Design Of concrete Structure – I

INTRODUCTION

1. To determine the modulus of rupture, the size of test specimen used is

a) 150 x150 x500 mm
b) 100 x100 x700 mm
c) 150 x150 x700 mm
d) 100 x100 x500 mm

2. The property of fresh concrete, in which the water in the mix tends to rise to the surface while placing and compacting is called

a) segregation
b) bleeding
c) bulking
d) creep

3. Select the incorrect statement
a) Lean mixes bleed more as compared to rich ones.
b) Bleeding can be minimized by adding pozzuolana finer aggregate.
c) Bleeding can be increased by addition ‘of calcium chloride.
d) none of the above

4. The property of the ingredients to separate from each other while placing the concrete is called

a) segregation
b) compaction
c) shrinkage
d) bulking

5. The factor of safety for

a) steel and concrete are same
b) steel is lower than that for concrete
c) steel is higher than that for concrete
d) none of the above

6. Examine the following statements :

i) Factor of safety for steel should be based on its yield stress,
ii) Factor of safety for steel should be based on its ultimate stress,
iii) Factor of safety for concrete should be based on its yield stress,
iv) Factor of safety for concrete should be based on its ultimate stress.

The correct statements are

a) (i) and (iii)
b) (i)and(iv)
c) (ii) and (iii)
d) (ii) and (iv)

7. Explain Working stress method and Limit state method of structural Design Philosophy

8. Define (i) Limit State (ii) Characteristic strength (iii) Partial Safety Factor

9. Write Advantages and Disadvantages of R.C.C. Structures.

10. Discuss different kinds of loads to be taken into account for the design

DESIGN OF BEAMS : SINGLY REINFORCED BEAM

1. If the depth of actual neutral axis in a beam is more than the depth of critical neutral axis, then the beam is called

a) balanced beam
b) under-reinforced beam
c) over-reinforced beam
d) none of the above

2 Maximum quantity of water needed per 50 kg of cement for M 15 grade of concrete is

a) 28 liters
b) 30 liters
c) 32 liters
d) 34 liters

3. Minimum grade of concrete to be used in reinforced concrete as per IS: 456-1978 is

a) M15
b) M20
c) M10
d) M25

4. Modulus of elasticity of steel as per IS : 456-2000 shall be taken as

a) 20 kN/cm^2
b) 200 kN/cm^2
c) 200kN/mm^2
d) $2x10^6 N/cm^2$

5. For a simply supported beam of span 15m, the minimum effective depth to satisfy the vertical deflection limits should be

a) 600 mm
b) 750 mm
c) 900 mm
d) more than 1 m

6. Explain the modes of failure for Under- reinforced and Over- reinforced beam

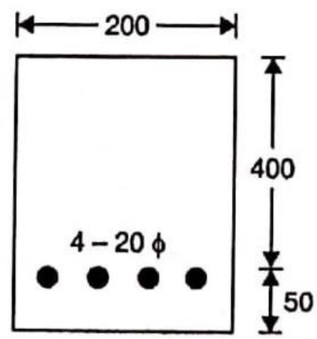
7. Sketch neatly the Design Stress and Strain Block Parameters and derive equation for Depth of Neutral Axis and Moment of Resistance for a balanced beam section

8. A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 no. 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. Materials used are: M20 and Fe 425

9. Design a rectangular beam to resist a bending moment equal to 45 KN-m using M20 mix and Fe 415 grade steel.

10. Design a rectangular beam for 7 m effective span which is subjected to a dead load of 15 KN/m and a live load of 12 KN /m. Use M20 mix and Fe 415 grade steel

11. Determine the actual stresses in steel for section shown in fig. if materials used are M20 and Fe 415 grade of steel.



Section

12. A 5 m long simply supported beam carries a superimposed load of 20 KN/m. Design the mid span section if its effective depth is kept constant at 500 mm using

    (i) Working stress method
    (ii) Limit state method.

Use M20 concrete and Fe 415 grade steel

13. A reinforced concrete beam has width equal to 300 nun and total depth equal to 800 mm with a cover of 40 mm to the Centre of reinforcement. Design the beam if it is subjected to a total bending moment of 140 KN-m. Use M20 concrete and Fe 415 grade steel. Compare the design with that obtained by working stress method.

DESIGN OF BEAM : DOUBLY REINFORCED BEAM

1. If the depth of actual neutral axis of a doubly reinforced beam

   (a) Is greater than the depth of critical neutral axis, the concrete attains its maximum stress earlier
   (b) Is less than the depth of critical neutral axis, the steel in the tensile zone attains its maximum stress earlier
   (c) Is equal to the depth of critical neutral axis; the concrete and steel attain their maximum stresses Simultaneously
   (d) All the above

2. The minimum thickness of the cover at the end of a reinforcing bar should not be less than twice the diameter of the bar subject to a minimum of

(a) 10 mm
(b) 15 mm
(c) 20 mm
(d) 25 mm

3. A higher modular ratio shows

(a) higher compressive strength of concrete
(b) lower compressive strength of concrete
(c) higher tensile strength of steel
(d) lower tensile strength of steel

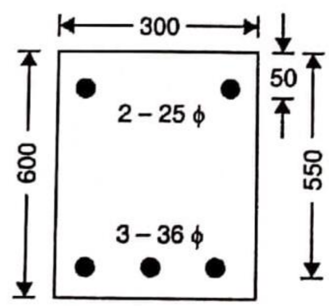
4. According to the steel beam theory of doubly reinforced beams

(a) Tension is resisted by tension steel
(b) Compression is resisted by compression steel
(c) Stress in tension steel equals the stress in compression steel
(d) All the above

5. Maximum strain at the level of compression steel for a rectangular section having effective cover to compression steel as d′ and neutral axis depth from compression face X_u is

(a) $0.0035(1-d'/X_u)$
(b) $0.002(1-d'/X_u)$
(c) $0.0035(1-X_u/d')$
(d) $0.002(1-X_u/d')$

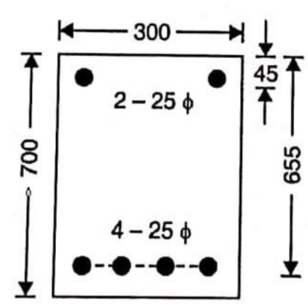
6. The cross-sectional dimensions of a doubly reinforced beam are shown in Fig. Determine the moment of resistance on the beam section. Assume M20 concrete and Fe 250 steel.



Section

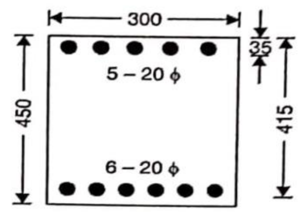
7. The cross-sectional dimensions of a doubly reinforced beam are shown in Fig .(refer question No-6). Determine the moment of resistance on the beam section. Assume M20 concrete and Fe 415.

8. Determine the ultimate moment of resistance of the doubly reinforced section shown in Fig. Assume M20 concrete and Fe 415 steel.



Section

9. Determine the ultimate moment of resistance of the doubly reinforced section shown in Fig. Assume M20 concrete and Fe 250 steel.



Section

10. Design a rectangular beam for an effective span of 6 m. The superimposed load is 80 KN/m and size of beam is limited to 300 mm x 700 mm overall. Use M20 mix and Fe 415 grade steel.

DESIGN OF BEAM: FLANGED – BEAM

1. A *T*-beam behaves as a rectangular beam of a width equal to its flange if it's neutral axis

(a) Remains within the flange
(b) Remains below the slab
(c) Coincides the geometrical centre of the beam
(d) None of these

2. For the design of a simply supported T-beam the ratio of the effective span to the overall depth of the Beam is limited to

(a) 10
(b) 15
(c) 20
(d) 25

3. The width of the rib of a T-beam, is generally kept between

(a) 1/7 to 1/3 of rib depth
(b) 1/3 to 1/2 of rib depth
(c) 1/2 to 3/4 of rib depth
(d) 1/3 to 2/3 of rib depth

4. The neutral axis of a T-beam exists

(a) Within the flange
(b) At the bottom edge of the slab
(c) Below the slab
(d) All the above

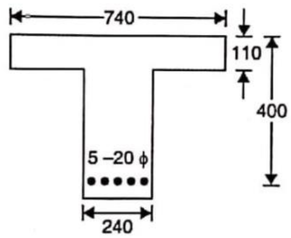
5. The width of the flange of a T-beam should be less than

(a) One-third of the effective span of the T-beam
(b) Distance between the centres of T-beam
(c) Breadth of the rib plus twelve times the thickness of the slab
(d) Least of the above

6. The width of the flange of a L-beam, should be less than

(a) One-sixth of the effective span
(b) Breadth of the rib + four times thickness of the slab
(c) Breadth of the rib + half clear distance between ribs
(d)Least of the above

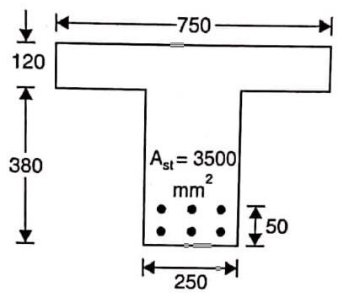
7. Find the moment of resistance of T-beam having following data in Fig. Use M20 mix of concrete and Fe 415 grade steel.



Section

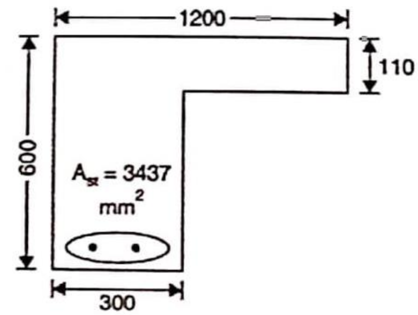
8. Determine the moment of resistance of T-beam having effective depth 400 mm ,flange width 740 mm ,flange thickness 90 mm ,web thickness 240 mm and area of tension steel is 5-20 mm dia .Use M 20 and Fe 415.

9. Calculate the moment of resistance of a T-beam as shown in Fig. assuming M20 mix and Fe 415 steel.



Section

10. Find the moment of resistance of L.-beam having following data in Fig. Use M20 mix and Fe 415 grade steel.



Section

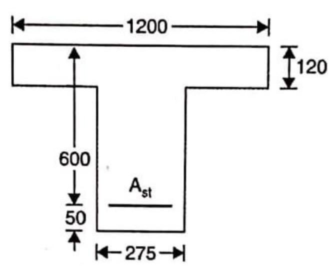
11. A T-beam has b_i = 1200 mm, b_{tu} -= 275 mm, d = 600 mm and D_f = 120 mm. Find the reinforcement for a factored moment of resistance of

(a) 375

(b) 625

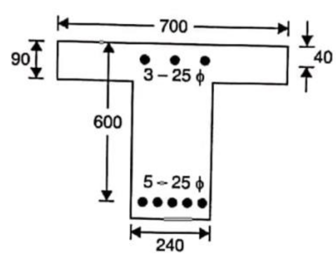
(c) 860 kN-m.

Use M20 **mix** and Fe 415 grade steel.



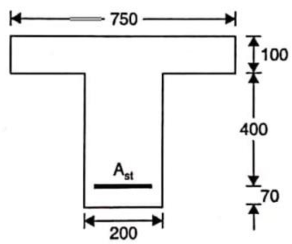
Section

12. Calculate the moment of resistance of a T-beam as shown in Fig. assuming M15 mix and Fe 415 grade steel.



13. Determine the moment of resistance of T-beam of question 12 if A_{st} = 6-25 mm dia. and A_{sc} = 2-20 mm dia. bars are used.

14. Calculate the amount of steel required in a T-beam to develop a factored moment of resistance of 450 KN-m .The dimensions of the beam section are given in Fig. Use M20 mix and Fe 415 grade steel.



Section

BOND AND TORSION

1. Torsion resisting capacity of a given reinforced concrete section

   (a) Decreases with decrease in stirrups spacing
   (b) Decreases with increase in longitudinal bars
   (c) Does not depend upon longitudinal bars and stirrups
   (d) Increases with increase in longitudinal bars and stirrups spacing

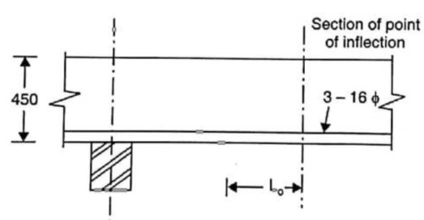
2. The transverse torsional reinforcement in RCC beams can be provided as

   (a) Like ties
   (b) Closed loops only
   (c) Open or closed loops
   (d) Helical loop only.

3. Discuss the procedure for the design of beam subjected to combined bending and torsion in concrete.

4. A simply supported beam is 260 mm by 520 mm and has 2-20 mm HYSD bars going in to the support. If the shear force at the center of support is 115 KN at working loads, determine the anchorage length. Assume M20 and Fe 415.

5. A continuous beam 250x 450 mm carries 3-16 mm longitudinal bars beyond the point of inflection in the lagging moment region as shown in Fig. The shear force at the point of inflection is 110 KN at working loads. Check if the beam is safe in bond. Use M20 and Fe 415. Fig. as shown below.



Section

6. A rectangular beam of size 230mm x 400 mm overall depth, is reinforced with 2-10 mm bars at top and 3-16 mm at bottom being tension reinforcement. It is subjected to characteristic loads, shear force of 18 KN, a torsional moment of 1.2 KN-M and bending moment of 18 KN-m. Check torsion reinforcement. Assume M 20 and Fe 415.

7. A beam of rectangular section is a multistory frame 250 mm x 500 mm deep. The section is subjected to an ultimate bending moment 55 KN-m, ultimate torsional moment 30 KN-m and ultimate shear force 40 KN. Using M 20 and Fe 415.Design suitable reinforcement in section. Effective cover to steel=50 mm.

DESIGN OF SHEAR

1. The maximum shear stress in concrete of are in forced cement concrete beam is

(a) Shear force/(Lever arm × Width)
(b) Lever arm/(Shear force × Width)
(c) Width /(Lever arm × shear force)
(d) None of these

2. The shear capacity of an RCC beam without shear reinforcement is

(a) τcbd
(b) τvbd
(c) τvbd^2
(d) τvbd^2

3. Shear reinforcement is provided in the form of :

(a) Vertical bars
(b) Inclined bars
(c) Combination of vertical and inclined bars
(d) Any one of the above

4. The minimum percentage of shear reinforcement in R.C.C beams is

(a) $0.85/f_y$
(b) 0.4
(c) 4
(d) $40S_v/0.87f_yd$

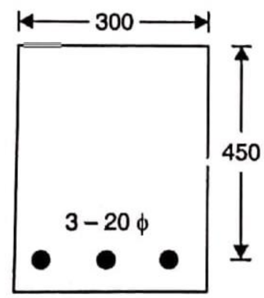
5. For M20 grade of concrete , the maximum shear stress has not exceed

(a) 1.6N/mm^2
(b)1.9N/mm^2
(c) 2.8N/mm^2
(d) 2.2N/mm^2

6. A reinforced concrete beam 230 mm wide and 460 mm effective depth is subjected to a shear force of 60 KN at support. The tensile reinforcement is 0.5%. Check the adequacy of the shear design, if M 20 mix and Fe 250 grade steel are used.

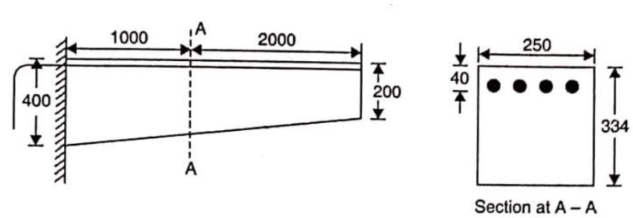
7. Determine the shear reinforcement for the beam section in question 6, if shear force of the section is 90 kN. Use M20 mix and Fe 415 grade steel.

8. The beam shown in Fig. is subjected to factored shear force of 150 KN. If 20 N/mm^2 and f = 415 N/mm^2, calculate the shear reinforcement, if bending of 1 bar of 20 mm dia. at an angle of 45°.



Section

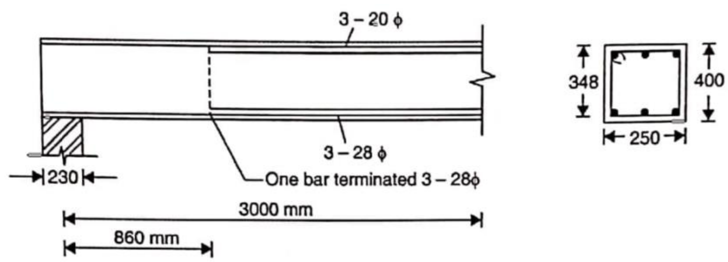
9. Design the shear reinforcement in a tapered cantilever beam of constant width 250 mm whose section at 1 m from the face of support as shown in Fig. It consists of 2-22 mm and 2-18 mm bars. Redesign the shear reinforcement i f 2-18 mm bars are curtailed at this section. Take shear force = 100 kN and bending moment at this section =150 kN-m. Assume M20 mix and 415 grade steel for shear stirrups.



Cantilever beam.

10. Design the shear reinforcement for one way slab of effective span 4.16 m, of thickness 200 mm with cover 35 mm are subjected to a factored load 15 kN/m^2. Slab is provided with 4-10 mm bars @ 250 mm c/c at support per meter. Assume M20 concrete and Fe 415 grade steel.

11. The beam shown in Fig. is subjected to factored shear force at support 101.25 KN and at mid span 33.75 kN. if f_{ck}, = 20 N/mm^2 and f_y = 415 N/mm^2. Calculate the shear reinforcement.



Section

SLAB DESIGN

1. Spacing of main bars in an RCC slab shall not exceed

(a) 3 times the effective depth
(b) 3 times then overall depth
(c) 30 times the dia. of main bar
(d) 30 cm

2. Minimum area of reinforcement in RCC slab shall be

(a)$\mathbf{5}$ = $\mathbf{0.12}$% $\boldsymbol{of\ total\ area}$
(b)$\boldsymbol{Fe250}$ = $\mathbf{0.15}$% $\boldsymbol{of\ total\ area}$
(c) Both a and b
(d) None

3.Maximum diameter of steel bar in RCC slab

(a)$\boldsymbol{Thickness\ of\ slab}$ $\mathbf{3}$
(b)$\boldsymbol{Thickness\ of\ slab}$ $\mathbf{8}$
(c)$\boldsymbol{kness\ \ of\ \ slab\ \ 4}$
(d)$\boldsymbol{Thickness\ of\ slab}$ $\mathbf{6}$

4. Half of the main steel in a simply supported slab is bent up near the support at a distance of x from the center of slab bearing where x is equal to

(a) 1/3
(b) 1/5
(c) 1/7
(d) 1/10

5. When shear stress exceeds the permissible limit in a slab, then it is reduced by

(a) increasing the depth
(b) providing shear reinforcement
(c) using high strength steel
(d) using thinner bars but more in number

6. Distinguish clearly between One way and Two way slab

7. What is meant by Aspect Ratio. State the limits of the same for One way and two way slabs. Also show the sharing of the loads on the adjacent beams of both the slabs by sketch.

