - 8.86 + 13.70 = \underline{+10.27} \ N/mm^2$$

at bottom

$$= K \cdot \frac{P_0}{A} + \frac{K P_0 e}{I} y_B - \frac{M_d + M_l}{I} y_B$$

$$= \frac{0.8 \times 2240 \times 10^3}{330000} + \frac{0.8 \times 224 \times 10^3 \times 313.64}{2.45 \times 10^{10}} * 513.64 - \frac{868.50 \times 10^6}{2.45 \times 10^{10}} \times 513.64$$

$$= 5.43 + 11.78 - 18.21$$

$$= \underline{-1.0}$$

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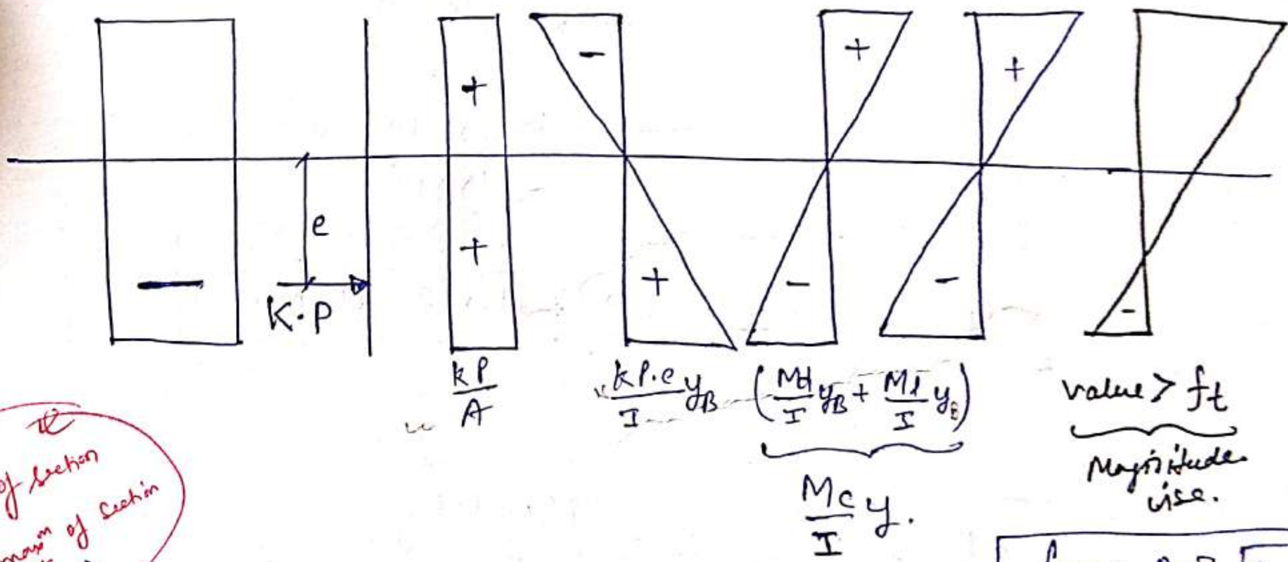
☆ Cracking Moment

→ Cracking Moment is the value of Moment due to (DL + L.L) all loads, for which stress in concrete at tension fibre reach to flexural tensile strength of concrete. (i.e $\left\{f_{cr} = 0.7\sqrt{f_{ck}}\right)$ also called Modulus of Rupture.

→ Beyond Cracking Moment Cracks will develope in the member.

At final stage.

{Mark: Bottom (y_B) dist. considered (we Tension of cracks to be checked. }



$$\boxed{f_{cr} = 0.7\sqrt{f_{ck}}}$$

I is of section
y_B is maxm dist of section from N.A

At Cracking Moment.

Mark ← Tensile stress Taken -ve sign

$$\frac{kP}{A} + \frac{K \cdot P.e}{I} y_B - \frac{M_c}{I} \cdot y_B = -f_t = f_{cr}$$

To be found

$$\boxed{M_c = \frac{W_c \cdot l^2}{8}} \rightarrow \text{For simply support. (Working Moment)}$$

Mc as a whole to be found which include (M_0+M_L)

Actual load on structure is kept less than cracking moment.

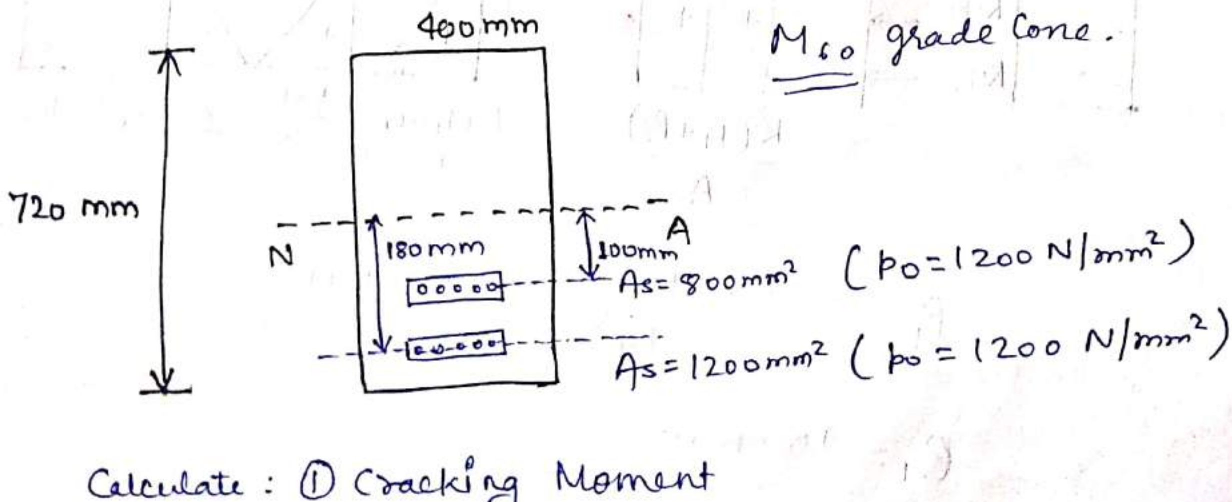
$$\left\{ \text{factor of safety against cracking} = \frac{\text{Cracking load}}{\text{Working load}} \right.$$

$$\therefore \boxed{F.O.S = \frac{\text{Cracking Moment}}{\text{Working Moment}}} \rightarrow M_c$$

<u>Q5</u> A rectangular Beam is provided with cables as shown in fig. The beam is simply supported over a span of 8m and is subjected to a live load of 40 KN/m.

Losses of stresses = 20%

<u>M_{60}</u> grade Conc.



Calculate : ① Cracking Moment

② Load factor against Cracking.

③ Stresses at Mid span due to working load.

① $I = \frac{BD^3}{12} = 1.24 \times 10^{10}$ mm

$P_{o1} = 960$ kN.

$P_{o2} = 1440$ kN.

② Dead load.

$D_L = 0.4 \times 0.72 \times 1 \times 25 = 7.2$ kN.

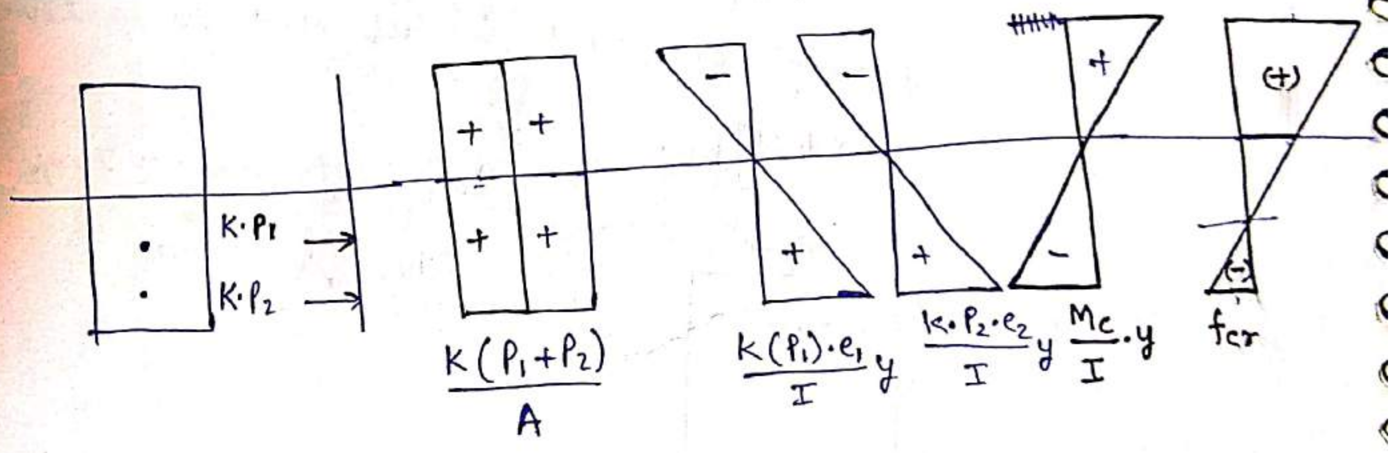
$L.L = \qquad = 40$ kN.

47.2 kN.

$K = 0.80$

Area $= 288000$ mm^2

At final stage.



$P_1 = 800 \times \frac{1200}{10^3} = 960$ kN

$e_1 = 100$ mm

$$P_2 = 1200 \times \frac{1200}{10^3} = \boxed{1440\ KN}$$

$$e_2 = 180\ mm$$

$$f_{cr} = 0.7\sqrt{f_{ck}} = 0.7\sqrt{60}$$

$$= 5.42\ N/mm^2$$

Mark ↓

① For Cracking Moment (at final stage)

$$\frac{K.P_1 + K.P_2}{A} + \frac{K.P_1.e_1}{I}.y + \frac{K.P_2.e_2}{I}.y - \frac{M_c}{I}.y = -5.42$$

$$\frac{0.8(960+1440)\times10^3}{400\times720} + \frac{0.8\times960\times10^3\times100}{400\times720^2/6} + \frac{0.8\times1440\times10^3\times180}{400\times\frac{720^2}{6}}$$

$$- \frac{M_c}{400\times\frac{720^2}{6}} = -5.42.$$

~~$M_c =$~~

$$6.67 + 2.22 + 6.00 - \frac{M_c}{400\times\frac{720^2}{6}} = -5.42.$$

$$M_c = 20.33\times400\times720^2/6 \times \frac{1}{10^6}$$

$$= \underline{\overline{702.605\ KN\text{-}m}}$$

② Factor of safety. (Working Moment)

$$D.L = 0.4 \times 0.72 \times 1 \times 25 = 7.2$$

$$L.L = \qquad\qquad = 40.0$$

$$47.2 \text{ kN.}$$

$$B.M = 47.2 \times \frac{8^2}{8} = 377.6 \text{ KN-m.}$$

$$\text{Factor of Safety} = \frac{M_c}{\underbrace{M_w}_{\text{working Moment}}} = \frac{702.605}{377.6} = 1.86.$$

③ Stresses at bottom fibre due to working Moment.

$$\frac{K.P_1 + K.P_2}{A} + \frac{K.P_1.e_1}{I}.y + \frac{K.P_2.e_2}{I}.y - \frac{M_w}{I}y.$$

$$\Rightarrow \frac{0.8(960+1440)\times10^3}{400\times720} + \frac{0.8\times960\times10^3\times100}{400\times720^2/6} + \frac{0.8\times1440\times10^3\times180}{400\times720^2/6}$$

$$- \frac{377.6\times10^6}{400\times720^2/6}$$

$$\Rightarrow 6.67 + 2.22 + 6.02 - 10.92 = +3.97 \text{ N/mm}^2$$

Stress at top fibre

$$= 6.67 - 2.22 - 6.02 + 10.92$$

$$= 9.35 \text{ N/mm}^2.$$

or 2nd Method.

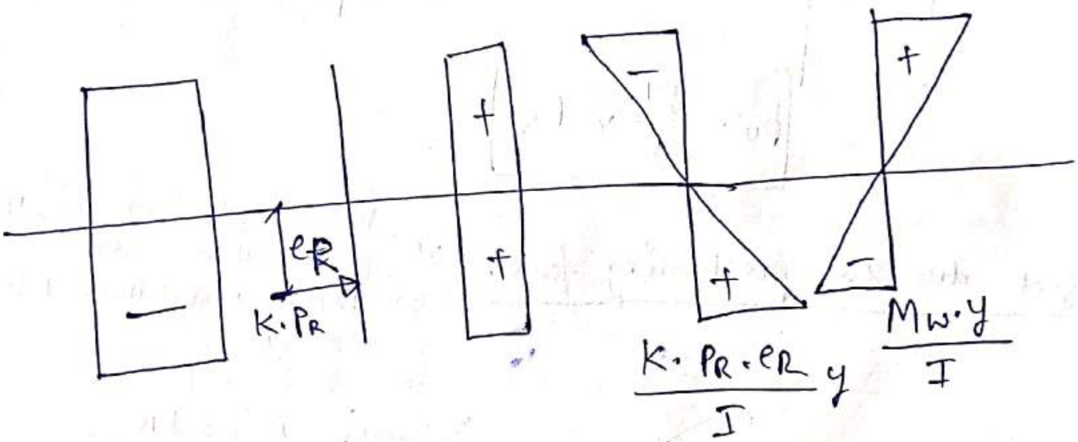
$$P_R = 960 + 1440 = 2400 \text{ KN}.$$

$$e_R = \frac{P_1 \cdot e_1 + P_2 \cdot e_2}{P_1 + P_2}$$

$$= \frac{960 \times 100 + 1440 \times 180}{2400}$$

$$= 148 \text{ mm}.$$

Stress at bottom due to Working.



$$\text{Stress} = \frac{0.8 \times 2400 \times 10^3}{400 \times 720} + \frac{0.8 \times 2400 \times 10^3 \times 148}{400 \times 720^2/6} - 10.92$$

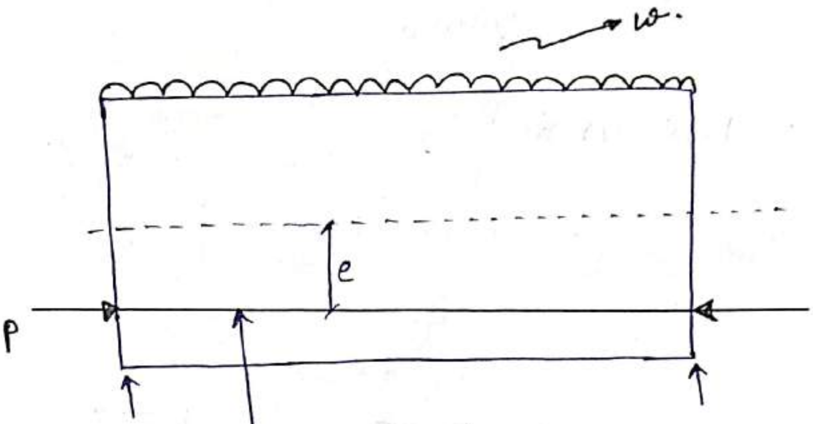
$$= 6.67 + 8.22 - 10.92$$

$$= +3.97 \text{ N/mm}^2$$

2-09-2015

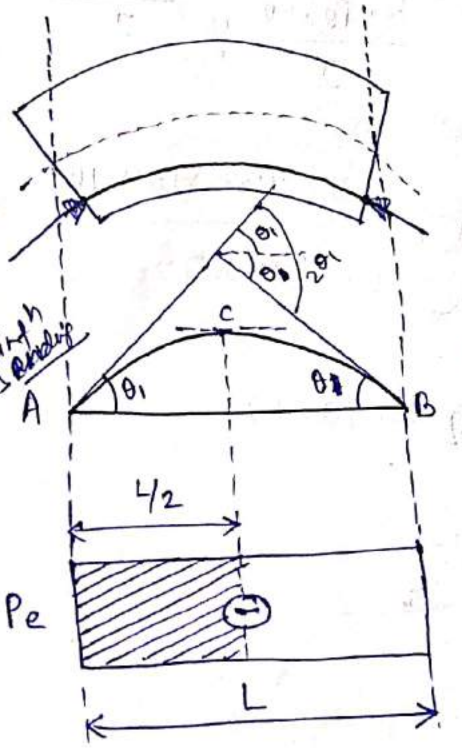
Slope and deflection.

(A) Effect of prestressing force and loading over stresses in reinforcement.



$$\boxed{p_0 = \frac{\delta l}{l} \times E_s}$$

① Effect due to prestressing force. ⇒ here due to prestressing only, Hogging effect will be seen
⇒ Let angle substained is θ_1 at each end.