8. Design a one way slab, with a clear span of 4.0 m simply supported on 300 mm thick masonry walls, and subjected to a live load of 5 kN/m^2 and surface finish 0.6 kN/m^2. Assume that the slab is subjected to moderate exposure conditions. Use Fe 415 grade of steel.

9. **Design a continuous R.C. slab for a hall 7.5 m wide and 14.5 in long (clear span). The slab is supported on R.C.C. beams, each 300 mm wide which are monolithic. The ends of the slab are supported on walls, 230 mm wide. The specified floor loading of a live load of 3 kN/m^2 and a dead load (due to floor finish, partitions etc.) of 1.5 KN/ m^2 in addition to self-weight. Assume Fe 415 steel and M.25 concrete.**

10. Design a two way slab for a residential roof with internal dimension 4.5 m x 6.0 m and 230 mm thick brick wall all around. Assuming live load 2 kl\l/m^2 and a finish load of I KN/m^2. Use M20 and Fe 425. Edge conditions: Simply supported on all the sides of wall. Assume that the slab corners are free to lift up and exposure conditions are mild.

11. Design a two way slab for a residential roof with internal dimension 4.5 m x 6.0 m and 230 mm thick brick wall all around. Assuming live load 2 KN/m^2 and a finish load of I KN/m2. Use M20 and Fe 425. Edge conditions: Simply supported on all the sides of wall. Assume that the slab corners are prevented from lifting and exposure conditions are mild.

COLUMN DESIGN

1. Spacing between longitudinal bars measured along the periphery of RCC columns should not exceed

(a) 150mm
(b) 250mm
(c) 300mm
(d) 500mm

2. The diameter of transverse reinforcement of columns should be equal to one fourth of the diameter of the main steel rods but not less than

(a) 4mm
(b) 5mm
(c) 6mm
(d) 7mm

3. The minimum number of longitudinal bars provided in rectangular RCC column

(a) 2
(b) 4
(c) 6
(d) 8

4. The pitch of lateral ties should not exceed

(a)The least lateral dimension
(b)16 times the diameter of longitudinal bars
(c)300mm
(d)All of these

5. The minimum number of main reinforcement bars provided in RC circular column

(a) 2
(b) 3
(c) 4
(d) 6

6. The limit of percentage of longitudinal reinforcement in a column is given by

(a) 0.15 -2%
(b) 0.8 -4%
(c) 0.8 -6%
(d) 0.8 –8%

7. In limit state of collapse for direct compression, the maximum axial compressive strain in concrete is

(a) 0.002
(b) 0.003
(c) 0.0035
(d) 0.004

8 . The diameter of ties in a column should be

(a) more than or equal to one fourth of diameter of main bar
(b) more than or equal to 5 mm
(c) more than 5 mm but less than one-fourth of diameter of main bar
(d) more than 5 mm and also more than one-fourth of diameter of main bar

9. Enumerate the difference between short and slender columns. State the code specifications for:

(a) minimum eccentricity for design of columns
(b) longitudinal reinforcement
(c) lateral ties.

10. What is difference in behavior of short and long compression members?

11. Calculate the area of steel required for a short R.C. column 400 mm x 450 mm in cross-section to carry an axial load of 1200 KN. Assume concrete grade M 20 and Fe 415 steel grade,

12. Design the reinforcement in a column of a 450 mm x 600 mm, subject to an axial load of 200 kN under service dead load and live loads. The column has an unsupported length of 3.0 in and is restrained in both directions. Use M 20 concrete and Fe 415 steel.

13. Design the reinforcement in a spiral column of 450 mm diameter subjected to service load of 1200 KN. The column has an unsupported length of 3.4 tn. Use M 25 concrete and Fe 415 steel. Assume effective length to be equal to unsupported length.

FOOTING DESIGN

1 .As per IS 456 , the minimum nominal cover specified for footing is

(a) 25mm
(b) 40mm
(c) 50mm
(d) 75mm

2. In a combined footing for two columns carrying unequal loads, the maximum hogging bending moment occurs at

(a) Less loaded column
(b) More loaded column
(c) A point of the maximum shear force
(d) A point of zero shear force

3. In a combined footing if shear stress exceeds 5 kg/cm^2, the nominal stirrups provided are:

(a) 6 legged
(b) 8 legged
(c) 10 legged
(d) 12 legged

4. Explain one way shear check and two way shear check for footing design.

5. Write the design steps for the RC combined footing.

6. Sketch reinforcement detail of a rectangular combined footing to be provided for two columns. Draw plan, longitudinal and cross section.

7. Design a footing for an axially loaded square column of 450 mm side, transmitting a load of P„ = 1000 KN and safe bearing capacity of soil is 300 KN/m^2. Use M 20 grade of concrete and Fe 415. Draw sectional elevation and plan showing reinforcement details.

8. A column carries axial load P„ 1200 KN. Design an isolated rectangular footing for the column. Safe bearing capacity of soil is 250 KN/m^2. The column size is 300 mm x 500 mm. Use M 20 grade of concrete and Fe 415 steel. Draw sectional elevation and plan showing reinforcement details.

9. Design a square footing for a 400 mm x 400 mm size column, carrying a direct load of 800 kN and subjected to a moment of 70 kN-m. The safe bearing capacity of soil is 150 KN/m^2. Use M 20 grade concrete and Fe 415.

SINGLY REINFORCED BEAM

Question - 8 : *A rectangular beam 230 mm wide and 520 mm effective depth is reinforced with 4 no. 16 mm diameter bars. Find out the depth of neutral axis and specify the type of beam. Materials used are: M20 and Fe 415.*

Solution. Given data

$$f_{ck} = 20 \text{ N/mm}^2 \quad b = 230 \text{ mm}$$
$$f_y = 415 \text{ N/mm}^2 \quad d = 520 \text{ mm.}$$

When section is reinforced with 4 no. 16 mm diameter.

Force of compression $= 0.36 f_{ck} \cdot b \cdot x_u$

$= 0.36 \times 20 \times 230 \times x_u$

$= 1656\, x_u$ N

Force of tension $= 0.87 f_y A_{st}$

$$= 0.87 \times 415 \times (4 \times 201) \left(A_{st} = 4 \times \frac{\pi}{4} \times 16^2 \text{ mm}^2 \right)$$

$= 290284.2$ N

or, $$1656\, x_u = 290284.2$$

$$x_u \simeq 175.3 \text{ mm}$$

Limiting value of the depth of neutral axis

$$x_{u,\max} = 0.48\, d = 0.48 \times 520 = 249.6 \text{ mm} > 175.3 \text{ mm}$$

Hence, given section is *under reinforced section.*

and depth of neutral axis = 175.3 mm

Qusetion - 9 : *Design a rectangular beam to resist a bending moment equal to 45 kN-m using M20 mix and Fe 415 grade steel.*

Solution. The beam will be designed so that under the applied moment both materials reach their maximum stresses. Let us assume ratio of effective depth to breadth of the beam equal to 2.

For a balanced design

Factored B. M. = load factor × B.M. = 1.5 × 45 = 67.5 kN-m

Limiting depth of neutral axis $x_{u,\max} = 0.48\,d$

Moment of resistance

$$M_{u,\lim} = 0.36 f_{ck} \cdot b \cdot x_{u,\max} (d - 0.42\, x_{u,\max})$$

$$67.5 \times 10^6 = 0.36 \times 20 \times b \times 0.48\, d\, (d - 0.42 \times .48\, d)$$

$$67.5 \times 10^6 = 2.76\, bd^2$$

Since, $\frac{d}{b} = 2 \quad \text{or} \quad b = \frac{d}{2}$

$\therefore \quad 67.5 \times 10^6 = 2.76 \times \frac{d}{2} \times d^2$

or, $d = 366$ mm and $b = 200$ mm

Adopt $D = 450$ mm, $d = 410$ mm

Balanced area of steel is given by

$$P_{\lim} = 0.414 \times \left(\frac{f_{ck}}{f_y}\right)\left(\frac{x_{u,\max}}{d}\right)$$

$$= 0.414 \times \left(\frac{20}{415}\right) \times 0.48 = 0.0096$$

$\therefore \quad A_{st} = 0.0096 \times 200 \times 410 = 788 \text{ mm}^2$

Minimum area of steel

$$A_{so} = \frac{0.85\, bd}{f_y} = \frac{0.85 \times 200 \times 410}{415}$$

$$= 168 \text{ mm}^2 < 788 \text{ mm}^2 \quad \text{(O.K.)}$$

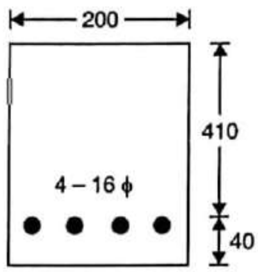
Provide 4–16 mm bars giving total area

$$= 4 \times 201 = 804 \text{ mm}^2 > 788 \text{ mm}^2 \quad \text{(O.K.)}$$

Actual area provided $= \frac{804}{200 \times 450} = 0.0089$

or, 0.98% < 4% (Maximum) (O.K.)

The reinforcement detail is shown in Fig.



Section

Question - 10 : *Design a rectangular beam for 7 m effective span which is subjected to a dead load of 15 KN/m and a live load of 12 kN/m.*

Use M20 mix and Fe 415 grade steel.

Solution. For a simply supported beam (as per clause 23.2 IS: 456–2000)

Let us take a value of $\frac{l}{d} = 12$

For $\quad l = 7000 \text{ mm} \quad d = \frac{7000}{12} = 584 \text{ mm}$

Let us adopt $\quad b = 300 \text{ mm} \quad d = 600 \text{ mm} \quad D = 650 \text{ mm}$

Density of concrete $\quad = 25 \text{ kN/m}^3$

Self weight of beam $\quad W_d = 25 \times 0.300 \times 0.600 \times 1$

$= 4.5 \text{ kN/m}$

Superimposed $\quad$ D.L. $= 15$ kN/m

Superimposed $\quad$ L.L. $W_L = 12$ kN/m

∴ Total load $\quad = 31.5$ kN/m

∴ Factored load $\quad W_u =$ load factor × total load

$= 1.5 \times 31.5 = 47.25 \text{ kN/m}$

Factored B.M. $\quad M_u = W_u \cdot \frac{l^2}{8} = 47.25 \times \frac{7^2}{8} = 289.40 \text{ KN-m}$

For a balanced design

limiting depth of neutral axis $x_{u,\,max} = 0.48\,d$

∴ Factored B.M. = Moment of resistance with respect to concrete

$= 0.138 f_{ck}\, bd^2$

$289.40 \times 10^6 = 0.138 \times 200 \times 300 \times d^2$

or, $\quad d = 591 \text{ mm} \simeq 600 \text{ mm}$

The area of steel is given by

$$P_{\lim} = 0.414 \times \frac{f_{ck}}{f_y} \times \frac{x_{u,\max}}{d}$$

$$= 0.414 \times \frac{20}{415} \times 0.48 = 0.0096$$

$\therefore$ $$A_{st} = 0.0096 \times 300 \times 600 = 1728 \text{ mm}^2$$

Minimum area of steel

$$A_{so} = \frac{0.85\,bd}{f_y} = \frac{0.85 \times 300 \times 600}{415}$$

$$= 369 \text{ mm}^2 < 1728 \text{ mm}^2 \quad \text{(O.K.)}$$

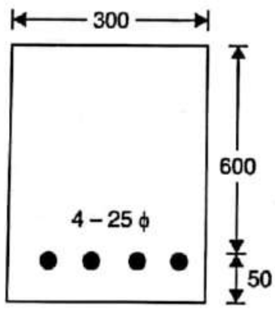
Provide 4–25 mm bars giving total area

$$= 4 \times 491 = 1964 \text{ mm}^2 > 1728 \text{ mm}^2 \quad \text{(O.K.)}$$

Actual area provided $$= \frac{1964}{300 \times 650} = 0.0109$$

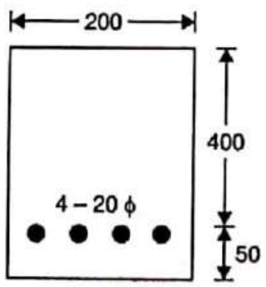
or, $$1.09\% < 4\% \text{ (maximum)} \quad \text{(O.K.)}$$

The reinforcement detail is shown in Fig.



Section

Question - 11 : *Determine the actual stresses in steel for section shown in Fig. if materials used are M20 and Fe 415 grade of steel.*



Section

Solution. Given data

$$b = 200 \text{ mm} \quad A_{st} = 4 \times 314 = 1256 \text{ mm}^2$$

$$d = 400 \text{ mm}$$

Stresses $\quad f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$

Limiting depth of neutral axis $\dfrac{x_{u,max}}{d} = 0.48$

Actual depth of neutral axis $\dfrac{x_u}{d} = 2.417 \dfrac{A_{st}}{bd} \dfrac{f_y}{f_{ck}}$

$$= 2.417 \times \frac{1256}{200 \times 400} \times \frac{415}{20} = 0.78$$

Thus, the actual neutral axis depth is more than the limiting one. Such a beam is *over-reinforced* and hence is undesirable, since the failure is sudden and without warning. IS: 456–2000 recommends that such a beam should be redesigned.

$$M_{u,\lim} = 0.36 f_{ck} \cdot \frac{x_{u,max}}{d}\left(1 - .42\frac{x_{u,max}}{d}\right) bd^2$$

$$= 0.36 \times 20 \times .48\,(1 - .42 \times .48)\,200 \times 400^2 = 88.3 \text{ kN-m}$$

It is very clear that if $\dfrac{x_u}{d} = 0.78$ is taken, the stresses in concrete will exceed $0.45 f_{ck}$. However at $M_u = M_{u,\lim}$, x_u will be equal to $x_{u,\max}$ and stress in concrete will be equal to $0.45 f_{ck}$. In such a circumstance, the stress f_s in steel will be less than f_y.

From the equilibrium of internal forces, we have

$$0.36 f_{ck} \cdot b \cdot x_u = 0.87 f_s A_{st}$$

$$f_s = \frac{0.36 \times 20 \times 200 \times 0.48 \times 400}{1256 \times 0.87}, \text{ where } \quad x_u = x_{u,\max}$$

$$= 253.0 \text{ N/mm}^2$$

$\therefore$ Actual stress in steel section

$$= 253 \times 0.87 = 220.2 \text{ N/mm}^2 \text{ (against a value of } f_y = 415 \text{ N/mm}^2\text{)}$$

The corresponding strain in steel is

$$\varepsilon_s = \frac{0.0035\,(d - x_{u,\max})}{x_{u,\max}}$$

$$= \frac{0.0035\,(d - .48\,d)}{0.48\,d} = 0.0038$$

Check:

Moment of resistance with respect to concrete

$$M_u = 0.36 f_{ck} \cdot b \cdot x_{u,\max}\,(d - 0.42\,x_{u,\max})$$

$$= 0.36 \times 20 \times 200 \times .48 \times 400\,(400 - 0.42 \times .48 \times 400)$$

$$= 88.3 \text{ kN-m}$$

Moment of resistance with respect to steel

$$M_u = 0.87 f_S A_{st} \times (d - 0.42\, x_{u,\max})$$
$$= 0.87 \times 253 \times 1256 \times (400 - 0.42 \times .48 \times 400)$$
$$= 88.3 \text{ kN-m}$$

Each one of these is equal to $M_{u,\lim}$.

Question - 12 : *A 5 m long simply supported beam carries a superimposed load of 20 kN/m. Design the mid span section if its effective depth is kept constant at 500 mm using*

(i) Working stress method

(ii) Limit state method.

Use M20 concrete and Fe 415 grade steel.

Solution: Given data

effective span $l = 5$ m

superimposed load $= 20$ kN/m

effective depth $d = 500$ mm

Stresses $f_{ck} = 20\ \text{N/mm}^2$ $f_y = 415\ \text{N/mm}^2$

(i) Working stress method:

Balanced depth of neutral axis n_c is given by

$$n_c = \frac{1}{1 + \dfrac{\sigma_{st}}{m\,\sigma_{cb}}}$$

Lever arm $$j = 1 - \frac{n_c}{3}$$

For a balanced design $M = R \,.\, bd^2$

where $$R = \frac{1}{2}\,\sigma_{cb} \cdot n_c \cdot j$$

σ_{cb} = permissible compressive stress in concrete
$= 7\ \text{N/mm}^2$ for $M20$ mix

σ_{st} = permissible tensile stress in steel
$= 230\ \text{N/mm}^2$ for Fe 415 grade steel

$$m = \text{modular ratio} = \frac{280}{3\,\sigma_{cb}} = 13 \text{ for } M20 \text{ mix}$$

$$n_c = \frac{1}{1 + \dfrac{230}{13 \times 7}} = 0.28$$

$$j = 1 - \frac{0.28}{3} = 0.90$$

$$R = \frac{1}{2} \times 7 \times 0.28 \times 0.9 = 0.88 \text{ N/mm}^2$$

Bending moment at mid span $M = \frac{Wl^2}{8} = 20 \times \frac{5^2}{8} = 62.5$ kN-m

or, $$62.5 \times 10^6 = 0.88 \times b \times 500^2$$

or $$b = 284 \text{ mm} \approx 290 \text{ mm}$$

Adopt a 290 mm × 500 mm effective section.

Force of compression $= \frac{1}{2} \sigma_{cb} . b . n_c . d$

Force of tension $= A_{st} \sigma_{st}$

or, Area of steel is given by

$$A_{st} = \frac{\sigma_{cb}.b.n_c.d}{2 \times \sigma_{st}} = \frac{7 \times 290 \times 0.28 \times 500}{2 \times 230}$$

$$A_{st} = 618 \text{ mm}^2.$$

(*ii*) **Limit state method:**

Factored load $= 1.5 \times 20 = 30$ kN/m

Factored bending moment $= 30 \times \frac{5^2}{8}$

or, $$M_u = 93.75 \text{ kN-m}$$

For a balanced section,

$$M_u = 0.138 f_{ck}\ bd^2$$

or $$b = \frac{93.75 \times 10^6}{0.138 \times 20 \times 500^2} = 135.9 \text{ mm}$$

Adopt a 140 mm × 500 mm effective section,

Area of tension steel $A_{st} = \frac{0.36 f_{ck} . b.x_{u,\max}}{0.87 f_y}$ where $x_{u,\max} = 0.48\, d$

or, $$A_{st} = \frac{0.36 \times 20 \times 140 \times 0.48 \times 500}{0.87 \times 415} = 670 \text{ mm}^2$$

Now a comparison can be made regarding the savings in concrete and steel using the limit state method with respect to Working stress method.

% Saving in	*Limit state design*
Concrete	$\left[\frac{290 \times 500 - 140 \times 500}{290 \times 500}\right] \times 100$ $= 51.7\%$
Steel	$\left[\frac{617 - 670}{617}\right] \times 100$ $= -8.6\%$

The negative sign shows that excess steel is required in limit state method for balanced section.

Question - 13 : *A reinforced concrete beam has width equal to 300 mm and total depth equal to 800 mm with a cover of 40 mm to the centre of reinforcement. Design the beam if it is subjected to a total bending moment of 140 kN-m. Use M20 concrete and Fe 415 grade steel.*

Compare the design with that obtained by working stress method.

Solution. Given data

$$b = 300 \text{ mm} \quad d = 800 - 40 = 760 \text{ mm}$$

Bending moment $= 140$ kN-m

Stresses $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2.$

(*i*) Limit state method:

Factored applied B.M. $M_{ua} = 1.5 \times 140 = 210$ kN-m

Limiting depth of neutral axis = 0.48 d for Fe 415

Limiting bending moment

$$M_{u,\text{lim}} = 0.138 f_{ck}\, bd^2 \quad \text{for} \quad \text{Fe } 415$$
$$= 0.138 \times 20 \times 300 \times (760)^2$$
$$= 478.25 \times 10^6 \text{ N-mm} \simeq 478.25 \text{ kN-m} > M_{ua} = 210 \text{ kN-m}$$

Hence, compression reinforcement is not needed.

Area of steel

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, M_{ua}}{f_{ck}\, bd^2}}\right] bd$$

$$A_{st} = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 210 \times 10^6}{20 \times 300 \times 760^2}}\right] \times 300 \times 760$$

$$= 828 \text{ mm}^2$$

(*ii*) Working stress method:

The balanced moment of resistance of a singly reinforced beam

$$M = R \,.\, b \,.\, d^2$$

Now, neutral axis depth factor n_c is given by

$$n_c = \frac{1}{1+\frac{\sigma_{st}}{m\,\sigma_{cb}}} = \frac{1}{1+\frac{230}{13\times 7}} = 0.28 \text{ for } M20 \text{ and Fe 415}$$

$$j = 1 - \frac{n_c}{3} = 1 - \frac{0.28}{3} = 0.90$$

$$\therefore \quad R = \frac{1}{2}\times\sigma_{cb}\times n_c\times j = \frac{1}{2}\times 7\times 0.28\times 0.9 = 0.88 \quad \text{N/mm}^2$$

$$M = 0.88\times 300\times 760^2 = 152.5\times 10^6\ \text{N/mm} = 152.5\ \text{kN-m} > 140\ \text{kN-m}$$

Since $M > M_a$, compression reinforcement is not required.

Area of steel is given by

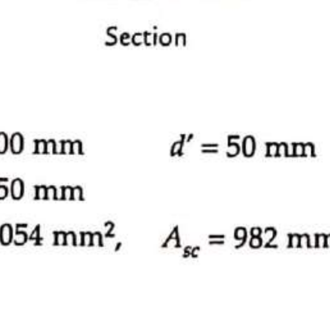
$$A_{st} = \frac{M}{\sigma_{st}\cdot j\,.\,d} = \frac{152.5\times 10^6}{230\times 0.90\times 760} = 971\ \text{mm}^2$$

Now a comparison can be made regarding the savings in steel using the limit state method with respect to working stress method.

% Saving in	*Limit state design*
Steel	$\left(\frac{971-828}{971}\right)\times 100 = 14.72\%$

DOUBLY REINFORCED BEAM

Question - 6 : *The cross-sectional dimensions of a doubly reinforced beam are shown in Fig. Determine the moment of resistance on the beam section. Assume M20 concrete and Fe 250 steel.*



Section

Solution. Given data

$b = 300$ mm $\quad d' = 50$ mm

$d = 550$ mm

$A_{st} = 3054\ \text{mm}^2, \quad A_{sc} = 982\ \text{mm}^2$

Stresses $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 250 \text{ N/mm}^2$

Limiting depth of neutral axis

$$x_{u,\max} = 0.53\, d \text{ for Fe 250}$$
$$= 0.53 \times 550 = 291.5 \text{ mm}$$

Assuming $f_{sc} = 0.87 f_y$ and considering force equilibrium $C_{uc} + C_{us} = T_u$; with

$$C_{uc} = 0.36 \times 20 \times 300 \times x_u = 2160\, x_u \text{ N}$$
$$C_{us} = (0.87 f_y - 0.45 f_{ck})\, A_{sc} = (0.87 \times 250 - 0.45 \times 20) \times 982 = 204747 \text{ N}$$
$$T_u = 0.87 \times 250 \times 3054 = 664245 \text{ N}$$

or, $2160\, x_u + 204747 = 664245$

$$x_u = 212.7 \text{ mm} < x_{u,\max} = 291.5 \text{ mm}$$

Hence, the assumption $f_{sc} = 0.87 f_y$ is justified, and the section is *under reinforced.*

Also; strain at the level of compression steel

$$\varepsilon_{sc} = 0.0035\left(\frac{x_u - d'}{x_u}\right) = 0.0035\left(1 - \frac{50}{212.7}\right)$$

$$= 0.00267 > \varepsilon_s = \frac{0.87 \times 250}{2 \times 10^5} = 0.001087$$

Hence, $f_{sc} = 0.87 f_y$ is also justified.

Moment of resistance

$$M_u = C_{uc}\,(d - 0.42\, x_u) + C_{us}\,(d - d')$$
$$= (2160 \times 212.7) \times (550 - 0.42 \times 212.7) + 204747 \times (550 - 50)$$
$$= 314 \times 10^6 \text{ N-mm} = 314 \text{ kNm.}$$

Question - 7 : Repeat the Question No. 6 , considering Fe 415 .

Solution. Given data

$$b = 300 \text{ mm} \quad d = 550 \text{ mm} \quad d' = 50 \text{ mm}$$
$$A_{st} = 3054 \text{ mm}^2 \quad A_{sc} = 982 \text{ mm}^2$$

Stresses $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$

Limiting depth of neutral axis

$$x_{u,\max} = 0.48\, d \quad \text{for} \quad \text{Fe 415}$$
$$= 0.48 \times 550 = 264 \text{ mm}$$

Assuming $f_{sc} = 0.87 f_y$ and considering force equilibrium $C_{uc} + C_{us} = T_u$; with

$$C_{uc} = 0.36 \times 20 \times 300\, x_u = 2160\, x_u \text{ N}$$
$$C_{us} = (0.87 f_y - 0.45 f_{ck})\, A_{sc} = (0.87 \times 415 - 0.45 \times 20) \times 982 = 345713 \text{ N}$$
$$T_u = 0.87 f_y \times A_{st} = 0.87 \times 415 \times 3054 = 1102646.7 \text{ N}$$

or, $2160\, x_u + 345713 = 1102646.7$

$$x_u = 350.4 \text{ mm} > x_{u,\max} = 264 \text{ mm}$$

So, the section is *over reinforced.*

Therefore, limiting x_u to $x_{u,\max} = 264$ mm

∴ Corresponding strain in compression steel is obtained as

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right)$$

$$= 0.0035\left(1 - \frac{50}{264}\right) = 0.00284$$

From stress-strain curve shown in Fig. 3.4, the corresponding value of stress $f_{sc} = 352.5\ \text{N/mm}^2$

This value is alternatively obtained from

Table for $\frac{d'}{d} = 0.09$ and Fe 415.

Accordingly, limiting the moment of resistance M_u to the limiting moment

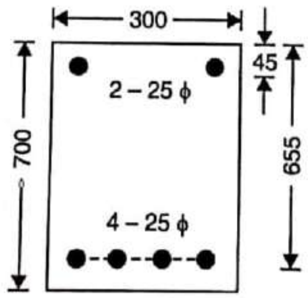
$$M_{u,\lim} = 0.36 f_{ck} \cdot b \cdot x_u (d - 0.42\,x_u) + (f_{sc} - 0.45 f_{ck})\, A_{sc}\, (d - d')$$

or, $M_{u,\lim} = 0.36 \times 20 \times 300 \times 264 \times (550 - .42 \times 264) + (352.5 - 0.45 \times 20) \times 982 \times (550 - 50)$

or, $M_{u,\lim} = 419.06 \times 10^6$ N-mm

$M_{u,\lim} = 419$ kN-m.

Question - 8 : *Determine the ultimate moment of resistance of the doubly reinforced section shown in Fig. . Assume M20 concrete and Fe 415 steel .*



Section

Solution. Given data

$$b = 300\ \text{mm} \quad d = 655\ \text{mm} \quad d' = 45\ \text{mm}$$

$$A_{st} = 4 \times \frac{\pi}{4} \times 25^2 = 1964\ \text{mm}^2 \quad A_{sc} = 2 \times 491 = 982\ \text{mm}^2$$

Stresses $f_{ck} = 20\ \text{N/mm}^2 \quad f_y = 415\ \text{N/mm}^2$

Limiting depth of neutral axis

$$= 0.48\,d = 0.48 \times 655 = 314.4\ \text{mm}$$

Assuming $f_{sc} = 0.87 f_y$

$$C_{uc} = 0.36 f_{ck} \cdot b \cdot x_u = 0.36 \times 20 \times 300 \times x_u = 2160\, x_u \text{ N}$$

$$C_{us} = (0.87 f_y - 0.45 f_{ck})\, A_{sc} = (0.87 \times 415 - 0.45 \times 20) \times 982 = 345713 \text{ N}$$

$$T_u = 0.87 f_y A_{st} = 0.87 \times 415 \times 1964 = 709102 \text{ N}$$

Considering force equilibrium

$$C_{uc} + C_{us} = T_u$$

$$\therefore \quad 2160\, x_u + 345713 = 709102$$

$$\therefore \quad x_u = 168.23 \text{ mm} < x_{u,\max} = 314.4 \text{ mm}$$

Hence the assumption $f_{sc} = 0.87 f_y$ is justified and section is *under reinforced*.

Further, strain at the level of compression steel

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right) = 0.0035\left(1 - \frac{45}{168.23}\right) = 0.00256$$

For Fe 415,

$$\varepsilon_s = \frac{0.87 f_y}{E_s} + 0.002 = \frac{0.87 \times 415}{2 \times 10^5} + 0.002 = 0.0038$$

As $\varepsilon_{sc} < \varepsilon_s$, the assumption $f_{sc} = 0.87 f_y$ is not justified. Whereby the calculated value of C_{us} (and hence of $x_u = 167.3$ mm) is also not correct. The correct value has to be obtained iteratively using strain compatibility.

First Cycle:

Assuming $\varepsilon_{sc} = 0.00256$

From stress-strain curve shown in Fig. 3.4, the corresponding value of stress $f_{sc} = 346.7$ N/mm²

$$\therefore \quad C_{us} = (346.7 - 0.45 \times 20) \times 982 = 331621.4 \text{ N}$$

$$\therefore \quad C_{uc} + C_{us} = T_u \;\Rightarrow\; x_u = \frac{709102 - 331621.4}{2160} = 174.8 \text{ mm}$$

$$\therefore \quad \varepsilon_{sc} = 0.0035\left(1 - \frac{45}{174.8}\right) = 0.00259$$

Second Cycle:

Assuming $\varepsilon_{sc} = 0.00259$

From stress-strain curve shown in Fig. 3.4, the corresponding value of stress $f_{sc} = 347.4$ N/mm²

$$\therefore \quad C_{uc} = (347.4 - 0.45 \times 20) \times 982 = 332308.8 \text{ N}$$

$$\therefore \quad x_u = \frac{709102 - 332308.8}{2160} = 174.4 \text{ mm (converged)}$$

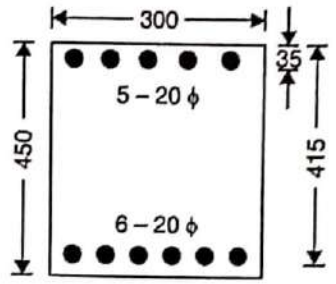
According to the ultimate moment of resistance

$$M_u = C_{uc}\,(d - 0.42\, x_u) + C_{us}\,(d - d')$$

$$= (2160 \times 174.4)\,(655 - 0.42 \times 174.4) + 332308.8 \times (655 - 45)$$

$$= 421.85 \times 10^6 \text{ N-mm} \simeq 422 \text{ kN-m.}$$

Question - 9 : *Determine the ultimate moment of resistance of the doubly reinforced section shown* in Fig. Assume M20 concrete and Fe 250 steel.



Section

Solution. Given data

$$b = 300 \text{ mm} \quad d = 415 \text{ mm} \quad d' = 35 \text{ mm}$$

$$A_{st} = 6 \times \frac{\pi}{4} \times 20^2 = 1884 \text{ mm}^2 \qquad A_{sc} = 5 \times 314 = 1570 \text{ mm}^2$$

Stresses $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 250 \text{ N/mm}^2$

Limiting depth of neutral axis

$$= 0.53d \text{ for Fe 250} = 0.53 \times 415 = 219.95 \text{ mm}$$

Assuming $f_{sc} = 0.87 f_y$

$$C_{uc} = 0.36 f_{ck}\, b \,.\, x_u = 0.36 \times 20 \times 300 \times x_u = 2160\, x_u \text{ N}$$

$$C_{us} = (f_{sc} - 0.45 f_{ck})\, A_{sc} = (0.87 \times 250 - 0.45 \times 20) \times 1570 = 327345 \text{ N}$$

$$T_u = 0.87 f_y A_{st} = 0.87 \times 250 \times 1884 = 409770 \text{ N}$$

Considering force equilibrium,

$$C_{uc} + C_{us} = T_u$$

$\therefore \quad 2160\, x_u + 327345 = 409770$

$$x_u = 38.2 \text{ mm} < x_{u,\max} = 219.95 \text{ mm}$$

Hence, the assumption $f_{sc} = 0.87 f_y$ is satisfied.

Further, strain at the level of compression steel

$$\varepsilon_{sc} = 0.0035\left(1 - \frac{d'}{x_u}\right) = 0.0035\left(1 - \frac{35}{38.2}\right) = 0.00029$$

For Fe 250, $\varepsilon_s = \frac{0.87 f_y}{2 \times 10^5} = 0.001087$

As $\varepsilon_{sc} < \varepsilon_s$, the assumption $f_{sc} = 0.87 f_y$ is not justified.

Where by the calculated value of C_{us} (and hence of $x_u = 38.2$ mm) is also not correct.

The correct value has to be obtained as;

Assuming $\varepsilon_{sc} = 0.0002932$

From Table , the corresponding value of stress

$$f_{sc} = 0.00029 \times \varepsilon_s = 0.0002932 \times 2 \times 10^5$$
$$= 58.64 \text{ N/mm}^2$$

$\therefore$ $$C_{us} = (58.64 - 0.45 \times 20) \times 1570 = 77934.8 \text{ N}$$

$\therefore$ $$C_{uc} + C_{us} = T_u \Rightarrow x_u = \frac{409770 - 77934.8}{2160} = 153.63 \text{ mm}$$

$\therefore$ $$\varepsilon_{sc} = 0.0035\left(1 - \frac{35}{153.63}\right) = 0.00270 > \varepsilon_s = 0.001087$$

Hence, $f_{sc} = 0.87 f_y$ is also justified.

$\therefore$ Moment of resistance

$$M_u = C_{uc}(d - 0.42\, x_u) + C_{us}(d - d')$$
$$= (2160 \times 153.63) \times (415 - 0.42 \times 153.63) + 77934.8 \times (415 - 35)$$
$$= 145.9 \times 10^6 \text{ N-mm} = 145.9 \text{ kN-m}.$$

Question - 10 : *Design a rectangular beam for an effective span of 6m. The superimposed load is 80 kN/m and size of beam is limited to 300 mm × 700 mm overall.* Use M20 mix and Fe 415 grade steel.

Solution. Given data

$d = 300$ mm, superimposed load $w_l = 80$ kN/m

$D = 700$ mm, effective span $l = 6$ m

Stresses $f_{ck} = 20 \text{ N/mm}^2$ $f_y = 415 \text{ N/mm}^2$.

Now, Self weight of beam $w_d = 0.3 \times 0.7 \times 25 = 5.25$ kN/m

Superimposed load $w_L = 80$ kN/m

$\therefore$ Total load $w = 85.25$ kN/m

Factored load $= 1.5 \times 85.25 = 127.88$ kN/m

$\therefore$ Factored B.M. (applied B.M.) $M_{ua} = 127.88 \times \frac{6^2}{8} = 575.45$ kN-m

Let, effective cover $= 0.1\, D = 70$ mm

$\therefore$ effective depth $d = 700 - 70 = 630$ mm

Limiting depth of neutral axis $= 0.48\, d$ (for Fe 415) $= 0.48 \times 630 = 302.4$ mm.

In design of doubly reinforced beam, the section is kept balanced to make full utilisation of resistance of concrete. Therefore A_{st_1} is calculated for balanced section with $x_u = x_{u,\max} = 0.48\, d$.

The limiting reinforcement

$$A_{st_1} = P_{\lim} \times bd = 0.414 \frac{f_{ck}}{f_y} \times \frac{x_{u,\max}}{d} \times bd$$

$$= 0.414 \times \frac{20}{415} \times 0.48 \times 300 \times 630 = 1810 \text{ mm}^2$$

Limiting Bending Moment

$$M_{u,\lim} = 0.138 f_{ck}\, bd^2 \quad \text{for} \quad \text{Fe 415}$$

$$= 0.138 \times 20 \times 300 \times 630^2 = 328.6 \text{ kN-m}$$

The remaining bending moment has to be resisted by a couple consisting of compression steel and the corresponding tension steel.

$$M_{ua} - M_{u,\lim} = (f_{sc} - 0.45 f_{ck})\, A_{sc}\, (d - d')$$

If $\frac{d'}{d} = 0.1$, then from Table 3.2, $f_{sc} = 351.9 \text{ N/mm}^2$

$$\therefore \quad A_{sc} = \frac{(575.45 - 328.6) \times 10^6}{(351.9 - 0.45 \times 20) \times (630 - 70)} = 1283 \text{ mm}^2$$

Corresponding tension steel A_{st_2}

$$0.87 f_y\, A_{st_2} = f_{sc} \cdot A_{sc}$$

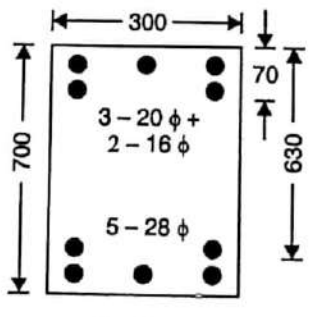
or,

$$A_{st_2} = \frac{351.9 \times 1283}{0.87 \times 415} = 1250 \text{ mm}^2$$

Total tension steel

$$A_{st} = A_{st_1} + A_{st_2} = 1810 + 1250 = 3060 \text{ mm}^2$$

Provide 5-28 bars in tension (A_{st} = 3078 mm^2) and 3-20 ϕ + 2-16 ϕ bars in compression (A_{sc} = 1344 mm^2) each in two layers as shown in Fig.