~~Deflection~~ length bending ~~Bendings~~ data

BMD
~~BMD~~

Between A and B

$$\theta_1 - (-\theta_1) = \frac{P.e.L}{EI}$$ → Area of BMD.

$$\boxed{2\theta_1 = \frac{Pe.L}{E_c . I_c}}$$

B/w A and C

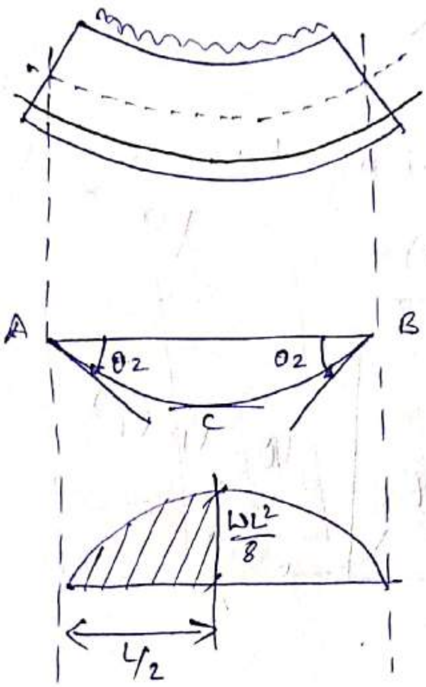
$$\theta_1 - 0 = \frac{Pe \cdot L/2}{EI}$$

$$\boxed{\theta_1 = \frac{Pe \cdot L}{2 E_c I_c}}$$

$\frac{ML}{2EI}$

② **Due to U.d.L.** (i.e effect of loading)

⇒ here due to load sagging will be seen, let θ_2 be the angle subtended at ends



Difference of slope b/w A and C

$$\theta_2 - 0 = \frac{\frac{2}{3} \cdot \frac{WL^2}{8} \cdot \frac{L}{2}}{E_c I_c}$$

$$\boxed{\theta_2 = \frac{WL^3}{24 E_c I_c}} \rightarrow ②$$

③ Net slope of the Beam at ends

(i) if $\theta_1 > \theta_2$ $\quad \theta = (\theta_1 - \theta_2)$

(ii) if $\theta_2 > \theta_1$ $\quad \theta = (\theta_2 - \theta_1)$

(It mean sagging effect due to load is more than Hogging effect due to Prestressing)

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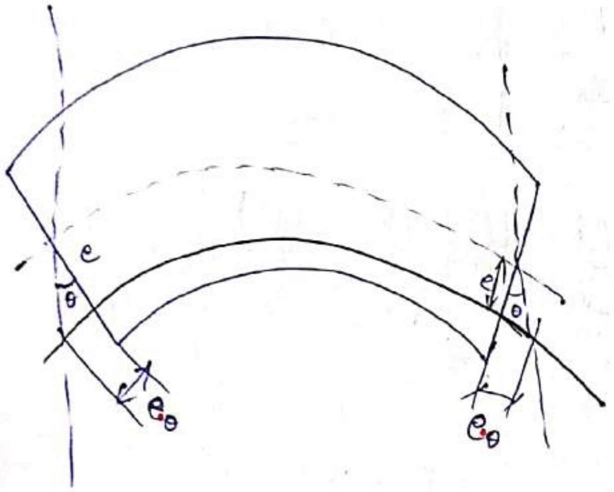
Case 1 When $\boxed{\theta_1 > \theta_2}$ i.e Prestressing force > loading effect/force.

$\boxed{\frown + \smile \text{ net} = \frown}$

Net slope $\theta = (\theta_1 - \theta_2)$ Hogging.



Total reduction of length in reinforcement.

$= e\theta + e\theta = \underline{(2e\theta)}$

$\boxed{2 \times (e \times \theta)}$

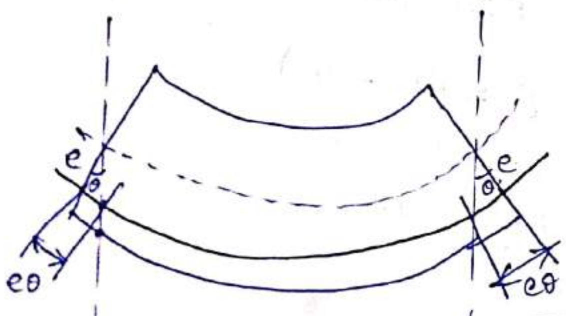
$(\theta = \theta_1 - \theta_2)$

$\boxed{\text{Loss of stress in reinforcement} = \frac{(2e\theta)}{l} \times E_s}$

of steel

Case 2. if $\theta_2 > \theta_1$

Net slope $= \underline{\theta} = (\theta_2 - \theta_1)$ Sagging.



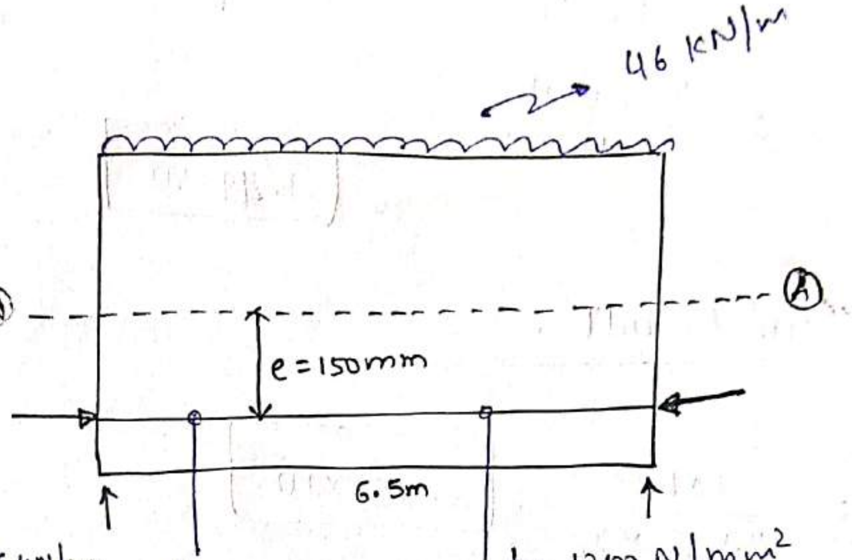
diff. of $(\theta_2 - \theta_1)$

$\boxed{\text{Gain of stress in steel} = \frac{2e\theta}{l} \times E_s}$

**

assumption says minor loss/gain

Q6. A S.S beam of size 300 mm X 560 mm is provided with a straight Cable at an eccentricity of 150mm below N.A



Live load = 46 kN/m

A_s = 1100 mm²

p = 1250 N/mm²

span = 6.5 m

Calculate Net increase / decrease of stress in reinforcement only due to the effect of P-force and loading.

$$E_s = 2.1 \times 10^5 \ N/mm^2.$$

$$E_c = 34000 \ N/mm^2$$

Ans: ① prestressing force

$$P = A_s \times p_o$$

$$= 1100 \times 1250 \times \frac{1}{10^3} = 1375 \ kN$$

$$e = 150 \ mm$$

② D.L $= 0.30 \times 0.56 \times 1 \times 25$

$= 4.20$ KN

L.L $=$ 46 KN

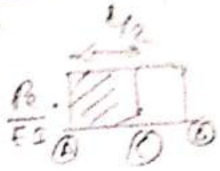
$$w = 50.20 \ kN/m$$

Scanned by CamScanner

③ Slope due to P-Force.

diff b/w A & C

$$\theta_1 = \frac{P \cdot e \cdot L}{\underbrace{2 E_c \cdot I_c}_{\text{keep units in N and mm.}}} = \frac{1375 \times 10^3 \times 6500}{2 \times 34000 \times \frac{300 \times 560^3}{12}}$$



$$= \boxed{4.49 \times 10^{-3}}$$

④ Slope due to udL.

diff b/w A and C

$$\theta_2 = \frac{wL^3}{24 E_c \times I_c} = \boxed{3.85 \times 10^{-3}}$$

So $\theta_1 > \theta_2$. 3.85×10^{-3}

So net slope will be hogging.

$$\boxed{\theta_1 - \theta_2 = 0.64 \times 10^{-3} \text{ (Hogging)}}$$

⑤ Loss of stress in reinforcement

$$\text{Stress} = \frac{2 e \theta}{l} \times E_s$$

$$= \frac{2 \times 150 \times 0.64 \times 10^{-3}}{6500} \times 2.1 \times 10^5$$

$$= \underline{6.2} \text{ N/mm}^2 \quad \text{Ans}$$

very less

however as per assumption loss of stress in reinf^n not considered.

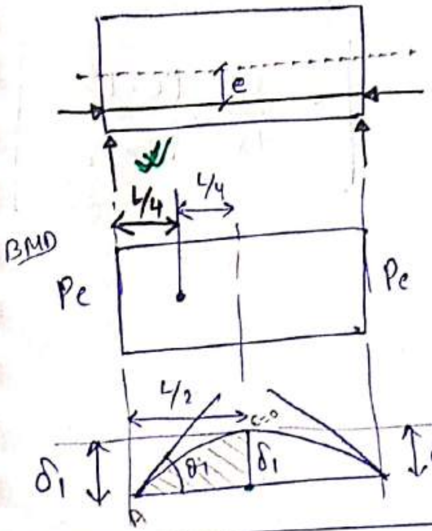
Slope and deflection.

(A) Due to Prestressing.

Cases

Slope | Deflection.