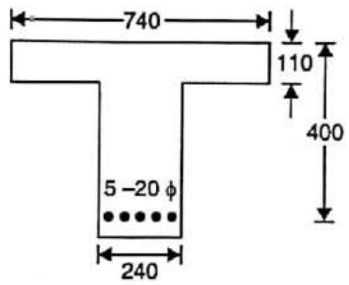


Section

Check: Maximum tension steel = 0.04 bD = 0.04 × 300 × 700 = 8400 mm^2 > 3078 mm^2. (O.K.)

Flanged Beam

Question - 7 : *Find the moment of resistance of T-beam having following data in Fig. Use M20 mix of concrete and Fe 415 grade steel.*



Section

Solution. Given data

$b_f = 740$ mm $\quad b_w = 240$ mm

$D_f = 110$ mm $\quad A_{st} = 5 \times \frac{\pi}{4}(20)^2 = 1570 \text{ mm}^2$

$d = 400$ mm

Stresses $\quad f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$

Let us assume that the neutral axis lies within the flange.

$$\therefore \quad x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b_f} = \frac{0.87 \times 415 \times 1570}{0.36 \times 20 \times 740} = 106.4 \text{ mm} < D_f$$

As the value of x_u is less than 110 mm, the neutral axis lies in the flange.

Also, $\quad x_{u,\max} = 0.48\, d \quad$ for $\quad$ Fe 415 steel

$= 0.48 \times 400$

$= 192 \text{ mm} > x_u$

Hence section is *under reinforced.*

Therefore, moment of resistance is given by

$$M_u = 0.87 f_y A_{st} (d - 0.42 x_u)$$
$$= 0.87 \times 415 \times 1570 \times (400 - 0.42 \times 106.4)$$
$$= 201.4 \times 10^6 \text{ N-mm} = 201.4 \text{ kN-m}.$$

Question - 8 : *Determine the moment of resistance of T-beam of* question 7 *if $D_f = 90$ mm.*

Solution. Let us assume that the neutral axis lies within the flange

$$0.36 f_{ck} b_f . x_u = 0.87 f_y A_{st}$$

or,

$$x_u = \frac{0.87 \times 415 \times 1570}{0.36 \times 20 \times 740} = 106.4 \text{ mm} > D_f = 90 \text{ mm}$$

As the value of x_u is more than 90 mm, the neutral axis lies in the web.

Let us obtain a new value of x_u by equating the total compressive force to the total tensile force.

Total tensile force $T_u = 0.87 f_y A_{st} = 0.87 \times 415 \times 1570$

$$= 566848.5 \text{ N}$$

Depth of equivalent stress block

$$Y_f = 0.15 x_u + 0.65 D_f$$
$$= 0.15 x_u + 0.65 \times 90$$
$$= 0.15 x_u + 58.5$$

Hence,

$$C_u = 0.36 f_{ck} . x_u . b_w + 0.45 f_{ck} (b_f - b_w) y_f$$
$$= (0.36 \times 20 \times x_u \times 240) + 0.45 \times 20 \times (740 - 240) \times (0.15 x_u + 58.5)$$
$$= 1728 x_u + 675 x_u + 263250$$
$$= 2403 x_u + 263250$$

Equating $C_u = T_{u,}$ we get

$$2403 x_u + 263250 = 566848.5$$

from which $x_u = 126.34$ mm.

This value will be acceptable only if $Y_f \le D_f$

Now, $Y_f = 0.15 x_u + 58.5 = 0.15 \times 126.34 + 58.5 = 77.5 \text{ mm} < D_f$

Hence, $x_u = 126.34$ mm is *acceptable.*

Maximum depth of neutral axis

$$x_{u,\max} = 0.48 d \text{ for Fe 415}$$
$$= 0.48 \times 400 = 192 \text{ mm} > x_u$$

Hence, section is *under reinforced.*

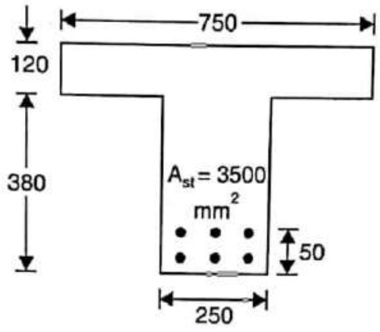
Now, $\frac{3}{7} x_u = \frac{3}{7} \times 126.34 = 54.2$ mm

$\therefore$ $D_f > \frac{3}{7} x_u \Rightarrow \frac{D_f}{x_u} > 0.43$

Therefore, the moment of resistance is given by

$$M_u = 0.36 f_{ck} b_w x_u (d - 0.42 x_u) + 0.45 f_{ck} (b_f - b_w) y_f (d - 0.5 y_f)$$
$$= 0.36 \times 20 \times 240 \times 126.34 (400 - 0.42 \times 126.34)$$
$$+ 0.45 \times 20 \times (740 - 240) \times 77.5 \times (400 - 0.5 \times 77.5)$$
$$= 75.74 \times 10^6 + 125.98 \times 10^6 \text{ N-mm}$$
$$= 201.72 \text{ kN-m.}$$

Question - 9 : *Calculate the moment of resistance of a T-beam as shown in Fig. assuming M20* mix and Fe 415 steel.



Section

Solution. Given data $b_f = 750$ mm $D_f = 120$ mm
$d = 450$ mm $b_w = 250$ mm
$A_{st} = 3500 \text{ mm}^2$

Stresses $f_{ck} = 20 \text{ N/mm}^2$ $f_y = 415 \text{ N/mm}^2$.

Let us assume that the neutral axis lies within the flange.

$$\therefore \quad x_u = \frac{0.87 \times 415 \times 3500}{0.36 \times 20 \times 750} = 234 \text{ mm} > D_f = 120 \text{ mm.}$$

As the value of x_u is more than 120 mm, the neutral axis lies in the web.

Let as obtain a new value of x_u by equating the total compressive force to the total tensile force.

Total tensile force $T_u = 0.87 f_y A_{st}$
$$= 0.87 \times 415 \times 3500 = 1263675 \text{ N}$$

Depth of equivalent stress block

$$y_f = 0.15 x_u + 0.65 D_f$$
$$= 0.15 x_u + 0.65 \times 120 = 0.15 x_u + 78$$

Hence,
$$C_u = 0.36 f_{ck} x_u . b_w + 0.45 f_{ck} (b_f - b_w) y_f$$
$$= 0.36 \times 20 \times x_u \times 250 + 0.45 \times 20 \times (750 - 250) \times (0.15 x_u + 78)$$
$$= 2475 x_u + 351000$$

Equating $C_u = T_u$, we get
$$2475 x_u + 351000 = 1263675$$

from which $x_u = 368.75$ mm

This value is acceptable only if $y_f \le D_f$

Now, $y_f = 0.15 \times 368.75 + 0.65 \times 120 = 133.3$ mm $> D_f = 120$ mm

Hence, $x_u = 368.75$ mm is not acceptable.

Now, $y_f \not> D_f$. Hence take $y_f = D_f = 120$ mm

$\therefore$ $C_u = 0.36 \times 20 \times x_u \times 250 + 0.45 \times 20 \times (750 - 250) \times 120$

$= 1800\, x_u + 540000$

$\therefore$ $1800\, x_u + 540000 = 1263675$, we get

$x_u = 402$ mm

Maximum depth of neutral axis

$$x_{u,\max} = 0.48\, d \quad \text{for Fe 415}$$

$$= 0.48 \times 450 = 216 \text{ mm} < x_u = 402 \text{ mm}$$

Hence, $x_u > x_{u,\max}$, thus the beam is *over reinforced*.

Though the code recommends that such a beam should be redesigned, its moment of resistance is limited to $M_{u,\lim}$ corresponding to the balanced section.

$\therefore$ We have $x_u = x_{u,\max} = 216$ mm

$\therefore$ $y_f = 0.15\, x_{u,\max} + 0.65\, D_f = 110.4$ mm < 120 mm (O.K.)

Again $\dfrac{D_f}{d} = \dfrac{120}{450} = 0.27 > 0.20.$

Hence moment of resistance is given

$$M_{u,\lim} = 0.36\, f_{ck} \cdot x_{u,\max}\, b_w\, (d - 0.42\, x_{u,\max}) + 0.45\, f_{ck}\, (b_f - b_w)\, y_f\, (d - 0.5\, y_f)$$

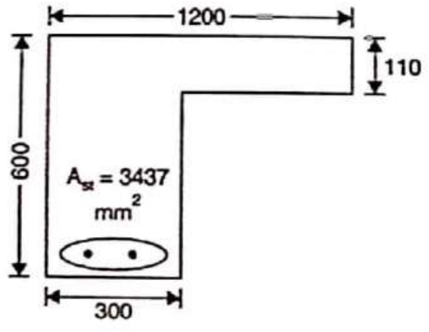
$$= 0.36 \times 20 \times 216 \times 250\, (450 - 0.42 \times 216)$$

$$+ 0.45 \times 20 \times (750 - 250) \times 110.4 \times (450 - 0.5 \times 110.4)$$

$$= 139.7 \times 10^6 + 196.3 \times 10^6$$

$$= 336 \text{ kN-m.}$$

Question - 10 : *Find the moment of resistance of L-beam having following data in Fig. Use M20* mix and Fe 415 grade steel.



Section

Solution. Given data

$$b_f = 1200 \text{ mm} \qquad d = 600 \text{ mm} \qquad A_{st} = 3437 \text{ mm}^2$$

$$D_f = 110 \text{ mm} \qquad b_\omega = 300 \text{ mm}$$

Stresses $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$.

It is assumed that neutral axis lies in the flange

or,
$$x_u = \frac{0.87 \times 415 \times 3437}{0.36 \times 20 \times 300} = 574.5 \text{ mm} > D_f = 110 \text{ mm}$$

As the value of x_u is more than 110 mm, the neutral axis lies in the web.

Let us obtain a new value of x_u by equating the total compressive force to the total tensile force.

Total tensile force $T_u = 0.87 f_y A_{st}$

$$= 0.87 \times 415 \times 3437 = 1240928.9 \text{ N}$$

Depth of equivalent stress block

$$y_f = 0.15\, x_u + 0.65 \times D_f$$

$$= 0.15\, x_u + .65 \times 110 = 0.15\, x_u + 71.5$$

Hence $C_u = 0.36 f_{ck}\, x_u\, b_w + 0.45 f_{ck} (b_f - b_w)\, y_f$

$$= 0.36 \times 20 \times x_u \times 300 + 0.45 \times 20 \times (1200 - 300) \times (0.15\, x_u + 71.5)$$

$$= 3375\, x_u + 579150$$

Equating $C_u = T_u$, we get

$$3375\, x_u + 579150 = 1240928.9$$

From which $x_u = 196$ mm

This value is acceptable only if $y_f \le D_f$

Now, $y_f = 0.15 \times 196 + 71.5 = 100.9 \text{ mm} < D_f = 110 \text{ mm}$

Hence $x_u = 196$ mm is acceptable.

Limiting depth of neutral axis

$$x_{u,\max} = 0.48\, d \text{ for Fe 415}$$

$$= 0.48 \times 600 = 288 \text{ mm} > x_u = 196 \text{ mm}$$

Hence section is under reinforced.

Now,
$$\frac{3}{7} x_u = \frac{3}{7} \times 196 = 84 \text{ mm}$$

$\therefore$
$$D_f > \frac{3}{7} x_u \quad \Rightarrow \quad \frac{D_f}{x_u} > 0.43$$

Therefore, the moment of resistance is given by

$$M_u = 0.36 f_{ck}\, b_w \,.\, x_u\, (d - 0.42\, x_u) + 0.45 f_{ck} (b_f - b_w)\, y_f\, (d - 0.5\, y_f)$$

$$= 0.36 \times 20 \times 300 \times 196 \times (600 - 0.42 \times 196)$$

$$+ 0.45 \times 20 \times (1200 - 300) \times 100.9 \times (600 - 0.5 \times 100.9)$$

$$= 219.2 \times 10^6 + 449.2 \times 10^6 \text{ N-mm}$$

$$= 668.4 \text{ kN-m.}$$

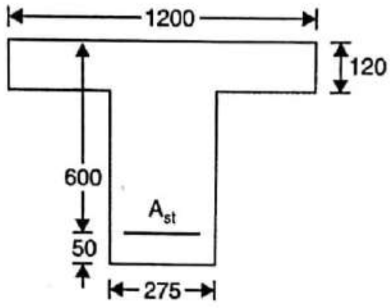
Question - 11 : *A T-beam has b_f = 1200 mm, b_w = 275 mm, d = 600 mm and D_f = 120 mm. Find the reinforcement for a factored moment of resistance of*

(*a*) *375 kN-m*

(*b*) *625 kN-m*

(*c*) *860 kN-m.*

Use M20 mix and Fe 415 grade steel.



Section

Solution.

Case (*a*): When factor moment of resistance M_{ua} = 325 kN-m.

Assuming

$$x_u = D_f, \text{ we get from the following equations}$$

$$M_{u_1} = 0.36 f_{ck} \cdot b_f \cdot D_f (d - 0.42\, D_f)$$

$$= 0.36 \times 20 \times 1200 \times 120 \times (600 - 0.42 \times 120)$$

$$= 569.82 \text{ kN-m} > M_{ua} = 325 \text{ kN-m.}$$

Therefore, $x_u < D_f$ = 120 mm *i.e.* Neutral axis lies in the flange.

Maximum depth of neutral axis

$$x_{u,\max} = 0.48\, d \quad \text{for} \quad \text{Fe 415}$$

$$= 0.48 \times 600 = 288 \text{ mm} > x_u \qquad \text{(Since } D_f > x_u\text{)}$$

Hence, section is *under reinforced.*

The reinforcement is found as

$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, M_u}{f_{ck} \cdot b_f \cdot d^2}}\right] b_f \cdot d$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 375 \times 10^6}{20 \times 1200 \times 600^2}}\right] \times 1200 \times 600$$

$$= 1828.3 \text{ mm}^2$$

Use 6–20 mm bars (A_{st} = 1884 mm^2 > 1828.3 mm^2)

Minimum area of steel

$$A_{s_0} = 0.85 \frac{b_w d}{f_y}$$

$$A_{s_0} = 0.85 \times \frac{275 \times 600}{415} = 338 \text{ mm}^2 < 1884 \text{ mm}^2 \quad \text{(O.K.)}$$

Maximum area of tension steel

$$A_{st} = 0.04\, b_{\omega} D$$
$$= 0.04 \times 275 \times 650 = 7150 \text{ mm}^2 > 1884 \text{ mm}^2 \quad \text{(O.K.)}$$

Check for neutral axis (x_u):

Force of compression $= 0.36 f_{ck} b_f \cdot x_u$

$= 0.36 \times 20 \times 1200 \times x_u$

Force of tension $= 0.87 f_y A_{st}$

$= 0.87 \times 415 \times 1828.3$

Equating

$$0.36 \times 20 \times 1200 \times x_u = 0.87 \times 415 \times 1828.3$$

$$x_u = \frac{0.87 \times 415 \times 1828.3}{0.36 \times 20 \times 1200} = 76.40 \text{ mm} < D_f < x_{u,\max} \quad \text{(O.K.)}$$

Case (*b*): When factored moment of resistance is 625 kN-m

applied factor M.R. $M_{ua} = 625$ kN-m

$M_{u_1} = 569.82$ kN-m (calculated as in case *a*)

Here, $M_{ua} > M_{u_1}$

therefore, neutral axis lies in web.

We calculate moment of resistance $\left(M_{u_2}\right)$ for $\frac{D_f}{x_u} = \frac{3}{7}$ *i.e.*, $x_u = \frac{7}{3} D_f$

$$M_{u_2} = 0.36 f_{ck} \cdot b_w \left(\frac{7}{3} D_f\right)\left(d - 0.42 \times \frac{7}{3} D_f\right) + 0.45 f_{ck} (b_f - b_w) D_f \times \left(d - \frac{D_f}{2}\right)$$

$$= 0.36 \times 20 \times 275 \times \left(\frac{7}{3} \times 120\right) \times \left(600 - 0.42 \times \frac{7}{3} \times 120\right)$$
$$+ 0.45 \times 20 \times (1200 - 275) \times 120 \times (600 - 120/2)$$

$$= 802.10 \text{ kN-m}$$

Here $M_{ua} < M_{u_2}$, It means that $D_f > \frac{3}{7} x_u$

Now, depth of equivalent stress block

$$y_f = 0.15\, x_u + 0.65\, D_f$$
$$= 0.15 \times x_u + 0.65 \times 120$$
$$= 0.15\, x_u + 78$$

The moment of resistance

$$M_u = M_{ua} = 0.36 f_{ck}\, b_w\, x_u\, (d - 0.42\, x_u) + 0.45 f_{ck}\, (b_f - b_w)\, y_f\, (d - 0.5\, y_f)$$
$$625 \times 10^6 = 0.36 \times 20 \times 275 \times x_u \times (600 - 0.42\, x_u) + 0.45 \times 20 \times (1200 - 275)$$
$$\times (0.15\, x_u + 78) \times \left(600 - \frac{0.15 x_u + 78}{2}\right)$$

Simplifying and rearranging, we get

$$x_u^2 - 1984.04\, x_u + 285683.4 = 0$$

from which $x_u = 156.30$ mm.