(1) Straight cable.



half Area cuz (slope is 0 at centre) (BMD - Pe) (360)

$$\theta_1 = \frac{Pe \cdot \frac{L}{2}}{E_c I_c}$$

$$= \frac{Pe \cdot L}{2 E_c \cdot I_c}$$

Moment of half area

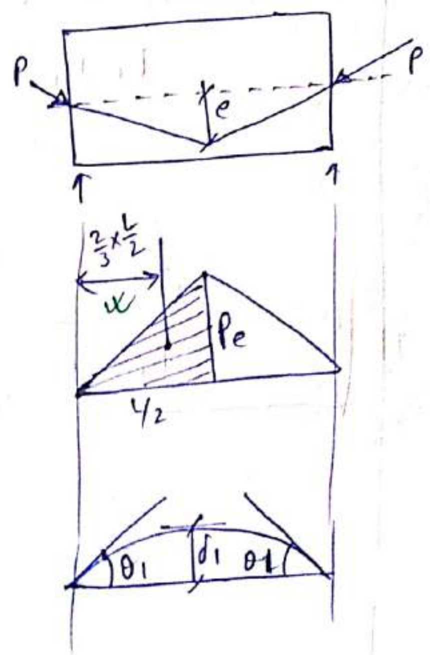
$$\delta_1 = \frac{Pe \cdot \frac{L}{2} \times \frac{L}{4}}{E_c \cdot I_c}$$

$$= \frac{Pe \cdot L^2}{8 E_c \cdot I_c}$$

Mohr for. Deflection:

Calculating Moment of Area.
which arm is taken.
Moment of half area, so that we get tangent at C ∥ to B.G. line, or beam

(2) Bend Cable.



$$\theta_1 = \frac{\frac{1}{2} \cdot Pe \times \frac{L}{2}}{E_c I_c}$$

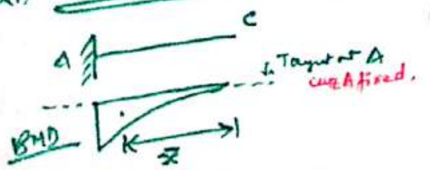
$$\boxed{\theta_1 = \frac{Pe \cdot L}{4 E_c \cdot I_c}}$$

$$\delta_1 = \frac{\frac{1}{2} \times Pe \cdot \frac{L}{2} \times \frac{2}{3} \times \frac{L}{2}}{E_c I_c}$$

$$\boxed{\delta_1 = \frac{Pe\, L^2}{12 E_c \cdot I_c}}$$

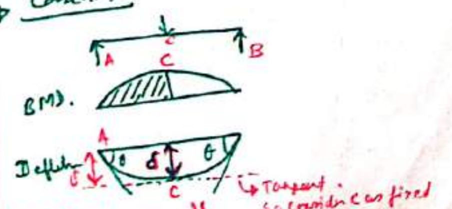
QSN QSN

(vii) To find Deflection



"x̄" is considered for cal. δ at C

⇒ Case (ii)

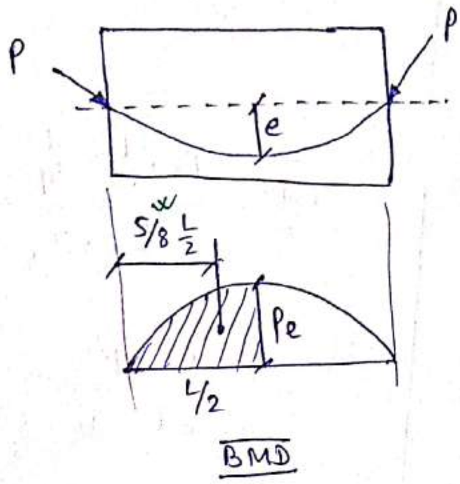


You can't get actual deflection at C if tangent at A taken

so consider C as fixed and calculate δ at A which is same as δ at C

Case	Slope	Deflection

③ Parabolic Cable



$$\theta_1 = \frac{\frac{2}{3} P \cdot e \times \frac{L}{2}}{E_c \cdot I_c}$$

$$\boxed{\theta_1 = \frac{Pe \cdot L}{3E_c \cdot I_c}}$$

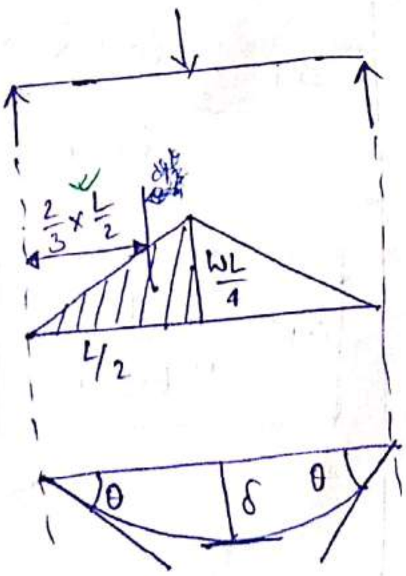
$$\boxed{\delta_1 = \frac{\frac{2}{3} \cdot Pe \cdot \frac{L}{2} \times \frac{5}{8} \times \frac{L}{2}}{E_c I_c}}$$

$$\boxed{= \frac{5}{48} \frac{Pe \cdot L^2}{E_c \cdot I_c}}$$

Ⓑ Due to loading.

Case	Slope	Deflection.

④ Point load

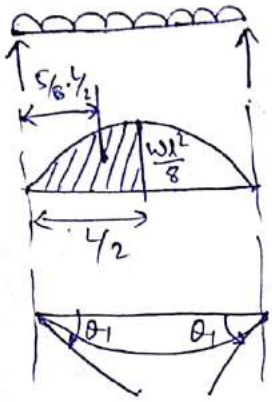


$$\theta = \frac{\overbrace{\frac{1}{2} \times \frac{WL}{4} \times \frac{L}{2}}^{\text{area}}}{E_c I_c}$$

$$\boxed{\theta = \frac{WL^2}{16 E_c I_c}}$$

$$\delta = \frac{\overbrace{\left(\frac{1}{2} \times \frac{WL}{4} \times \frac{L}{2}\right)}^{\text{area}} \times \left(\frac{2}{3} \times \frac{L}{2}\right)}{E_c I_c}$$

$$\boxed{\delta = \frac{WL^3}{48\, E_c \cdot I_c}}$$

Case.	Slope.	Deflection
(5) UdL	$\theta = \dfrac{\frac{2}{3} \times \frac{wl^2}{8} \times \frac{L}{2}}{E_c I_c}$	$\delta = \dfrac{\frac{2}{3} \times \frac{wl^2}{8} \times \frac{L}{2} \times \frac{5}{8} \times \frac{L}{2}}{E_c I_c}$ (centroid dist)
	$\boxed{\theta = \dfrac{wL^3}{24 E_c I_c}}$	$\boxed{\delta = \dfrac{5}{384} \dfrac{wL^4}{E_c I_c}}$

Q7. For Q6 Calculate Net deflection at Centre of Beam

due to prestressing st.. Cable

$$\delta_1 = \frac{Pe \cdot L^2}{8 E_c I_c} = \frac{1375 \times 10^3 \times 150 \times (6500)^2}{8 \times 34000 \times 300 \times \frac{500^3}{12}} = 7.30\,mm \uparrow \text{ (due to hogg)}$$

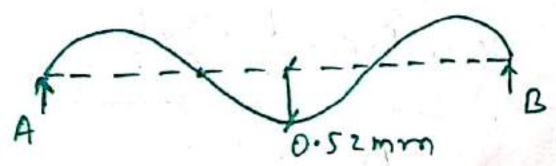
due to udl.

$$\delta_2 = \frac{5}{384} \times \frac{wL^4}{E_c I_c} = 7.82 \downarrow \text{ (due to hogg.) downward.}$$

$\therefore$ $\delta \rightarrow$ net deflectn at Centre.

$$\boxed{\delta_2 - \delta_1 = 0.52\ mm \downarrow}$$ Max

Note: Net moment was hogging but we got deflection ↓ it is due to we got two Point of Contraflexure.



Q8. A simply supported beam is subjected to a live load of 65 KN/m, over a span of 10m.

Size of beam = 400mm × 700mm.

Prestressing force = 1800 KN

Prabolic cable

eccentricity at ends = 0

at Centre = 180 mm

Calculate ~~xxx~~ :-

(I) net deflection of beam at Centre.

(II) net slope at ends

$E_s = 2\times10^5\ N/mm^2$

M_{60} Conc. used.

Ans:

$E_c = 5000\sqrt{f_{ck}}$

$= 5000 \times \sqrt{60}$

$= 38729.83\ N/mm^2$

load Calculation:

$D.L = 0.4\times0.7\times1\times25 = 7\ KN/m.$

$L.L = 65\ KN/m$

$w = 72\ KN/m.$

(I) Net deflection due to prestressing

$$\delta = \frac{\frac{2}{3}\cdot P.e\cdot\frac{L}{2}\cdot\frac{5}{8}\times\frac{L}{2}}{E_c\, I_c}$$

$$\delta = \frac{\frac{2}{3} \times 1800\times10^3 \times 180 \times \frac{10\times10^3}{2} \cdot \frac{5}{8} \times \frac{5000}{2}}{38729.93 \times \frac{400\times700^3}{12}}$$ in.

Calculator

$$\delta = \frac{5}{48} \times \frac{P \cdot e \cdot L^2}{E_c \cdot I_c}$$

$$= \frac{5}{48} \times \frac{1800\times10^3 \times 180 \times 10000^2}{5000\sqrt{60} \times \frac{400\times700^3}{12}}$$

$$= 7.62 \text{ mm} \uparrow$$

(II) deflection due to load.

$$\delta_2 = \frac{5}{384} \frac{w L^4}{E_c I_c}$$

$$= \frac{5}{384} \frac{72 \times 10000^4 \times 12}{5000\sqrt{60} \times 400 \times 700^3} = 21.17 \downarrow \text{ mm}$$

Net deflection

$$= \delta_2 - \delta_1 = 21.17 - 7.62 = 13.55 \text{m} \downarrow$$

(2) Slope.

(a) Due to Pe.

$$\theta_1 = \frac{Pe \cdot L}{3 E_c \cdot I_c} = \frac{1800\times10^3 \times 180 \times 10000}{3\times 5000\sqrt{60} \times \frac{400\times700^3}{12}}$$

$$= 2.44 \times 10^{-3}.$$

(b) due w

$$\theta_2 = \frac{wl^3}{24 E_c I_c} = \frac{72 \times 10000^3}{24 \times 5000\sqrt{60} \times \frac{400 \times 700^3}{12}}$$

$$= \underline{\overline{6.77 \times 10^{-3}}}$$

Net slope.

$$\boxed{\theta = \theta_2 - \theta_1}$$

$$= 6.77 \times 10^{-3} - 2.44 \times 10^{-3}$$

$$= \underline{\overline{4.33 \times 10^{-3}}}$$

2nd Method.

(1) Balancing u.d.l.

$$w_p = \frac{8Ph}{l^2} = \frac{8 \times 1800 \times 0.18}{10^2}$$

$$= \underline{\overline{25.92 \text{ kN/m}}} \uparrow$$

$$\underline{\overline{w = 72 \text{ kN/m}}}$$

Net load.

$$w_{net} = 72 - 25.92$$

$$= 46.08 \text{ kN/m}.$$

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(i) deflection

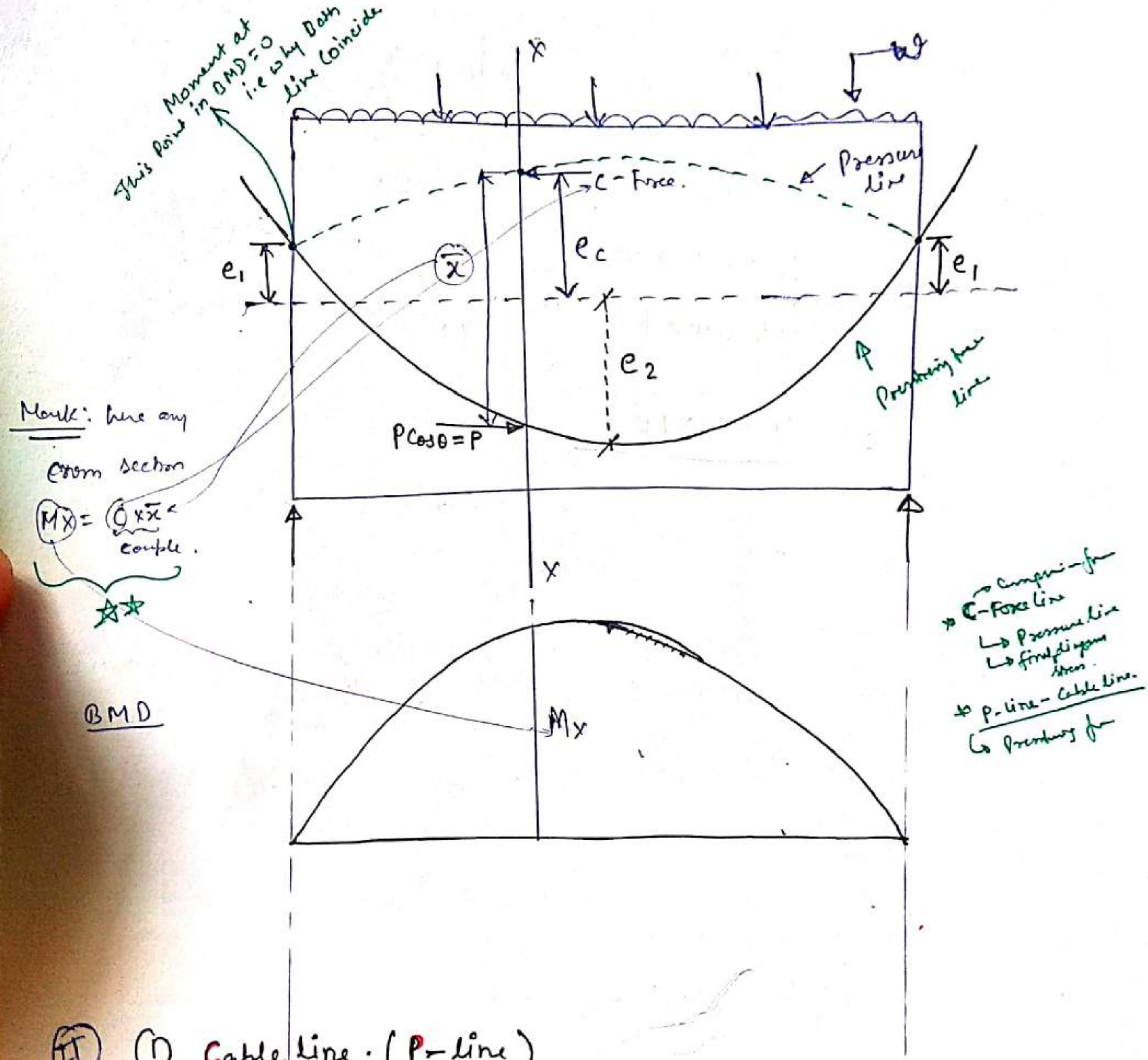
$$\delta_{net} = \frac{5}{384} \times \frac{46.08 \times 10000^4}{5000\sqrt{60} \times \frac{400 \times 700^3}{12}} = 13.55 \text{ mm.}$$

(ii) slope =

$$\theta = \frac{46.08 \times 10000^3}{24 \times 5000\sqrt{60} \times \frac{400 \times 700^3}{12}}$$

$$= \underline{4.33 \times 10^{-3}}$$

Pressure Line and Cable line.



(II) (I) Cable line. (P-line)

Prestressing force line.

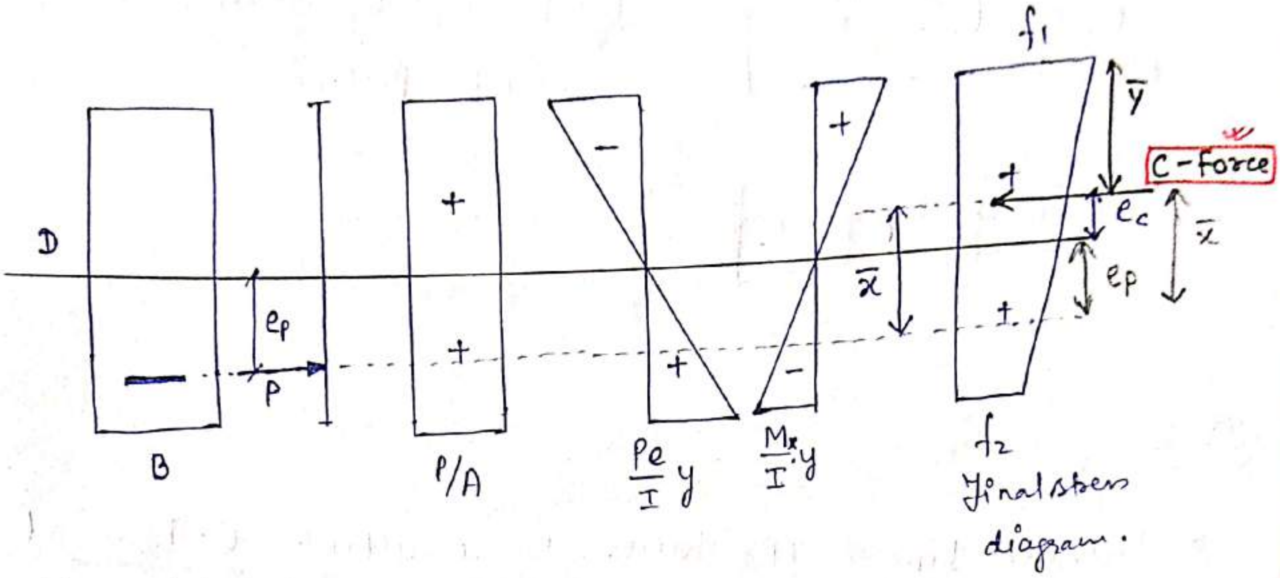
Impt → Stress Concept method is based on this force. (P-force)

→ Cable line (P-line) is the location of prestressing force in the beam at different cross section.