This value of x_u will be acceptable only if the value of y_f is less than D_f.

$$y_f = 0.15\, x_u + 78$$
$$= 0.15 \times 156.30 + 78 = 101.4 \text{ mm} < D_f = 120 \text{ mm}$$

Also, $D_f = 120 \text{ mm} > \frac{3}{7} \times 156.3 = 66.98 \text{ mm}$

Therefore the required area of steel is obtained as

$$T_u = C_u\, (= C_{uw} + C_{uf})$$
$$0.87 f_y\, A_{st} = 0.36 f_{ck}\, b_w\, x_u + 0.45 f_{ck}\, (b_f - b_w)\, y_f$$

$\therefore$
$$A_{st} = \frac{0.36 \times 20 \times 275 \times 156.3 + 0.45 \times 20 \times (1200 - 275) \times 101.4}{0.87 \times 415}$$

$$A_{st} = 3174.43 \text{ mm}^2$$

Use 4–32 mm bars ($A_{st} = 3215 \text{ mm}^2 > 3174.43 \text{ mm}^2$)

Minimum area of steel

$$A_{so} = 0.85\, \frac{b_w\, d}{f_y} = 338 \text{ mm}^2 < 3215 \text{ mm}^2 \quad \text{(O.K.)}$$

Maximum area of tension steel

$$A_{st} = 0.04\, b_w\, D = 7150 \text{ mm}^2 > 3215 \text{ mm}^2 \quad \text{(O.K.)}$$

Case (*c*): When applied factored moment of resistance is 860 kN-m

applied factor moment of resistance $M_{ua} = 860$ kN-m.

This is more than $M_{u_1} = 569.82$ kN-m (calculated as in case *a*)

Hence $x_u > D_f$

Therefore neutral axis lies in web.

Now, calculate moment of resistance $\left(M_{u_2}\right)$ for $x_u = \frac{7}{3} D_f$

$$M_{u_2} = 802.10 \text{ kN-m (calculated as in case } b)$$

Since $\quad M_{ua} > M_{u_2}$, therefore $D_f < \frac{3}{7} x_u$

calculate x_u corresponding to M_{ua} for $\frac{D_f}{x_u} < 0.43$

The moment of resistance

$$M_u = M_{ua} = 0.36 f_{ck}\, b_w\, x_u\, (d - 0.42\, x_u) + 0.45 f_{ck}\, (b_f - b_w)\, D_f\, (d - 0.5\, D_f)$$

$$860 \times 10^6 = 0.36 \times 20 \times 275 \times x_u \times (600 - 0.42 \times x_u)$$
$$+ 0.45 \times 20 \times (1200 - 275) \times 120 \times (600 - 0.5 \times 120)$$

Simplifying and rearranging, we get

$$x_u^2 - 1428.57\, x_u + 391293.9 = 0$$

from which $\quad x_u = 369.45$ mm.

Maximum depth of neutral axis

$$x_{u,\max} = 0.48\, d \text{ for Fe 415}$$
$$= 0.48 \times 600 = 288 \text{ mm} < x_u$$

Here $\quad x_u > x_{u,\max}$.

Thus the beam is *over-reinforced*. Though IS: 456-2000 recommends that such a beam should be redesigned, its moment of resistance is limited to $M_{u,\lim}$ corresponding to the balanced section.

$\therefore \quad x_u = x_{u,\max} = 288$ mm

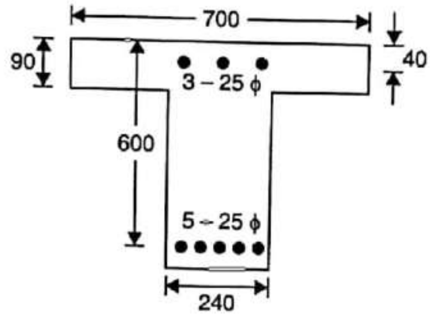
Also; $\quad \frac{D_f}{d} = \frac{120}{600} = 0.2,$

Then moment of resistance

$$M_{u,\lim} = 0.36 f_{ck}\, b_w\, x_{u,\max}\, (d - 0.42\, x_{u,\max}) + 0.45 f_{ck}\, (b_f - b_w)\, D_f\, (d - 0.5\, D_f)$$
$$= 0.36 \times 20 \times 275 \times 288 \times (600 - 0.42 \times 288)$$
$$+ 0.45 \times 20 \times (1200 - 275) \times 120 \times (600 - 0.5 \times 120)$$
$$= 812.4 \text{ kN-m} < M_{ua} = 860 \text{ kN-m}$$

Since $M_{u,\lim} < M_{ua}$, a doubly reinforced section is required.

Question - 12 : *Calculate the moment of resistance of a T-beam as shown in Fig. assuming M15* mix and Fe 415 grade steel.



Section

Solution. Given data

$$b_f = 700 \text{ mm} \qquad d = 600 \text{ mm} \qquad d' = 40 \text{ mm}$$

$$b_w = 240 \text{ mm} \qquad D_f = 90 \text{ mm}$$

$$A_{st} = 5 \times \left(\frac{\pi}{4} 25^2\right) = 2453.2 \text{ mm}^2$$

$$A_{sc} = 3 \times \left(\frac{\pi}{4} 25^2\right) = 1471.9 \text{ mm}^2$$

Stresses $f_{ck} = 15 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$

Maximum depth of neutral axis

$$x_{u,\max} = 0.48\, d \quad \text{for Fe 415}$$

$$= 0.48 \times 600 = 288 \text{ mm}$$

$$\frac{d'}{d} = \frac{40}{600} = 0.06, \text{ from Table 3.2}$$

∴ $f_{sc} = 353 \text{ N/mm}^2$

Let us assume that neutral axis lies in flange.

Force of tension = force of compression

or, $$0.87 f_y A_{st} = 0.36 f_{ck} b_f x_u + (f_{sc} - 0.45 f_{ck}) A_{sc}$$

or, $$x_u = \frac{0.87 \times 415 \times 2453.2 - (353 - 0.45 \times 15) \times 1471.9}{0.36 \times 15 \times 700}$$

$$x_u = 100.66 \text{ mm} > D_f = 90 \text{ mm}$$

(Note that d' (= 40 mm) is less than $\frac{3}{7} x_u$ (= 43.14 mm) so that the stress in concrete at A_{sc} level is $0.45 f_{ck}$).

Hence neutral axis lies outside the flange. To obtain a new value of x_u, equate total compressive force to that of total tensile force.

Now, Depth of equivalent stress block

$$y_f = 0.15\, x_u + 0.65\, D_f$$
$$= 0.15\, x_u + 0.65 \times 90$$
$$= 0.15\, x_u + 58.5, \text{ with a maximum equal to } D_f.$$

$\therefore$ Force of tension = Force of compression

$$0.87 f_y A_{st} = 0.36 f_{ck}\, b_w \cdot x_u + (f_{sc} - 0.45 f_{ck})\, A_{sc} + 0.45 f_{ck}\, (b_f - b_w)\, y_f$$

or,
$$0.87 \times 415 \times 2453.2 = 0.36 \times 15 \times 240 \times x_u + (353 - 0.45 \times 15) \times 1471.9$$
$$+ 0.45 \times 15 \times (700 - 240) \times (0.15\, x_u + 58.5)$$

or,
$$x_u = 110.4 \text{ mm} > D_f = 90 \text{ mm}$$
$$< x_{u,\max} = 288 \text{ mm}$$

This value of x_u is acceptable if $y_f < D_f$

Since $x_u < x_{u,\max}$, therefore section is an *under reinforced*.

Now,
$$\frac{D_f}{x_u} = \frac{90}{110.4} = 0.81 > 0.43$$

$\therefore$
$$y_f = 0.15 \times 110.4 + 58.5 = 75.06 \text{ mm} < D_f \quad \text{(O.K.)}$$

The moment of resistance is given by

$$M_u = 0.36 f_{ck}\, x_u \cdot b_w\, (d - 0.42\, x_u) + (f_{sc} - 0.45 f_{ck})\, A_{sc}\, (d - d')$$
$$+ 0.45 f_{ck}\, (b_f - b_w)\, y_f\, (d - 0.5\, y_f)$$

$$M_u = 0.36 \times 15 \times 110.4 \times 240 \times (600 - 0.42 \times 110.4) + (353 - 0.45 \times 15) \times 1471.9$$
$$\times (600 - 40) + 0.45 \times 15 \times (700 - 240) \times 75.06 \times (600 - 0.5 \times 75.06)$$

$$M_u = 495.7 \times 10^6 \text{ N-mm} = 495.7 \text{ kN-m}.$$

Question - 13 : *Determine the moment of resistance of T-beam of* **ques. 12** *if $A_{st} = 6 - 25$ mm ϕ and $A_{sc} = 2 - 20$ mm ϕ bars are used.*

Solution. Given data

$$b_f = 700 \text{ mm} \qquad b_w = 240 \text{ mm} \qquad d = 600 \text{ mm}$$
$$D_f = 90 \text{ mm} \qquad d' = 40 \text{ mm}$$

$$A_{st} = 6 \times \frac{\pi}{4} \times 25^2 = 2946 \text{ mm}^2$$

$$A_{sc} = 2 \times \frac{\pi}{4} \times 20^2 = 628 \text{ mm}^2$$

Stresses

$$f_{ck} = 15 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2.$$

Maximum depth of neutral axis

$$x_{u,\max} = 0.48\,d \text{ for Fe 415}$$
$$= 0.48 \times 600 = 288 \text{ mm}$$

Also,
$$\frac{d'}{d} = \frac{40}{600} = 0.06, \text{ from Table 3.2}$$
$$f_{sc} = 353 \text{ N/mm}^2$$

Let us assume that neutral axis lies in flange.

Force of tension = Force of compression

$$0.87 f_y A_{st} = 0.36 f_{ck} \cdot b_f x_u + (f_{sc} - 0.45 f_{ck}) A_{sc}$$

or,
$$x_u = \frac{0.87 \times 415 \times 2946 - (353 - 0.45 \times 15) \times 628}{0.36 \times 15 \times 700}$$
$$x_u = 223.86 \text{ mm} > D_f = 90 \text{ mm.}$$

$$\left(\begin{array}{l}\text{Note that } d' \text{ (= 40 mm) is less than } \frac{3}{7} x_u \text{ (= 95.9 mm) so the} \\ \text{stress in concrete at } A_{sc} \text{ level is } 0.45 f_{ck}\text{).}\end{array}\right)$$

Hence, neutral axis lies outside the flange. To obtain a new value of x_u equate T_u and C_u.

Force of tension $T_u = 0.87 f_y A_{st} = 0.87 \times 415 \times 2946 = 1063653.5$ N

Depth of equivalent stress block

$$y_f = 0.15\,x_u + 0.65\,D_f$$
$$= 0.15 \times x_u + 0.65 \times 90$$
$$= 0.15\,x_u + 58.5, \text{ with a maximum equal to } D_f$$

Force of compression $C_u = 0.36 f_{ck}\, b_w \cdot x_u + (f_{sc} - 0.45 f_{ck}) A_{sc} + 0.45 f_{ck} (b_f - b_w)\, y_f$

or,
$$C_u = 0.36 \times 15 \times 240 \times x_u + (353 - 0.45 \times 15) \times 628$$
$$+ 0.45 \times 15 \times (700 - 240) \times (0.15 x_u + 58.5)$$

or,
$$1063653.5 = 1296\,x_u + 466\,x_u + 399087.5$$

or,
$$x_u = 377.2 \text{ mm} > D_f \qquad \text{(O.K.)}$$

but this value of x_u is acceptable only if $y_f \ngtr D_f$

$$y_f = 0.15 \times 377.2 + 58.5 = 115.08 \text{ mm} \ngtr D_f.$$

take
$$y_f = D_f = 90 \text{ mm}$$

$\therefore$ Force of tension = Force of compression

$$0.87 f_y A_{st} = 0.36 f_{ck} \cdot x_u \cdot b_w + (f_{sc} - 0.45 f_{ck}) A_{sc} + 0.45 f_{ck} (b_f - b_w) \times D_f$$

or,
$$0.87 \times 415 \times 2946 = 0.36 \times 15 \times x_u + 240 + (353 - .45 \times 15) \times 628$$
$$+ 0.45 \times 15 \times (700 - 240) \times 90$$

or,
$$x_u = 437.3 \text{ mm} > x_{u,\max} = 288 \text{ mm}$$

Hence $x_u > x_{u,\max}$. Thus the beam is over reinforced. Though IS: 456-2000 recommends that such a beam should be redesigned, its moment of resistance is limited to $M_{u,\lim}$ corresponding to balanced section.

$\therefore$ take $\quad x_u = x_{u,\max} = 288$ mm

Now, $\quad \frac{D_f}{d} = \frac{90}{600} = 0.15 < 0.2$. Hence moment of resistance is given by

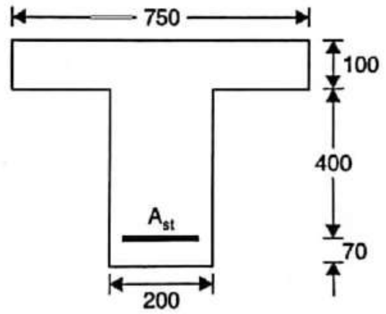
$$M_{u,\lim} = 0.36 f_{ck}\, x_{u,\max}\, b_w (d - 0.42\, x_{u,\max}) + (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d') + 0.45 f_{ck} (b_f - b_w) D_f (d - D_f/2)$$

$$M_{u,\lim} = 0.36 \times 15 \times 288 \times 240 \times (600 - 0.42 \times 288) + (353 - 0.45 \times 15) \times 628 \times (600 - 40) + 0.45 \times 15 \times (700 - 240) \times 90 \times \left(600 - \frac{90}{2}\right)$$

$$M_{u,\lim} = 178.8 \times 10^6 + 121.8 \times 10^6 + 155 \times 10^6 = 455.6 \times 10^6 \text{ N-mm}$$

$$M_{u,\lim} = 455.6 \text{ kN-m.}$$

Question - 14 : *Calculate the amount of steel required in a T-beam to develop a factored moment of resistance of 450 kN-m. The dimensions of the beam section are given in Fig. Use M20 mix and Fe 415 grade steel.*



Section

Solution. Given data

$$b_f = 750 \text{ mm} \quad D_f = 100 \text{ mm}$$
$$b_w = 200 \text{ mm} \quad d = 500 \text{ mm}$$

Factored B.M., $M_{ua} = 450$ kN-m

Stresses are

$$f_{ck} = 20 \text{ N/mm}^2 \qquad f_y = 415 \text{ N/mm}^2$$

Assuming $\quad x_u = D_f$, we get

$$M_{u_1} = 0.36 f_{ck}\, b_f D_f (d - 0.42\, D_f)$$
$$= 0.36 \times 20 \times 750 \times 100 \times (500 - 0.42 \times 100)$$
$$= 247.3 \text{ kN-m} < M_{ua} = 450 \text{ kN-m}$$

Therefore, $x_u > D_f$ *i.e.*, Neutral axis lies in web.

We calculate moment of resistance (M_{u_2}), taking $\frac{D_f}{x_u} = 0.43 = \frac{3}{7}$ i.e., $x_u = 2.34\ D_f$

$$M_{u_2} = 0.36 f_{ck} b_w \left(\frac{7}{3} D_f\right)\left(d - 0.42 \times \frac{7}{3} D_f\right) + 0.45 f_{ck} (b_f - b_w) D_f (d - 0.5 D_f)$$

$$= 0.36 \times 20 \times 200 \times \left(\frac{7}{3} \times 100\right)\left(500 - 0.42 \times \frac{7}{3} \times 100\right)$$

$$+ 0.45 \times 20 \times (750 - 200) \times 100 \times (500 - 50)$$

$$= 357.82 \text{ kN-m} < M_{ua} = 450 \text{ kN-m}$$

Since $\quad M_{ua} > M_{u_2}$, therefore $\frac{D_f}{x_u} < 0.43$

then we calculate $M_{u,\lim}$ by taking $x_u = x_{u,\max} = 0.48 \times 500 = 240$ mm.

$$M_{u,\lim} = 0.36 f_{ck} b_w x_{u,\max} (d - 0.42 x_{u,\max}) + 0.45 f_{ck} (b_f - b_w) D_f (d - 0.5 D_f)$$

$$= 0.36 \times 20 \times 200 \times 240 \times (500 - 0.42 \times 240) + 0.45 \times 20 \times (750 - 200) \times 100 \times (500 - 50)$$

$$= 360.72 \text{ kN-m} < M_{ua}$$

Since $M_{ua} > M_{u,\lim}$, a doubly reinforced section is required.

Area of tension steel corresponding to a moment equal to 360.72 kN-m is given by

$$A_{st_1} = \frac{360.72 \times 10^6}{0.87 \times 415 \times (500 - 0.42 \times 240)} = 2503 \text{ mm}^2$$

The remaining moment to be resisted = 450 – 360.72 = 89.28 kN-m.

This moment is to be resisted by compression steel A_{sc} and additional tension steel A_{st_2}.

If effective cover to compression steel is 50 mm

$$\frac{d'}{d} = \frac{50}{500} = 0.1 \quad \therefore \quad f_{sc} = 351.9 \text{ N/mm}^2$$

Therefore,

Additional moment $= (f_{sc} - 0.45 f_{ck}) A_{sc} (d - d')$

or, $\quad 89.28 \times 10^6 = (351.9 - 0.45 \times 20) \times A_{sc} \times (500 - 50)$

or, $\quad A_{sc} = 578.6 \text{ mm}^2$

Corresponding tensile steel is given by

$$0.87 f_y A_{st_2} = (f_{sc} - 0.45 f_{ck}) A_{sc}$$

or, $$A_{st_2} = \frac{(351.9 - 0.45 \times 20) \times 578.6}{0.87 \times 415} = 549.5 \text{ mm}^2$$

$\therefore$ Total tension steel $A_{st_2} = A_{st_1} + A_{st_2} = 3053 \text{ mm}^2$

Provide 5–28 mm bars in tension (A_{st} = 3079 mm^2 > 3053 mm^2) at an effective cover of 70 mm, and

3–16 mm bars in compression (A_{sc} = 603 mm^2 > 578.6 mm^2) at an effective cover of 50 mm.

Minimum tension steel

$$A_{so} = \frac{0.85\, b_w\, d}{f_y} = \frac{0.85 \times 200 \times 500}{415} = 210 \text{ mm}^2 < 3079 \text{ mm}^2 \quad \text{(O.K.)}$$

Maximum tension steel

$$A_{st} = 0.04\, b_w\, D = 0.04 \times 200 \times 570 = 4560 \text{ mm}^2 > 3079 \text{ mm}^2 \quad \text{(O.K.)}$$

Maximum compression steel

$$= 0.04\, b_w\, D = 4560 \text{ mm}^2 > 603 \text{ mm}^2. \quad \text{(O.K.)}$$

Design of Shear

Question - 6 : *A reinforced concrete beam 230 mm wide and 460 mm effective depth is subjected to a shear force of 60 kN at support. The tensile reinforcement is 0.5%.*

Check the adequacy of the shear design, if M 20 mix and Fe 250 grade steel are used.