(II) Make a Note: that lever arm changes for every location X-X but force is Constant.

At x - x

By stress Concept Method.



Stress at top:

$$\boxed{f_1 = \frac{P}{A} - \frac{P \cdot e}{I} \cdot y + \frac{M_x}{I} \cdot y}$$

Stress at bottom

$$\boxed{f_2 = \frac{P}{A} + \frac{Pe}{I} y - \frac{M_x}{I} \cdot y}$$

C-Force: "Total Net compressive force acting over the section due to final stress diagram."

$$\boxed{C \text{ Force}} = B \times D \times \left(\frac{f_1 + f_2}{2}\right)$$

$$= \boxed{P - \text{Force}}$$

at all cross section

Location of C-force.

$$\bar{y} = \left(\frac{f_1 + 2 f_2}{f_1 + f_2}\right) \times \frac{D}{3}$$

eccentricity of C-Force

$$\boxed{e_c = \left(\frac{D}{2} - \bar{y}\right)}$$

& ⇒ Total shift of C-line w.r.t P-line

$$\boxed{\bar{x} = e_p + e_c}$$

② Pressure line

→ Pressure line is the locus of resultant C-Force at different cross-section of beam.

→ Pressure line will show the location of Net Compressive force in the beam at different cross-section.

Important properties.

① $\boxed{\text{C-Force} = \text{P Force}}$ at all cross-section of beam.

or

$$\boxed{C = P\cos\theta}$$

② The shift of C-line is proportional to the moment in beam at the particular c/s.

i.e at any cross section

$$\boxed{M_x = P \times \bar{x} = G \times \bar{x}}$$ see fig

$\bar{x}$ = shift of c line. w.r.t P-line

$$\therefore \quad \bar{x} = \frac{M_x}{P} = \frac{M_x}{C_1}$$

→ Shift of C-line w.r.t P-line.

→ eccentricity of C-line

$$\boxed{e_c = \bar{x} - e_p}$$

③ At ends

moment = 0

Shift of C line = 0

→ C-line and P line shall be at same location at cross section where $\boxed{M=0}$

Strength Concept Method.

① Calculate Moment due to all given loads at the section (without any load Balancing)

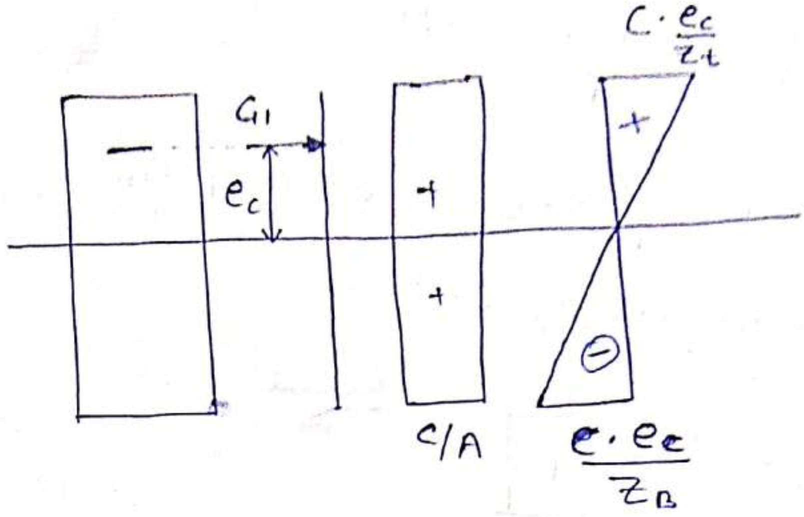
$$= M_x$$

② Shift of C-line w.r.t P-line

$$\boxed{\bar{x} = \frac{M_x}{P}}$$

③ Eccentricity of C-Force.

$$\boxed{e_c = (\bar{x} - e_p)}$$

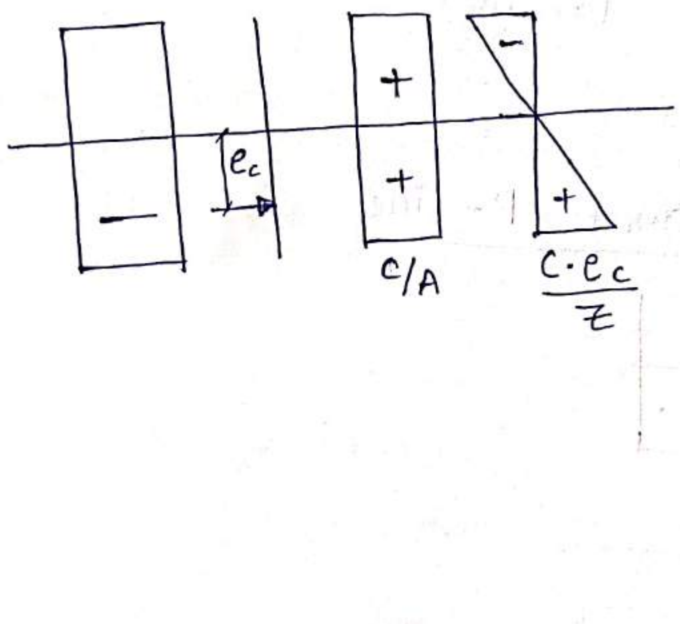


Case 1: if e_c is above NA.

Stress at top:	$= \frac{C}{A} + \frac{C \cdot e_c}{Z_t}$
Stress at bottom:	$= \frac{C}{A} - \frac{C \cdot e_c}{Z_B}$

No effect of Moment due to load is to be added here,

Case 2. if e_c is below N.A

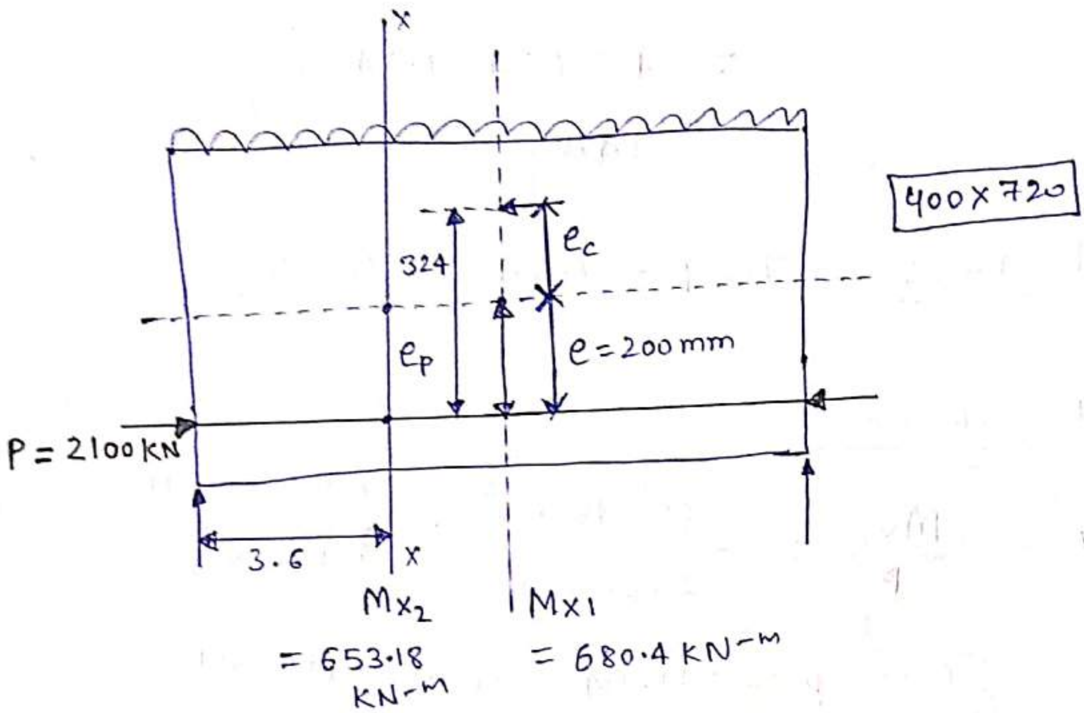


Stress at top	$= \frac{C}{A} - \frac{C \cdot e_c}{Z}$
at bottom	$= \frac{C}{A} + \frac{C \cdot e_c}{Z}$

Q1. Solve by strength concept.

(I) at mid span.

(II) at 3.6 m from support.



ⓐ at mid span.

(I) $M_x = 680.40$ kN-m

(II) shift of C-line w.r.t P-line

$$\bar{x} = \frac{M_x}{P} = \frac{680.40 \times 10^6}{2100 \times 10^3} = 324 \text{ mm}$$

(III) Eccentricity of C-force.

$$e_c = \bar{x} - e_p = 324 - 200$$

$= 124$ mm above NA

(IV) Stress

at top: $\frac{C}{A} + \frac{C \cdot e_c}{Z} = \frac{2100 \times 10^3}{400 \times 720} + \frac{2100 \times 10^3 \times 124}{\frac{400 \times 720^2}{6}}$

$= 7.29 + 7.54$

$= \boxed{14.83}$

at bottom: $= 7.29 - 7.54 = -0.25$

(b) at 3.60 m

(I) $\bar{x} = \frac{M_{x2}}{P} = \frac{653.18 \times 10^6}{2100 \times 10^3} = \frac{653.18 \times 10^6}{2100 \times 10^3} = 311.04$ mm

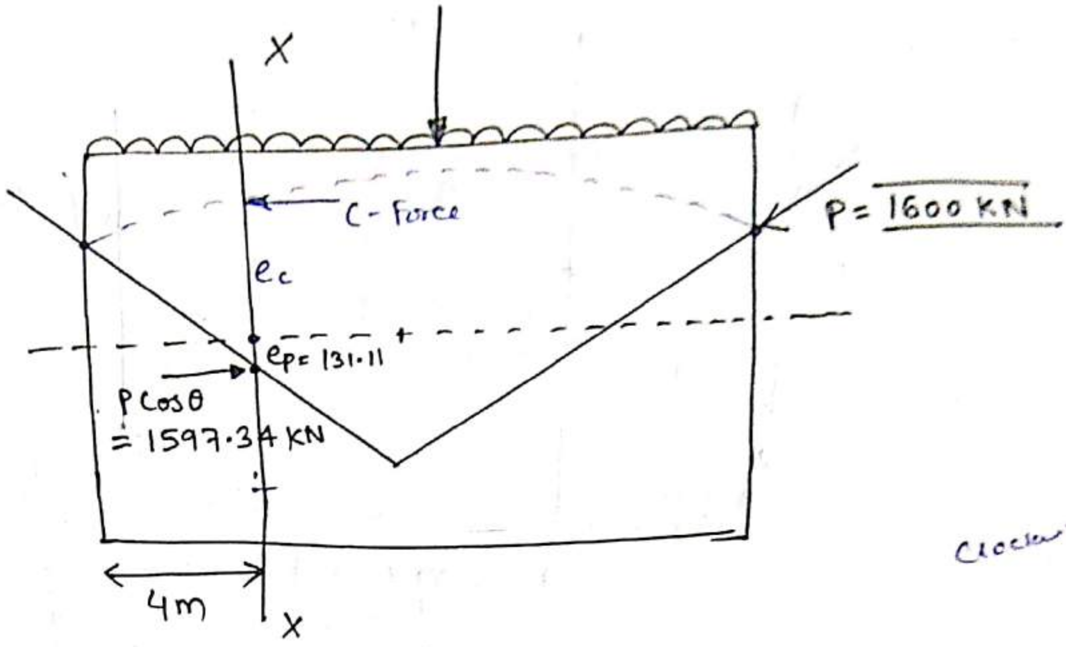
(II) $e_c = \bar{x} - e_p = 311.04 - 200 = 111.04$

(III) at top $= \frac{2100 \times 10^3}{400 \times 720} + \frac{2100 \times 10^3 \times 111.04}{400 \times 720^2/6}$

$= 7.29 + 6.75 = 14.04$

at bottom $= 7.29 - 6.75 = 0.54.$

Q2.



Clockwise

$$\boxed{M_x = 556.90 \text{ KN-m}}$$

Strength concept:

① $\bar{x} = \dfrac{M_x}{P\cos\theta} = \dfrac{556.90 \times 10^6}{1597.34 \times 10^3}$

$= \underline{348.642 \text{ mm}}$

② $e_c = \bar{x} - e_p$

$= 348.642 - 131.11$

$= \underline{217.532} \text{ mm}$ $\quad$ above NA

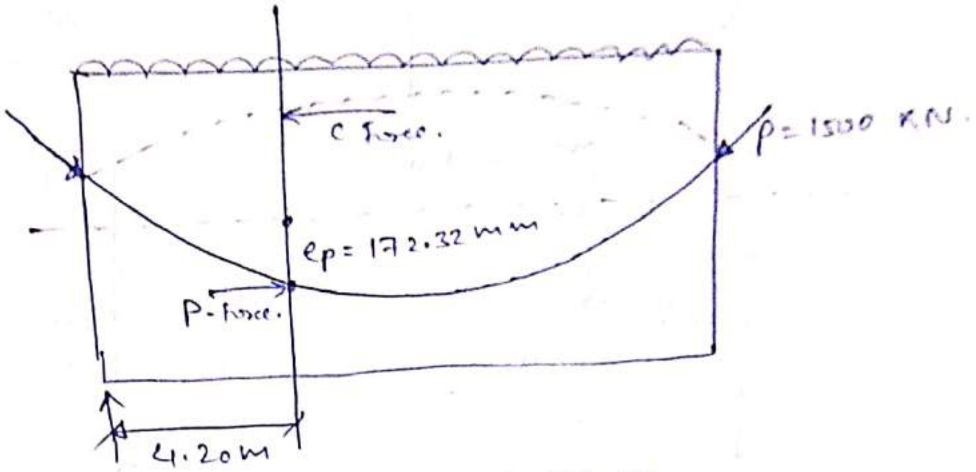
③ Stress

at top $= \dfrac{C}{A} + \dfrac{C \cdot e}{Z} = \dfrac{1597.34 \times 10^3}{350 \times 600} + \dfrac{1597.34 \times 10^3 \times 217.532}{350 \times \frac{650^2}{6}}$

$= 7.02 + 14.10 = \underline{21.12} \text{ N/mm}^2$

at bottom $= 7.02 - 14.10 = -7.08 \text{ N/mm}^2$

Q.3



$M_x = 504.25$ KN-m.

(1) $\bar{x} = \frac{M_x}{P} = \frac{504.25 \times 10^6}{1500 \times 10^3}$

$= 336.167$ mm.

(2) $e_c = \bar{x} - e_p$

$= 336.167 - 172.32$

$= 163.85$ mm.

$\frac{C}{A} = \frac{1500 \times 10^3}{320 \times 800} = 5.86$

$\frac{C \cdot e_c}{Z} = \frac{1500 \times 10^3 \times 163.85}{320 \times 800^2/6} = 7.20$

at top $= 5.86 + 7.20 = 13.06$ N/mm^2

at bottom $= 5.86 - 7.20 = -1.34$ N/mm^2

Q4 at mid span (T-beam)

$A = 330000 \text{ mm}^2$

$I = 2.45 \times 10^{10} \text{ mm}^4$

$\bar{y}_t = 386.36 \text{ mm}$

$y_B = 513.64 \text{ mm}.$

$M_d = 148.50 \text{ kN-m}$

$M_l = 720 \text{ kN·m}.$

$P = 2240 \text{ kN}$

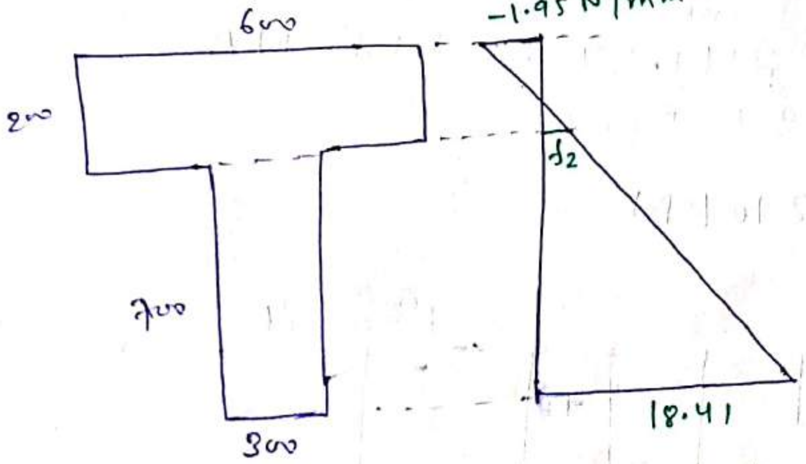
$k = \text{loss} = 20\ \%.$

$P_f = 2240 \times .8$

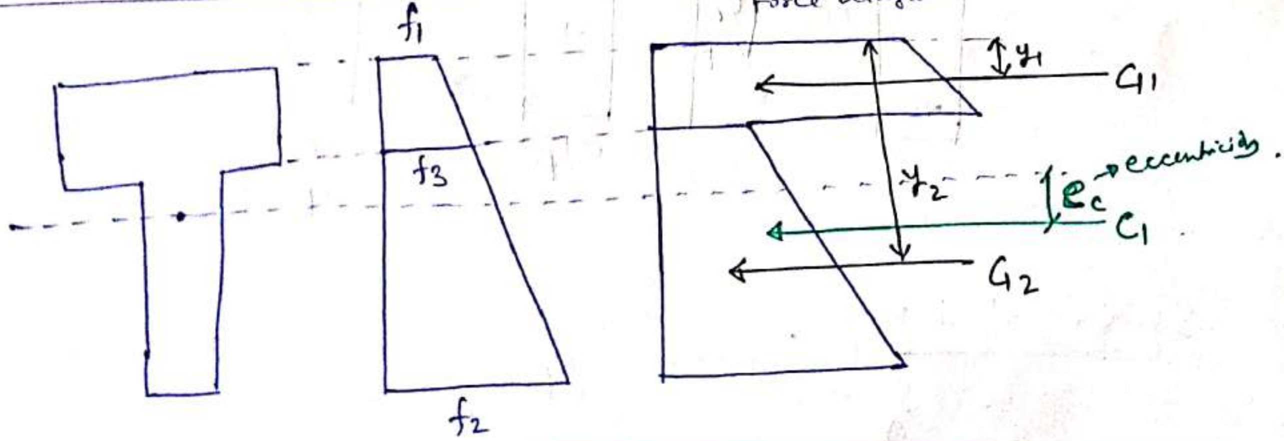
$= 1792 \text{ kN}$

$e = 313.64 \text{ mm}$

at Transfer.



very complex



(#) (i) At Transfer

$M_x = 148.50$ kN-m.