Solution. Given data

$$b = 230 \text{ mm}, \ d = 460 \text{ mm},$$

$$V = 60 \text{ kN}, \ \ p_t = 0.5\%$$

Stresses are $f_{ck} = 20 \text{ N/mm}^2 \ \ f_y = 250 \text{ N/mm}^2$

Factored shear force $V_u = 1.5 \times 60 = 90 \text{ kN}$

* Check adequacy of section

Nominal shear stress $\tau_v = \dfrac{V_u}{bd} = \dfrac{90 \times 10^3}{230 \times 460} = 0.85 \text{ N/mm}^2 < \tau_{c,\max}$

$= 2.8 \text{ N/mm}^2$ (for M 20 concrete).

Hence, size of section is adequate.

* Design shear resistance at given section

At given section, $p_t = 0.50$

shear strength of concrete (from Table 5.1, for M 20 concrete)

$$\tau_c = 0.48 \text{ N/mm}^2$$

$\Rightarrow$ Shear resisted by concrete $V_{uc} = \tau_c\, bd$

$$= 0.48 \times 230 \times 460 = 50.8 \text{ kN}$$

Shear resisted by minimum stirrups

$$V_{us,\min} = 0.4\, bd$$

$$= 0.4 \times 230 \times 460 = 42.3 \text{ kN}$$

Shear resisted by concrete and minimum stirrups

$$= V_{uc} + V_{us,\min} = 93.1 \text{ kN} > V_u = 90 \text{ kN}$$

Therefore, *minimum shear reinforcement is provided.*

Provide 8 mm bar of 2-legged stirrup Fe 250 grade

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

* Check minimum stirrup requirements (maximum spacing)

$$(S_v)_{\max} = \frac{0.87\, f_y\, A_{sv}}{0.4\, b} = \frac{2.175 \times 250 \times 100.6}{230} = 238 \text{ mm}$$

$$(S_V)_{\text{provided}} = 200 \text{ mm} < (S_v)_{\max}$$

Further; $$S_v \le \begin{cases} 0.75d = 0.75 \times 460 = 345 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Which are evidently satisfied by $S_v = 200$ mm.

Question - 7 : *Determine the shear reinforcement for the beam section in* question 6 , *if shear force of the section is 90 kN. Use M20 mix and Fe 415 grade steel.*

Solution. Given data $b = 230$ mm $p_t = 0.5\%$

$$d = 460 \text{ mm} \quad V = 90 \text{ kN}$$

stresses are $f_{ck} = 20 \text{ N/mm}^2$ $f_y = 415 \text{ N/mm}^2$

Factored shear force $V_u = 1.5 \times 90 = 135$ kN

* Check adequacy of section

Nominal shear stress $$\tau_v = \frac{V_u}{bd} = \frac{135 \times 10^3}{230 \times 460} = 1.28 \text{ N/mm}^2 < \tau_{c,\max}$$

$$= 2.8 \text{ N/mm (for } M \text{ 20 Concrete)}$$

Hence, since of section is adequate.

* Design shear resistance of section

shear resisted by concrete and minimum stirrups

$$= V_{uc} + V_{us,\min} = (50.8 + 42.3) \text{ kN}$$

$$= 93.1 \text{ kN} < V_u = 135 \text{ kN}$$

Therefore, *shear reinforcement is required.*

* Design of Vertical Stirrups

Shear to be resisted by stirrups

$$V_{us} = V_u - V_{uc} = 135 - 50.8 = 84.2 \text{ kN}$$

Assuming 2-legged closed stirrups of 8 mm dia.

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

$$\Rightarrow \quad \text{required spacing} \quad S_v \le \frac{0.87 f_y A_{sv} \cdot d}{V_{us}} = \frac{0.87 \times 415 \times 100.6 \times 460}{84.2 \times 10^3}$$

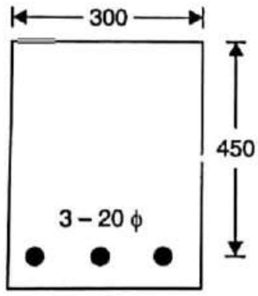
$$= 198.4 \text{ mm}$$

Code requirements for maximum spacing

$$S_v = \begin{cases} \dfrac{0.87 f_y A_{sv}}{0.4b} = \dfrac{0.87 \times 415 \times 100.6}{0.4 \times 230} = 395 \text{ mm} \\ 0.75\, d \quad = 0.75 \times 460 \quad = 345 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Provide 8 ϕ two legged closed stirrups at 190 mm c/c spacing.

Question - 8 : *The beam shown in Fig. is subjected to factored shear force of 150 KN. If $f_{ck} = 20$ N/mm² and $f_y = 415$ N/mm², calculate the shear reinforcement, if bending of 1 bar of 20 ϕ at an angle of 45°.*



Section

Solution. Given data

$$b = 300 \text{ mm}, \quad d = 450 \text{ mm} \quad V_u = 150 \text{ kN}$$

Stresses $\quad f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2.$

Now, one bar of 20 mm is bent up to resist shear

Therefore, available area of tension steel

$$A_{st} = 2 \times \left(\frac{\pi}{4} \times 20^2\right) = 628 \text{ mm}^2$$

$$p_t = \frac{A_{st}}{bd} \times 100 = 0.47\%$$

shear strength of concrete (from Table 5.1, for M20 concrete)

$$\tau_c = 0.46 \text{ N/mm}^2$$

⇒ Shear resisted by concrete

$$V_{uc} = \tau_c\, bd = 0.46 \times 300 \times 450 = 62.1 \text{ kN}$$

⇒ Shear resisted by minimum stirrups

$$V_{us,\,\min} = 0.4\, bd = 0.4 \times 300 \times 450 = 54.0 \text{ kN}$$

⇒ Shear resisted by one bent-up bar

$$V_{usb} = 0.87\, f_y\, A_{sv} \times \sin\alpha$$

$$= 0.87 \times 415 \times \left(1 \times \frac{\pi}{4} \times 20^2\right) \times \sin 45° = 80.2 \text{ kN}$$

As per code (cl. 40.4)

$$V_{usb} \le \frac{V_{us}}{2}, \quad \text{Where} \quad V_{us} = \text{shear to be carried by stirrup}$$

$$= V_u - V_{uc} = (150 - 62.1) = 87.9 \text{ kN}$$

$$\therefore \quad V_{us,\,b} \le \frac{87.9}{2} = 43.95 \text{ kN}$$

the useful contribution by bent-up bar is 43.95 kN only.

Shear to be resisted by concrete, minimum stirrup and bent-up bar

$$= (62.1 + 54.0 + 43.95) = 160.05 \text{ kN} > V_u = 150 \text{ kN}$$

Therefore minimum shear reinforcements are required.

Provide 8 mm bar of 2-legged stirrups

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

* Check minimum stirrup requirements (maximum spacing)

$$(S_v)_{\max} = \frac{0.87\, f_y\, A_{sv}}{0.4\, b} = \frac{2.175 \times 415 \times 100.6}{300} = 303 \text{ mm}$$

$$(S_v)_{\text{provided}} = 275 \text{ mm} < (S_v)_{\max}$$

Further; $$S_v \le \begin{cases} 0.75\, d = 0.75 \times 450 = 337.5 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

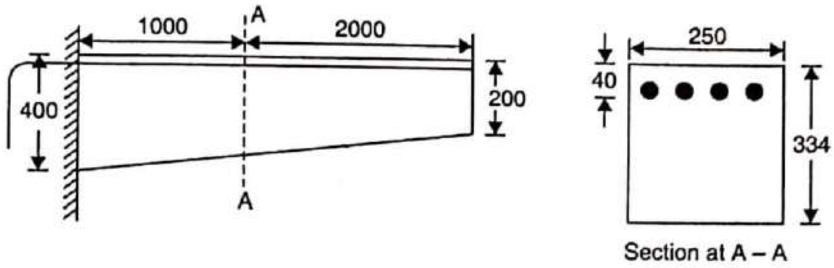
which are evidently satisfied by $S_v = 275$ mm *c/c* spacing. Bend one bar of 20 mm at a distance of between d (= 450 mm) to $2d$ (= 900 mm) from face of support.

Question - 9 : *Design the shear reinforcement in a tapered cantilever beam of constant width 250 mm whose section at 1 m from the face of support as shown in Fig. It consists of 2–22 mm and 2–18 mm bars.*

Redesign the shear reinforcement if 2–18 mm bars are curtailed at this section.

Take shear force = 100 kN and bending moment at this section = 150 kN-m.

Assume M20 mix and 415 grade steel for shear stirrups.



Cantilever beam.

Solution : Given data, $b = 250$ mm

effective depth at A-A, $d = (334 - 40) = 294$ mm

Area of tension steel $A_{st} = 2\left(\frac{\pi}{4} \times 18^2 + \frac{\pi}{4} \times 22^2\right) = 1268 \text{ mm}^2$

$V = 100$ kN, $M = 150$ kN-m

Stresses $f_{ck} = 20 \text{ N/mm}^2$ $f_y = 415 \text{ N/mm}^2$

* Check adequacy of section

Nominal shear stress $\tau_v = \frac{V_u - (M_u/d)\tan\beta}{bd}$.

In a cantilever, B.M. increases towards the support. In the given case, depth of beam is also increasing with the increase in B.M. Therefore, adopt negative sign in the formula for nominal shear stress.

Now,
$$V_u = 1.5 \times 100 = 150 \text{ kN}$$
$$M_u = 1.5 \times 150 = 225 \text{ kN}$$
$$\tan\beta = \frac{(400 - 200)}{3000} = 0.067$$

$\therefore$
$$\tau_v = \frac{150 \times 1000 - (225 \times 10^6/294) \times 0.067}{250 \times 294}$$

or,
$$\tau_v = 1.34 \text{ N/mm}^2 < \tau_{c,\,max} = 2.8 \text{ N/mm}^2 \text{ (for } M20 \text{ concrete)}$$

Hence, size of section is adequate.

(*i*) Design of shear reinforcement with no curtailment

At the critical section, $A_{st} = 1268 \text{ mm}^2$

$$p_t = \frac{100\, A_{st}}{bd} = \frac{100 \times 1268}{250 \times 294} = 1.72\%$$

$\Rightarrow$ Design shear strength of concrete (from Table 5.1, for $M20$ concrete)

$$\tau_c = 0.74 \text{ N/mm}^2$$

$\Rightarrow$
$$V_{uc} = \tau_c\, bd = 0.74 \times 250 \times 294 = 54.4 \text{ kN}$$

$\Rightarrow$ Shear resisted by minimum stirrups

$$V_{us,\,min} = 0.4\, bd = 0.4 \times 250 \times 294 = 29.4 \text{ kN}$$

∴ Shear resisted by concrete and minimum stirrups

$$= V_{uc} + V_{us,\,min} = (54.4 + 29.4) = 83.8 \text{ kN} < V_u = 150 \text{ kN}$$

Therefore, shear reinforcement is required.

Design of vertical stirrups

Shear stress to be resisted by stirrups

$$\tau_{us} = \tau_v - \tau_c$$
$$= 1.34 - 0.74 = 0.60 \text{ N/mm}^2$$

Assuming 2-legged vertical shear stirrup of 8 mm bars

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

⇒ required spacing $S_v \le \dfrac{0.87 f_y A_{sv}.d}{V_{us}} = \dfrac{0.87 \times 415 \times 100.6 \times 29}{(0.6 \times 250 \times 294)}$

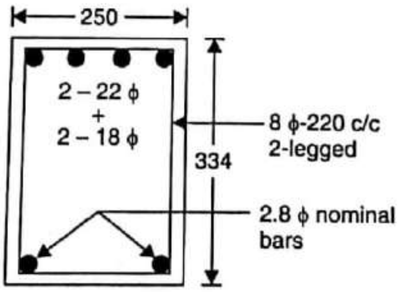
$$= 242 \text{ mm}$$

$$(S_v)_{provided} = 220 \text{ m}$$

code requirements for maximum spacing

$$S_v = \begin{cases} \dfrac{0.87 f_y A_{sv}}{0.4 \times b} = \dfrac{0.87 \times 415 \times 100.6}{0.4 \times 250} = 363 \text{ mm} \\ 0.75\, d \quad = 0.75 \times 294 \quad = 220.5 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Provide 8 ϕ-two legged stirrups at 220 mm c/c spacing.



Details of shear reinforcement.

(*ii*) Design of shear reinforcement with curtailment 2–18 mm longitudinal bars are curtailed at 1 m from the face of support.

The code requires that any one of the three conditions given in clause 26.2.3.2 must be satisfied at the point of cut off to minimise the stress concentration. Here the use of first two conditions is illustrated which are commonly used.

Condition I:

$$1.5\, \tau_v \le (\tau_c + \tau_{us})$$

τ_{us} = shear strength of web reinforcement.

Area of tension steel continuing beyond the cut off point

$$= 2 \times \left(\frac{\pi}{4} \times 22^2\right) = 760 \text{ mm}^2$$

$$p_t = \frac{100 \times 760}{250 \times 294} = 1.03\%$$

⇒ Design shear strength of concrete (from Table , for $M20$ concrete) $\tau_c = 0.62 \text{ N/mm}^2$

Nominal shear strength $\tau_v = 1.34 \text{ N/mm}^2$

Design shear stress $= 1.5\,\tau_v - \tau_c = 1.39 \text{ N/mm}^2$

Assuming 2-legged vertical shear stirrup of 10 mm bars

$$A_{sv} = 2 \times 78.5 = 157.0 \text{ mm}^2$$

required spacing

$$S_v \leq \frac{0.87 f_y A_{sv}.d}{V_{us}} = \frac{0.87 \times 415 \times 157 \times 294}{(1.39 \times 250 \times 294)}$$

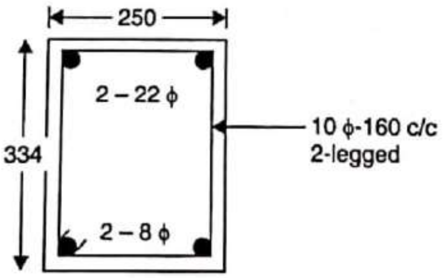
$$= 163 \text{ mm}$$

$$(S_v)_{\text{provided}} = 160 \text{ mm}$$

Code requirement for maximum spacing

$$S_v = \begin{cases} \dfrac{0.87 f_y A_{sv}}{0.4b} = \dfrac{0.87 \times 415 \times 157}{0.4 \times 250} = 567 \text{ mm} \\ 0.75\, d = 0.75 \times 294 = 220.5 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

Provide 10 φ 2-legged stirrups at 160 mm c/c spacing. The shear reinforcement as shown in Fig.



Details of shear reinforcement.

Condition II:

Additional stirrups must be provided over a distance of $0.75\,d = 220$ mm (along the terminated bar) with a spacing $< \dfrac{d}{8\beta_b} = \dfrac{294}{8 \times 0.4} = 91.6 \text{ mm} \simeq 90 \text{ mm}$

$$\left(\text{since, } \beta_b = \frac{\text{area of cut-off bars}}{\text{total area}} = \frac{2 \times \frac{\pi}{4} \times 18^2}{2 \times \frac{\pi}{4}(18^2 + 22^2)} = 0.4 \quad \text{as per clause 26.2.3.2}(b) \text{ of IS: 456-2000}\right)$$

$$\Rightarrow \quad \text{spacing} = \frac{220}{3} = 73.3 \text{ mm} \simeq 70 \text{ mm which is less than 90 mm.}$$

Now, Excess stirrup area required $= \frac{0.4 b\, S_v}{f_y} = \frac{0.4 \times 250 \times 70}{415} = 16.86 \text{ mm}^2$

Area of 2-legged 6 mm stirrups $= 2 \times \frac{\pi}{4} \times 6^2 = 56.5 \text{ mm}^2 > 16.86 \text{ mm}^2$.

Therefore, provide 2-legged 6 mm mild steel bars, 3 nos. @ 70 mm c/c at the cut-off point (towards the free end) in addition to 2-legged-8 mm Fe 415 grade vertical stirrups @ 220 mm c/c. But from a practical viewpoint, it is convenient to use the same 8 mm ϕ ($A_{sv} = 100.6 \text{ mm}^2$) for the additional stirrups.

Question - 10 : *Design the shear reinforcement for one way slab of effective span 4.16 m, of thickness 200 mm with cover 35 mm are subjected to a factored load 15 kN/m². Slab is provided with 4–10 mm bars @ 250 mm c/c at support per meter. Assume M20 concrete and Fe 415 grade steel.*

Solution. In general, slabs do not require shear reinforcement, as the depth provided (based on deflection criteria) is usually adequate to meet shear strength requirements.

Given that

slab thickness $= 200$ mm, Effective depth $= 165$ mm

$$A_{st} = 4 \times \left(\frac{\pi}{4} \times 10^2\right) = 314 \text{ mm}^2/\text{m, effective span} = 4.16 \text{ m}$$

Factored load $= 15 \text{ kN/m}^2$.

Stresses are $f_{ck} = 20 \text{ N/mm}^2 \quad f_y = 415 \text{ N/mm}^2$

Now; $p_t = \frac{100\, A_{st}}{bd} = \frac{100 \times 314}{1000 \times 165} = 0.19$

$\Rightarrow$ Design shear stress of concrete (from Table 5.1, for M 20 concrete)

$$\tau_c = 0.31 \text{ N/mm}^2$$

This value may further be enhanced by a multiplying factor (K) as per cl. 40.2.1.1. for overall depth 200 mm; (from Table 5.2), we get

$$K = 1.2$$

$$\Rightarrow \quad K\tau_c = 1.2 \times 0.31 = 0.372 \text{ N/mm}^2$$

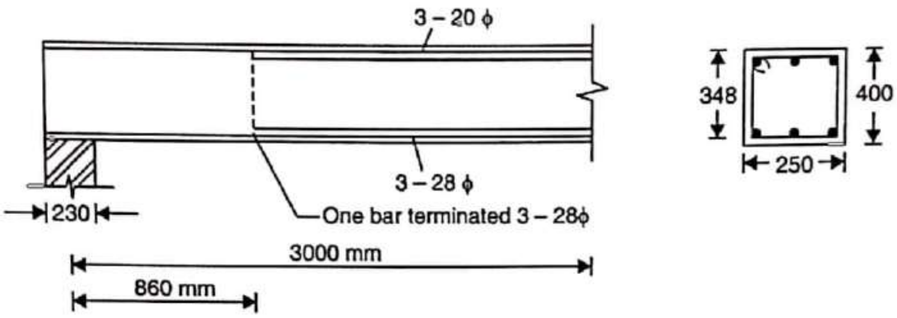
Also; It is convenient to prove that the section has adequate shear strength at the support it self, which has the maximum factored shear, rather than at d from the face of support.

$$V_u = W_u \frac{l}{2} = \frac{15 \times 4.16}{2} = 31.2 \text{ kN/m}$$

$\therefore$ Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{31.2 \times 10^3}{1000 \times 165} = 0.189 \text{ N/mm}^2$

since $K\tau_c >> \tau_v$, so support section is safe itself (where shear is maximum), there is no need to confirm this at the 'critical section' located '*d*' away from the face of support.

Question - 11 : *The beam shown in Fig. is subjected to factored shear force at support 101.25 kN and at mid span 33.75 kN. if f_{ck} = 20 N/mm² and f_y = 415 N/mm². Calculate the shear reinforcement.*



Section

Solution. Given data $b = 250$ mm $d = 348$ mm

At support $V_u = 110.25$ kN

At mid span $V_u = 33.75$ kN

Stresses are $f_{ck} = 20 \text{ N/mm}^2$ $f_y = 415 \text{ N/mm}^2$

Factored shear force at critical section

The critical section is $d = 348$ mm from the face of support $\frac{230}{2} + 348 = 463$ mm from the centre of support.

$$\therefore \quad V_u = 33.75 + (101.25 - 33.75) \times \frac{3000 - 463}{3000} = 90.8 \text{ kN}$$

Check adequacy of section

Nominal shear stress $\tau_v = \frac{V_u}{bd} = \frac{90.8 \times 10^3}{250 \times 348}$

$= 1.04 \text{ N/mm}^2 < \tau_{c,\max} = 2.8 \text{ N/mm}^2$ (for *M* 20 concrete)

Hence, the size of section is adequate.

Design shear strength

At the critical section,

A_{st} (due to 2–28 ϕ) = 2 × 616 = 1232 mm²

$$\Rightarrow \quad p_t = \frac{100\, A_{st}}{bd} = 1.416$$

$\Rightarrow$ Design shear strength of concrete (from Table , for *M* 20 concrete)

$$\tau_c = 0.71 \text{ N/mm}^2$$

Shear resisted by concrete $V_{uc} = \tau_c \, bd$

$$= 0.71 \times 250 \times 348 = 61.7 \text{ kN}$$

Shear resisted by minimum stirrups

$$V_{us,\min} = 0.4 \, bd = 0.4 \times 250 \times 348 = 34.8 \text{ kN}$$

Total shear resistance

$$V_{uc} + V_{us,\min} = (61.7 + 34.8) = 96.5 \text{ KN} > V_u = 90.8 \text{ kN}$$

Hence, the section is safe in shear.

But in good practice, minimum shear reinforcement is provided.

Provide 8 mm bar of 2-legged closed stirrup

$$A_{sv} = 2 \times 50.3 = 100.6 \text{ mm}^2$$

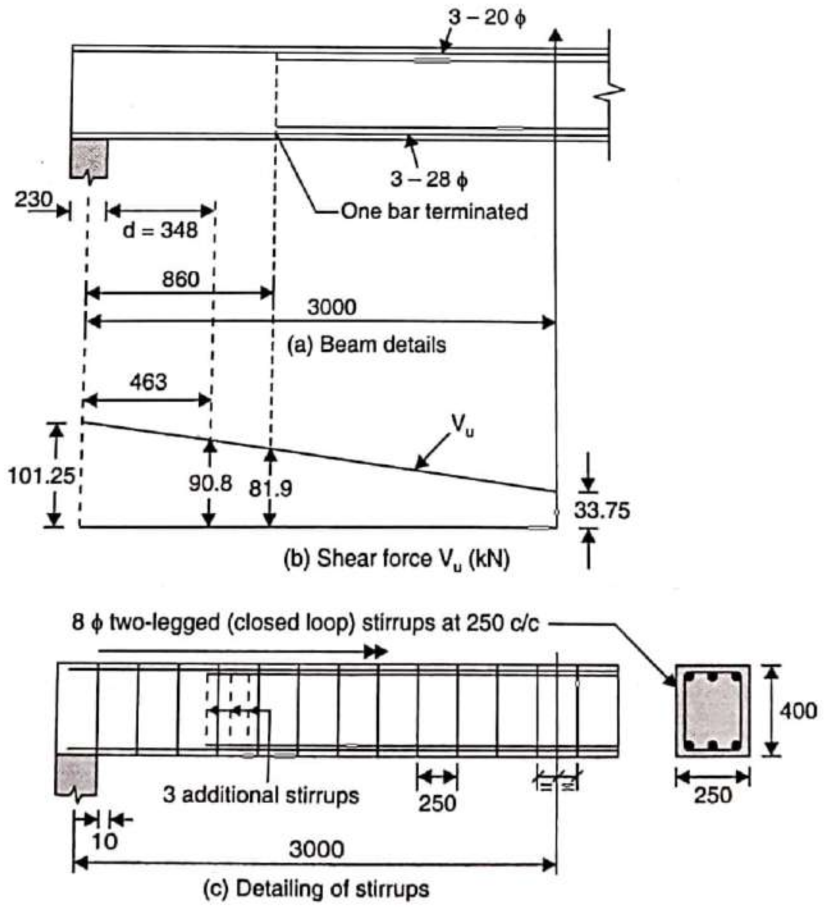
Check minimum stirrup requirements (maximum spacing)

$$(S_v)_{\max} = \frac{2.175 \, f_y \, A_{sv}}{b} = \frac{2.175 \times 415 \times 100.6}{250} = 363 \text{ mm}$$

Further,
$$S_v \leq \begin{cases} 0.75d = 0.75 \times 348 = 261 \text{ mm} \\ 300 \text{ mm} \end{cases}$$

From above, clear that

Provide 8 ϕ two legged closed stirrup at 250 mm c/c spacing



Detailing of stirrups

Check shear strength at bar cut off point

The cut off point is located at 860 mm from centre of support

∴ Factored shear force at this section

$$V_u = 33.75 + (101.25 - 33.75) \times \frac{(3000 - 860)}{3000} = 81.9 \text{ kN}$$

Shear resistance by concrete with stirrups

$$= \tau_c\, b\, d + \frac{0.87 f_y A_{sv}\, d}{S_v}$$

$$= (0.71 \times 250 \times 348) + \frac{0.87 \times 415 \times 100.6 \times 348}{250} = 114 \text{ kN}$$

$$\frac{2}{3} \text{ shear resistance} = \frac{2}{3} \times 114 = 76 \text{ kN} < 81.9 \text{ kN}$$

As, shear strength requirements (cl. 26.2.3.2(*a*)) of the code are not satisfied, therefore, additional stirrups must be provided over a distance of 0.75 d = 261 mm ('along the terminated bar') with a spacing $< \frac{d}{8\beta_b} = \frac{348}{\left(8 \times \frac{1}{3}\right)} = 130$ mm [cl. 26.2.3.2.(*b*) of the code]

This is achieved by adding three additional stirrups along the last portion of the cut-off bar.

$\Rightarrow$ spacing $= \frac{261}{3} = 87$ mm $\simeq$ 85 mm which is less than 130 mm.

∴ Excess stirrup area required $= \frac{0.4\, b\, S_v}{f_y} = \frac{0.4 \times 250 \times 85}{415} = 20.48$ mm.

Therefore, provide 2-legged 6 mm ϕ bars, 3 nos. @ 85 mm c/c at the cut-off point (along the terminated bar) in addition to 2-legged 8 mm Fe 415 grade vertical stirrups @ 250 mm c/c. But from a practical view point, it is convenient to use the same 8 mm ϕ (A_{sv} = 100.6 mm^2) for the additional stirrups.

Bond and Torsion

Question - 4 : *A simply supported beam is 260 mm by 520 mm and has 2–20 mm HYSD bars going into the support. If the shear force at the centre of support is 115 kN at working loads determine the anchorage length.*

Assume M20 mix and Fe 415.

Solution. Assuming 25 mm clear cover to the longitudinal bars effective depth

$$= 520 - 25 - \frac{20}{2} = 485 \text{ mm}$$

Moment of resistance $M_1 = 0.87 f_y A_{St} (d - 0.42 x_u)$

$$x_u = \frac{0.87 f_y A_{St}}{0.36 f_{Ck} . b} = \frac{0.87 \times 415 \times (2 \times 314)}{0.36 \times 20 \times 260} = 121.2 \text{ mm} < x_{u, \max}$$

or,

$$M_1 = 0.87 \times 415 \times (2 \times 314) \times (485 - 0.42 \times 121.2)$$

$$M_1 = 98.43 \times 10^6 \text{ N-mm.}$$

Bond stress

$$\tau_{bd} = 1.2 \text{ N/mm}^2 \text{ for M20.}$$

It can be increased by 60% in case of HYSD bars.

∴ Development length

$$L_d = \frac{0.87 f_y \phi}{4 \tau_{bd}} = \frac{0.87 \times 415 \times \phi}{4 \times 1.2 \times 1.6} = 47 \phi$$

If the bar is given a 90° bend at the centre of support, its anchorage value

$$L_o = 8 \phi = 8 \times 20 = 160 \text{ mm}$$

$$L_d \leq 1.3 \frac{M_1}{V} + L_o$$

$$47 \phi \leq \left[\frac{1.3 \times 98.43 \times 10^6}{(1.5 \times 115) \times 1000}\right] + 160$$

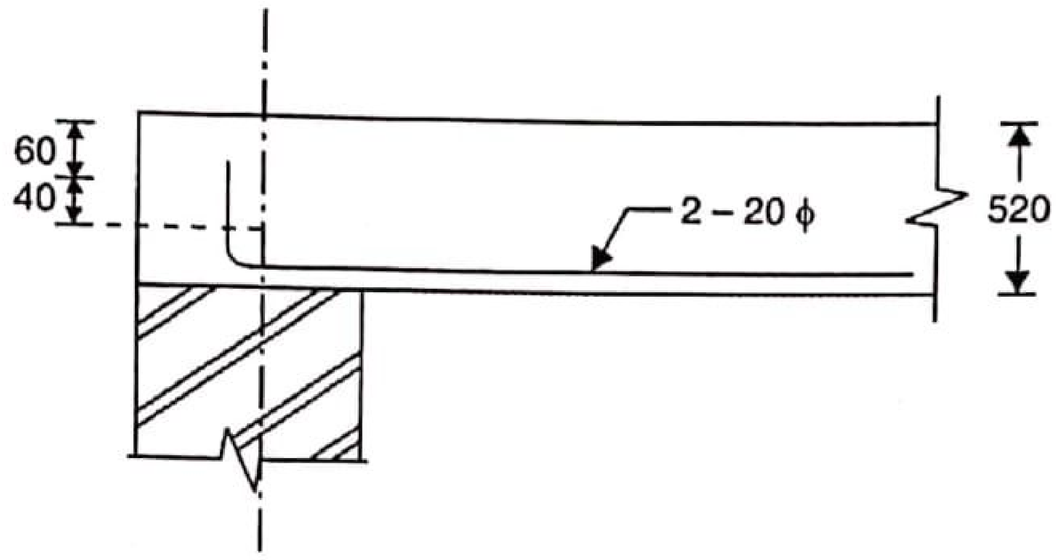
$$\phi \leq 19.2 \text{ mm}$$

Since the actual bar diameter of 20 mm is greater than 19 mm, there is a need to increase the anchorage length.

Let us increase the anchorage length L_o to 220 mm.

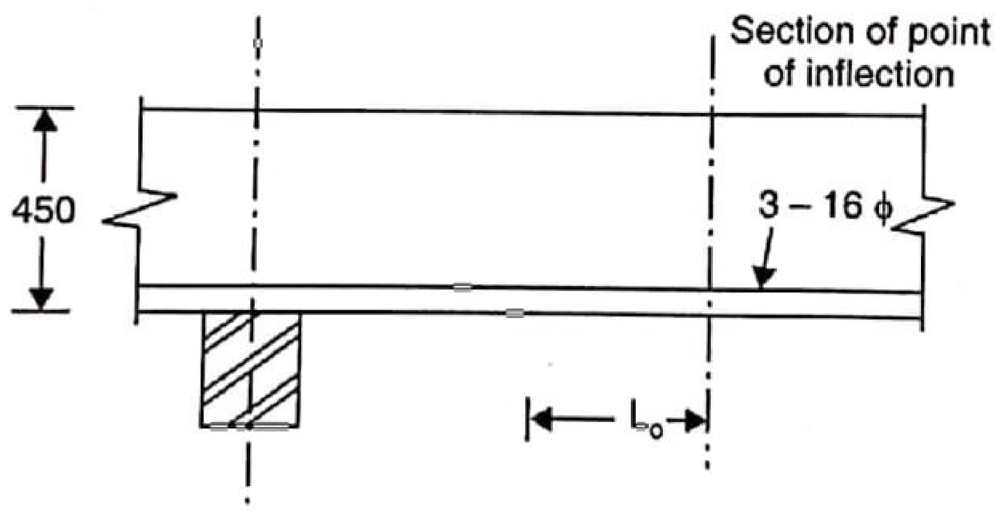
It gives $\phi \leq 20.46$ mm (O.K.)

The arrangement of 90° bend is shown in Fig.



Question - 5 : *A continuous beam 250 mm × 450 mm carries 3–16 mm longitudinal bars beyond the point of inflection in the lagging moment region as shown in Fig. The S.F. at the point of inflection is 110 KN at working loads, check if the beam is safe in bond ?*

Assume M20 and $f_y = 415$ *N/mm²*.



Section

Solution. Assuming 25 mm clear cover to the longitudinal bars

Effective depth $$d = 450 - 25 - \frac{16}{2} = 417 \text{ mm}$$

Depth of neutral axis $$x_u = \frac{0.87\, f_y\, A_{st}}{0.36\, f_{ck}\, b}$$

$$= \frac{0.87 \times 415 \times (3 \times 201)}{0.36 \times 20 \times 250} = 120 \text{ mm} < x_{u,\,max}\ (= 0.48\, d).$$

Moment of resistance $$M_1 = 0.87\, f_y\, A_{St}\, (d - 0.42\, x_u)$$

$$= 0.87 \times 415 \times (3 \times 201) \times (417 - .42 \times 120)$$

$$\approx 79.81 \times 10^6 \text{ N-mm}.$$

Development length

$$L_d = \frac{0.87\, f_y\, \phi}{4\, \tau_{bd}} = 47\, \phi \text{ (for } M20 \text{ and Fe 415)}$$

Anchorage length L_o = greater of d or 12 ϕ

= greater of 417 mm or 12 × 16 = 192 mm = 417 mm

$$\therefore \quad L_d \leq \frac{M_1}{V} + L_0$$

or,

$$47\, \phi \leq \frac{79.82 \times 10^6}{(1.5 \times 110) \times 1000} + 417$$

or,

$$\phi \leq 19.2 \text{ mm}$$

Thus, 16 mm bars are safe in bond at point of inflection.

Question - 6 : *A rectangular beam of size 230 mm × 400 mm overall depth, is reinforced with 2–10 mm bars at the top and 3–16 mm bars at bottom being tension reinforcement. It is subjected to characteristics loads, shear force of 18 kN, a torsional moment of 1.2 kN-m and a bending moment of 18 kN-m. Check for torsion reinforcement. Assume M20 mix and Fe 415.*

Solution. Assuming 25 mm clear cover to the longitudinal bars effective depth

$$d = 400 - 25 - \frac{16}{2} = 367 \text{ mm}$$

For a load factor equal to 1.5, the factored

$$\text{S.F.} = 1.5 \times 18 = 27 \text{ kN}$$

$$M_u = 1.5 \times 18 = 27 \text{ kN-m}$$

$$\tau_u = 1.5 \times 1.2 = 1.8 \text{ kN-m}$$

Now, equivalent shear force

$$V_e = V_u + 1.6\, \frac{T_u}{b}$$

$$= 27 + 1.6 \times \frac{1.8 \times 10^3}{230} = 39.52 \text{ kN}$$

Equivalent nominal shear stress

$$\tau_{Ve} = \frac{V_e}{b.d} = \frac{39.52 \times 10^3}{230 \times 367} = 0.468 \text{ N/mm}^2$$

$$\text{percentage of steel} = \frac{100\, A_{St}}{b.d}$$

$$p_t = \frac{100 \times (3 \times 201)}{230 \times 367} = 0.71$$

For $p_t = 0.71\%$ and M20 concrete

$\tau_c = 0.54\ N/mm^2$ (From table 19 of code)

or, $\tau_c = 0.54\ N/mm^2 > \tau_{Ve} = 0.468\ N/mm^2$

Therefore, torsion reinforcement is not required. However, minimum stirrup shall be provided.

Using 6 mm bars with 2 legged stirrups

Spacing of stirrups

$$S_V \le \frac{0.87 f_y A_{Sv}}{0.4 b} = \frac{0.87 \times 415 \times (2 \times 28)}{0.4 \times 230}$$

or, $$S_V \le 132.4 \text{ mm}$$

Provide 6 mm bars of 2 legged stirrups @ 130 mm c/c.

Question - 7 : *A beam of rectangular section is a multistory frame is 250 mm × 500 mm deep. The section is subjected to an ultimate bending moment of M_u = 55 kN-m, ultimate torsional moment T_u = 30 kN-m, and ultimate shear force V_u = 40 kN. Using M20 grade concrete and Fe 415. Design suitable reinforcement in section. Effective cover to steel = 50 mm.*

Solution. Equivalent bending moment

$$M_{e1} = M_u + M_t$$

where $M_u = 55$ kN-m

$M_t = T_u$

$$\left(\frac{1 + D/b}{1.7}\right) = 30\left(1 + \frac{500/250}{1.7}\right) = 53 \text{ kN-m}$$

Here, $M_t < M_u$, therefore no additional steel is required or compression side.

$$M_u = 55 + 53 = 108 \text{ kN-m}$$

Limiting bending moment

$$M_{u,\lim} = 0.138 f_{ck} bd^2$$

$$= 0.138 \times 20 \times 250 \times 450^2 = 139.7 \text{ kN-m}$$

Since, $M_{e1} < M_{u,\lim}$ section is under reinforced

$$\therefore \quad M_{e1} = 0.87 f_y \ A_{st} \cdot d \left[1 - \frac{f_y \cdot A_{St}}{f_{ck} \cdot bd}\right]$$

$$108 \times 10^6 = 0.87 \times 415 \times A_{St} \times 450 \left[1 - \frac{415 \times A_{St}}{20 \times 250 \times 450}\right]$$

Solving, $$A_{St}^2 - 5421.6\, A_{St} + 3.6 \times 10^6 = 0$$

$$\therefore \quad A_{St} = 774.8 \text{ mm}^2$$

This is required longitudinal reinforcement.