(ii) Shift of c-line w.r.t p-line

$$\bar{x} = \frac{M_x}{P} = \frac{148.50 \times 10^6}{2240 \times 10^3} = 66.295 \text{ mm.}$$

(iii) Eccentricity of c-force.

$e_c = \bar{x} - e_p$

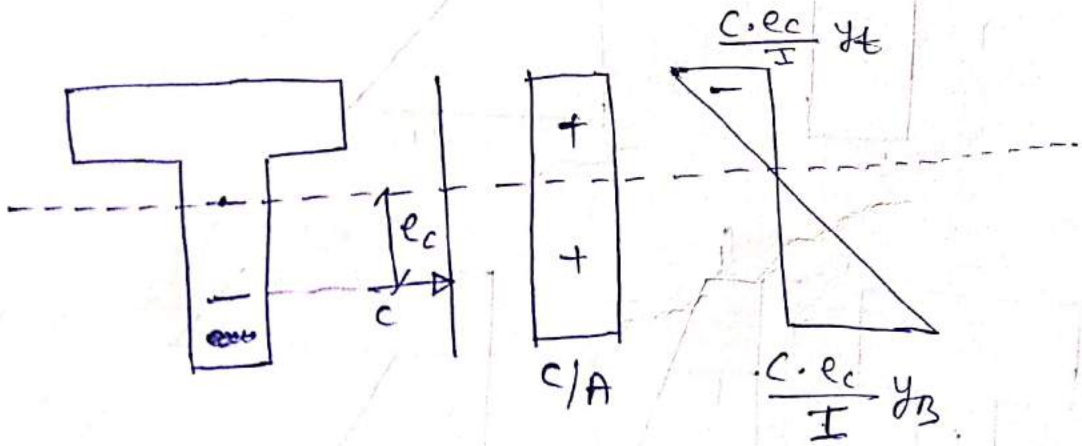
$= 66.295 - 313.64$ mm

$= \underline{-247.345}$ mm.

i.e $= 247.345$ mm below N.A

$C = P = 2240$ KN

(iv)



Stress at top.

$$= \frac{C}{A} - \frac{C \cdot e_c}{I} y_t$$

$$= \frac{2240 \times 10^3}{330000} - \frac{2240 \times 10^3 \times 247.345}{2.45 \times 10^6} \times 386.36$$

$=$

$= 6.79 - 8.74 = -1.95 \ N/mm^2.$

Stress at Bottom.

$$= \frac{2240 \times 10^3}{334000} + \frac{2240 \times 10^3 \times 297.345}{2.45 \times 10^{10}} \times 513.64$$

$= 6.79 + 11.62$

$= \boxed{18.41} \ N/mm^2$

② At final stage.

① Total Moment = 148.5 + 720

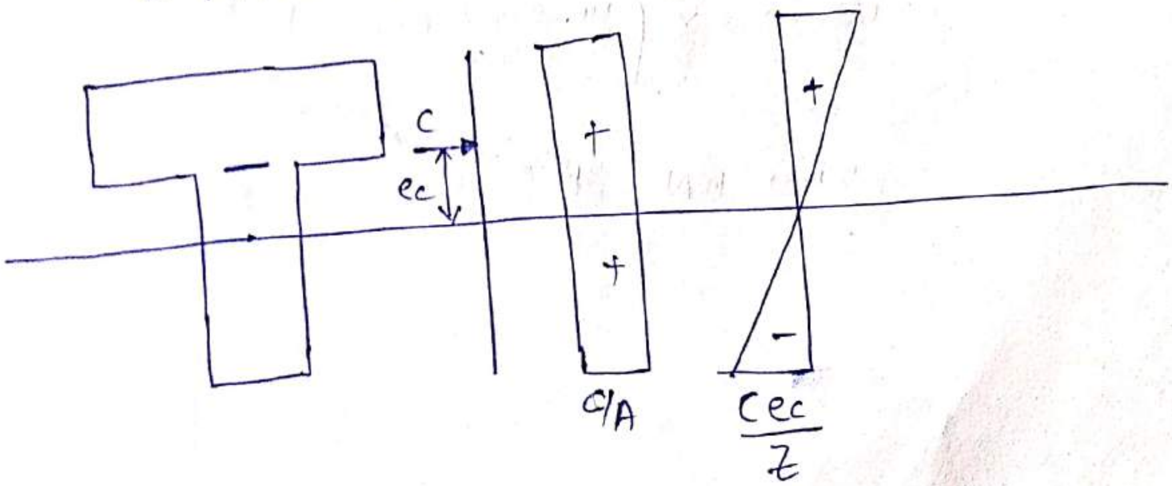
= 868.50 KN-m.

② $$\bar{x} = \frac{868.50 \times 10^6}{1792 \times 10^3} = 484.654 \text{ mm}$$

③ $e_c = \bar{x} - e_p$

$= 484.654 - 313.64$

$= 171.014$ (above NA)



Stresses at top.

$$= \frac{C}{A} + \frac{C \cdot e_c}{I} \cdot y_t$$

$$= \frac{1792 \times 10^3}{330000} + \frac{1792 \times 10^3 \times 171.014}{2.45 \times 10^{10}} \times 386.36$$

$$= 5.43 + 4.83 = 10.26$$

Stress at Bottom -

$$= \frac{1792 \times 10^3}{330000} - \frac{1792 \times 10^3 \times 171.014}{2.45 \times 10^{10}} \times 513.64$$

$$= 5.43 - 6.42$$

$$= -0.99$$

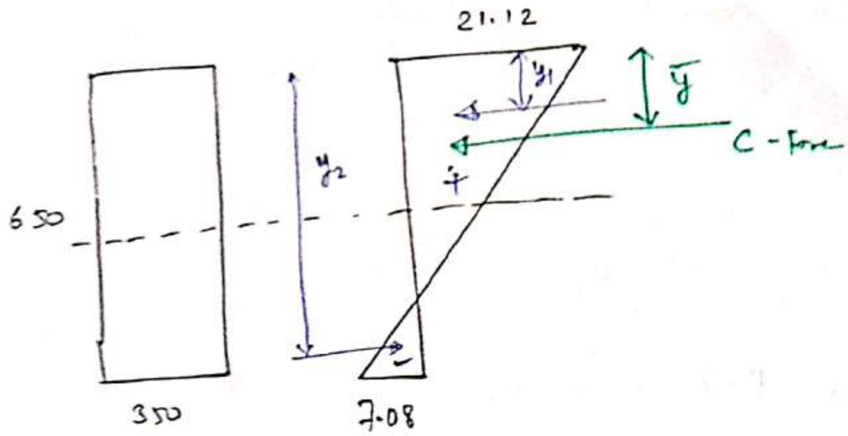
Proof C Force = P Force

In Q1 $\quad C = B \times D \times \left(\frac{f_1 + f_2}{2}\right)$

$$= 400 \times 720 \times \left(\frac{14.825 + (-0.25)}{2}\right)$$

$$= 2100 \text{ kN} = P \text{ Force.}$$

Q2.



$$C\text{-force} = 350 \times 650 \times \left(\frac{21.12 - 7.08}{2}\right) \times \frac{1}{10^3}$$

$$= 1597 \text{ kN} = P \text{ force}$$

$$\boxed{\bar{y} = \frac{f_1 + 2f_2}{f_1 + f_2} \times \frac{D}{3}}$$

$$= \frac{21.12 + 2 \times (-7.08)}{21.12 + (-7.08)} \times \frac{650}{3}$$

$$= \underline{\overline{107.407}} \text{ mm}$$

$\bar{y}$ can be directly calculated from this formula considering it a Trapezoid. But put ~~compressive~~ Tensile stress (-ve) magnitude

$$e_c = \frac{D}{2} - \bar{y} = 325 - 107.407$$

$$= \underline{\overline{217.593}}$$