Equivalent shear force

$$V_e = \left(V_u + 1.6\frac{T_u}{b}\right)$$

$$= \left(40 + 1.6 \times \frac{30}{250}\right) = 232 \text{ kN}$$

Equivalent nominal shear stress

$$\tau_{Ve} = \frac{V_e}{bd} = \frac{232 \times 10^3}{250 \times 450} = 2.06 \text{ N/mm}^2$$

$$\text{percentage of steel} = \frac{100\, A_{St}}{bd}$$

$$p_t = \frac{100 \times 774.8}{250 \times 450} = 0.69$$

For $p_t = 0.69\%$ and $M20$ concrete

Shear strength of concrete $\tau_c = 0.6 \text{ N/mm}^2$

∴ Shear resisted by concrete $V_c = \tau_c \,.\, bd$

$$= 0.6 \times 250 \times 450 = 67.5 \text{ kN}$$

Shear resisted by minimum stirrups

$$V_{us,\,min} = 0.4\,bd = 0.4 \times 250 \times 450 = 45 \text{ kN}$$

∴ Shear resisted by Concrete and stirrups

$$= V_c + V_{us,\,min} = 112.5 \text{ kN} < V_e = 232 \text{ kN}$$

Therefore, transverse reinforcement is to be designed. Using 10 mm diameter 2-legged stirrups.

The spacing of stirrups is given by

$$S_V = A_{SV} \times 0.87 f_y \left[\frac{b_1 d_1}{T_u} + \frac{2.5\, d_1}{V_u}\right] \quad \text{or} \quad \text{not less than}$$

$$S_V = \left[\frac{A_{SV} \times 0.87\, f_y}{(\tau_{Vc} - \tau_c)\, b}\right]$$

∴

$$S_V = (2 \times 78.5) \times 0.87 \times 415 \times \left[\frac{150 \times 400}{30 \times 10^6} + \frac{2.5 \times 400}{40 \times 10^3}\right]$$

$$= 1530.5 \text{ mm} \quad \text{or} \quad \text{not less than}$$

$$S_V = \left[\frac{(2 \times 78.5) \times 0.87 \times 415}{(2.06 - 0.6) \times 250}\right] = 155.3 \text{ mm}$$

Now, details of reinforcement in section are:

main tensile steel – 4 bars of 16 mm diameter ($A_{St} = 804 \text{ mm}^2$)

hanger bar (compression side) – 2 bars of 12 mm diameter

transverse steel – 10 mm diameter 2 legged stirrups at 150 mm c/c.

Slab Design

Question - 8 : *Design a one way slab, with a clear span of 4.0 m. Simply supported on 300 mm thick masonry walls, and subjected to a live load of 5 kN/m² and surface finish 0.6 kN/m². Assume that the slab is subjected to moderate exposure conditions. Use Fe 415 grade of steel.*

Solution. Design constants and limiting depth of N.A.

For moderate exposure conditions, considering M25 grade concrete,

$$f_{ck} = 25 \text{ N/mm}^2$$

$$\frac{x_{u,\max}}{d} = 0.48 \text{ for Fe 415 steel}$$

$$\therefore \quad R_u = 0.36 f_{ck} \cdot \frac{x_{u,\max}}{d}\left(1 - 0.42\frac{x_{u,\max}}{d}\right)$$

$$= 0.36 \times 25 \times 0.48\,(1 - 0.42 \times 0.48) = 3.45$$

Determining factored B.M.

Assume an effective depth, $d = \dfrac{4000}{25} = 160$ mm

and an overall depth $D = 160 + 30 = 190$ mm

$\therefore$ Effective span $l = \begin{cases} 4000 + 300 = 4300\ mm\ (c/c \text{ distance}) \\ 4000 + 160 = 4160 \text{ mm} \end{cases}$

Taking the lesser values (as per code) $l = 4.16$ m

Self weight of slab $= 0.19 \times 25 = 4.75$ kN/m²

Floor finish $= 0.6$ kN/m²

$\therefore$ Total dead loads $w_{DL} = 5.35$ kN/m²

Live load on slab, $w_{LL} = 5.0$ kN/m² (given)

$\therefore$ Factored load (as per code) $w_u = 1.5\,(w_{DL} + w_{LL})$

$= 1.5\,(5.35 + 5.0) = 15.53$ kN/m²

$\therefore$ Factored B.M. (Maximum at mid span)

$$M_u = \frac{w_u l^2}{8} = \frac{15.53 \times 4.16^2}{8} = 33.6 \text{ kN-m/m}$$

Computation of effective depth d for a balanced section;

$$M_u = R_u \,.\, bd^2$$

$$\therefore \quad d = \sqrt{\frac{M_u}{R_u . b}} = \sqrt{\frac{33.6 \times 10^6}{3.45 \times 1000}} = 98.6 \text{ mm}$$

Actual d available = 160 mm (based on the limit state of serviceability requirements) (O.K.)

Steel reinforcement (Main bars):

The area of main bar

$$A_{St} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6\, M_u}{f_{ck}\, bd^2}}\right] bd$$

$$= \frac{0.5 \times 25}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 33.6 \times 10^6}{25 \times 1000 \times 160^2}}\right] \times 1000 \times 160$$

$$(A_{St})_{reqd} = 622 \text{ mm}^2/\text{m}$$

$$(A_{St})_{\min} = \frac{0.12}{100} \times 1000 \times 190 = 228 \text{ mm}^2/\text{m}$$

Using 8 mm bars;

Hence, spacing of 8 ϕ bars as main reinforcement is

$$S = \frac{1000 \times a_{st}}{A_{St}} = \frac{1000 \times \left(\frac{\pi}{4} \times 8^2\right)}{622} = 81 \text{ mm c/c}$$

Maximum spacing limits

$$3d = 3 \times 160 = 480 \text{ mm}$$

or 300 mm, whichever is less

$\therefore$ provided 8 ϕ @ 75 mm c/c for main reinforcement.

$$(A_{St})_{\text{provided}} = \frac{1000 \times 50.3}{75} = 671 \text{ mm}^2/\text{m}$$

Distribution bars: (to be provided at right angles, in plan to the main reinforcement)

$$(A_{St})_{\text{dist.}} = \frac{0.12}{100} \times 1000 \times 190 = 228 \text{ mm}^2/\text{m}$$

using 8 mm bars as distribution reinforcement spacing $S = \frac{1000 \times 50.3}{228} = 221$ mm

Maximum spacing limits.

$$5d = 5 \times 160 = 800 \text{ mm}$$

or 450 mm; whichever is less

$\therefore$ provide 8 ϕ @ 200 mm c/c for distribution reinforcement.

Check for control of Deflection:

At mid-span;

$$\frac{100\, A_{St}}{bd} = \frac{100 \times 671}{1000 \times 160} = 0.42$$

and $$f_S = 0.58 f_y \frac{\text{area of cross-section of steel required}}{\text{area of cross-section of steel provided}}$$

$$= 223 \text{ N/mm}^2$$

$$K_f = 1.44 \quad \text{(From IS code)}$$

$$\therefore \quad \left(\frac{l}{d}\right)_{\max} = 20 \times 1.44 = 28.8$$

$$\left(\frac{l}{d}\right)_{\text{provided}} = \frac{4160}{160} = 26.0 < 28.8$$

Check of shear:

Normally slabs are found to be safe in shear but it should be checked.

Factored shear force $V_u = \frac{w_u \cdot l}{2} = \frac{15.53 \times 4.16}{2} = 32.3 \text{ kN/m}$

The critical section for shear is at a distance of d (= 160 mm) from the face of support

$$\therefore \quad V_{uD} = V_u - w_u (0.160) = 32.3 - 15.53 \times 0.160$$

$$V_{uD} = 29.8 \text{ kN/m}$$

Nominal shear stress

$$\therefore \quad \tau_V = \frac{V_{uD}}{bd} = \frac{29.8 \times 10^3}{1000 \times 160} = 0.17 \text{ N/mm}^2$$

percentage of steel at supports

$$\frac{100 A_{St}}{bd} = \frac{100 \times 671/2}{1000 \times 160} = 0.21\%$$

for $p_t = 0.21\%$ and M-25 concrete, shear strength of concrete

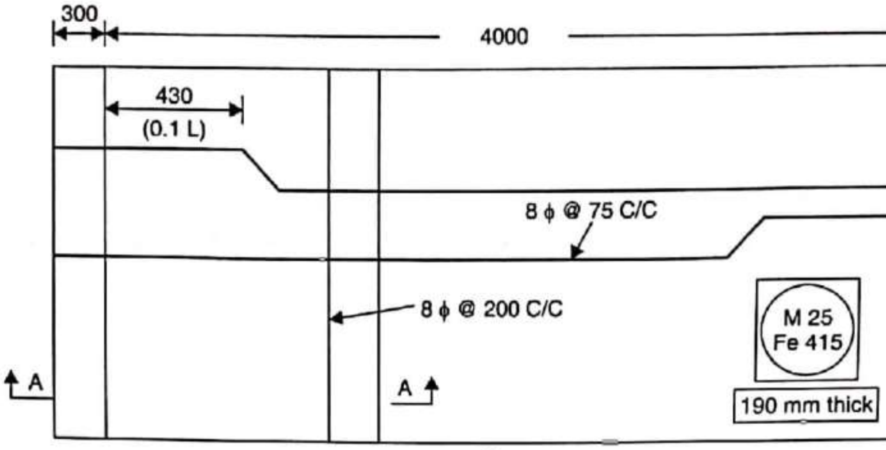
$$\tau_C \approx 0.35 \text{ N/mm}^2$$

For 190 mm thick slab $K = 1.21$ (from Table 8.1)

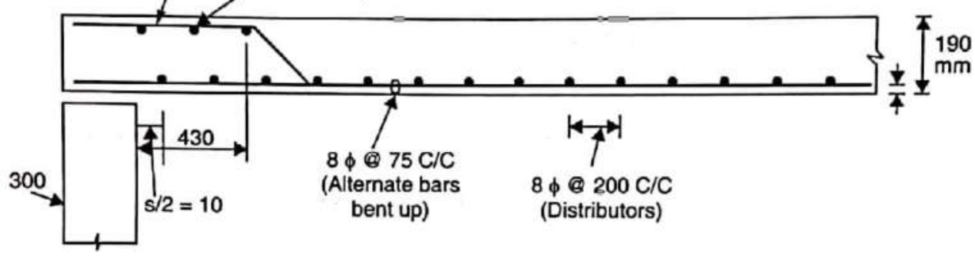
$$\therefore \quad \tau'_C = K \cdot \tau_c = 1.21 \times 0.35 = 0.42 \text{ N/mm}^2$$

Since $\tau'_C < K.\ \tau_c$, *the section is safe in shear.*

Detailing of reinforcement: The complete detailing of slab as shown in Fig. Alternate bars of the main reinforcement are bent up near the supports at a distance of 0.1 l from the support (as per clause D-1.6 of code) in order to resist any flexural tension.



Plan

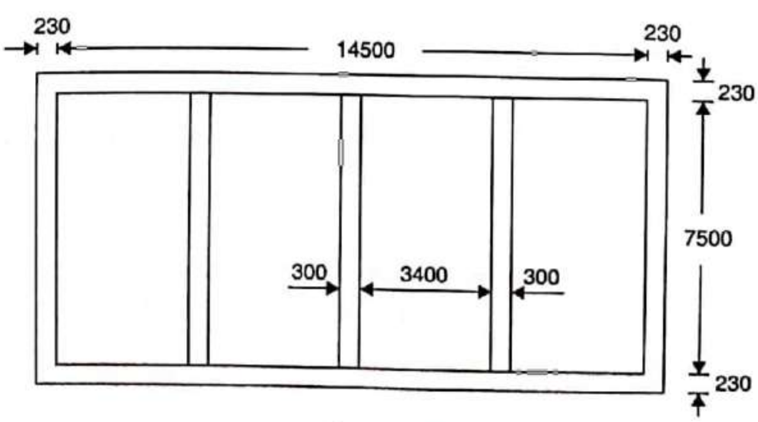


Section A-A

Reinforcement detail of one way slab.

Question - 9 : *Design a continuous R.C. slab for a hall 7.5 m wide and 14.5 m long (clear span). The slab is supported on R.C.C. beams, each 300 mm wide which are monolithic. The ends of the slab are supported on walls, 230 mm wide. The specified floor loading of a live load of 3 kN/m² and a dead load (due to floor finish, partitions etc.) of 1.5 kN/m² in addition to self weight. Assume Fe 415 steel and M.25 concrete.*

Solution.



Arrangement of spans.

Arrangement of spans: Each beam wide is 300 mm.

Clear spacing between beams is equal to $(14.5 - 0.3 \times 3)/4 = 3.4$ m

Each slab panel (with clear spans 3.4 m × 7.5 m) has an $\frac{L}{B}$ ratio greater than 2 and hence may be treated as one-way continuous slab

Fixation of *d* and *D*: From the limit state of serviceability, the effective depth is given by

$$\frac{l}{d} = 26 \text{ for continuous spans}$$

$$\frac{l}{d} = 20 \text{ for simply supported spans.}$$

Hence for end spans

$\frac{l}{d} = \frac{1}{2}(26 + 20) = 23$, since the end span which is critical is discontinues on one edge and continuous at the other.

Assume a modification factor $K_f = 1.34$ (corresponding to $p_t \simeq 0.4$ and $f_S = 240\ \text{N/mm}^2$)

$$\therefore \quad \left(\frac{l}{d}\right)_{max} = 23 \times 1.34 = 30.8$$

$$\therefore \quad d_{min} = \frac{3500}{30.8} = 114 \text{ mm (for an assumed effective span of } l = 3.5 \text{ m)}$$

Assumed overall depth

$$D = 114 + 40 = 155 \text{ mm for all spans and } d \approx 120 \text{ mm}$$

Effective length: As the beam width (300 mm) exceeds $\frac{1}{12} \times 3400 = 283$ mm. Hence, the effective span will be as under, (clause 22.2(*b*) of code)

(*i*) **For end spans:**

$$l = \text{clear span} + \frac{d}{2} = 3400 + \frac{120}{2} = 3460 \text{ mm}$$

hence, $l = 3.46$ m.

(*ii*) **For intermediate spans:**

$$l = \text{clear span} = 3400 = 3400 \text{ mm}$$

hence $l = 3.40$ m

Thus, in this case, the length of effective span is not equal.

Design constant and limiting depth of N.A.

For Fe 415 steel, $\frac{x_{u,max}}{d} = 0.48$

$\therefore \quad R_u = 0.36 f_{ck} \frac{x_{u,\max}}{d}\left(1 - 0.42 \frac{x_{u,\max}}{d}\right)$

$= 0.36 \times 25 \times 0.48 \times (1 - 0.42 \times 0.48) = 3.45.$

Computation of factor bending moment.

Self weight of slab $= 25 \times 0.155 = 3.875$ kN/m²

Floor finish $= 1.5$ kN/m²

$\therefore$ Total $w_{DL} = 5.38$ kN/m²

Live load $w_{LL} = 3.0$ kN/m² (given)

As the effective span is not equal but difference is very much less than 15% of the longer spans; hence they may be treated as being equal as per Cl. 22.5.1 of code.

As the spans are equal, uniformly loaded and more than three in number, using the B.M. coefficients given in Table 12 of IS code.

For end span ($l = 3.460$ m)

$$M_u = \begin{cases} -\left(\dfrac{w_{uD.L.} + w_{uL.L.}}{24}\right) l^2 = -6.27 \text{ kN-m/m at end support} \\ +\left(\dfrac{w_{uD.L.}}{12} + \dfrac{w_{uL.L.}}{10}\right) l^2 = +13.44 \text{ kN-m/m at mid span} \\ -\left(\dfrac{w_{uD.L.}}{10} + \dfrac{w_{uL.L.}}{9}\right) l^2 = -15.65 \text{ kN-m/m at interior span} \end{cases}$$

For intermediate span ($l = 3.400$ m)

$$M_u = \begin{cases} -\left(\dfrac{w_{uD.L.}}{10} + \dfrac{w_{uL.L.}}{9}\right) l^2 = -15.10 \text{ kN-m/m at first interior support} \\ +\left(\dfrac{w_{uD.L.}}{16} + \dfrac{w_{uL.L.}}{12}\right) l^2 = +11.61 \text{ kN-m/m at mid span} \\ -\left(\dfrac{w_{uD.L.}}{12} + \dfrac{w_{uL.L.}}{9}\right) l^2 = -13.55 \text{ kN-m/m at interior support} \end{cases}$$

At the first interior support, an average value of M_u should be considered.

$$M_u = -(15.65 + 15.10)/2 = -15.38 \text{ kN-m/m}$$

Computation of effective depth: Out of the above values of moments; the effective depth will be determined for max. value

$\therefore$ factored B.M. $= -15.38$ kN-m/m

$$\therefore \quad d = \sqrt{\frac{M_u}{R_u \cdot b}} = \sqrt{\frac{15.38 \times 10^6}{3.45 \times 1000}} = 67 \text{ mm}$$

available $d = 120$ mm, determined from limit state of serviceability.

Hence the section is (O.K.)

Determining the steel reinforcement (Main bars).

For maximum moment $M_u = -15.38$ kN-m/m the area of steel is calculated as;

$$A_{St} = \frac{0.5 f_{ck}}{f_y}\left[1-\sqrt{1-\frac{4.6\,M_u}{f_{ck}\,bd^2}}\right] bd$$

$$A_{St} = \frac{0.5 \times 25}{415}\left[1-\sqrt{1-\frac{4.6 \times 15.38 \times 10^6}{25 \times 1000 \times 120^2}}\right] \times 1000 \times 120 \text{ mm}^2/\text{m}$$

$$= 375 \text{ mm}^2/\text{m}$$

Using 8 ϕ bar ($a_{St} = 50.3$ mm^2)

spacing required $= \dfrac{1000 \times 50.3}{375} = 134$ mm

provide 8 ϕ bars 110 mm c/c spacing

$$(A_{St})_{provided} = \frac{1000 \times 50.3}{110} = 457 \text{ mm}^2/\text{m}$$

Maximum spacing allowed = 3 × 120 = 360 mm

or 300 mm, whichever is less. (O.K.)

Distribution bars:

$$(A_{St})_{min} = \frac{0.12}{100} \times b \times D = \frac{0.12}{100} \times 1000 \times 155$$

$$= 186 \text{ mm}^2/\text{m}$$

provided 8 mm ϕ @ 250 mm c/c

maximum spacing allowed = 5 × 120 = 600 mm

or 450 mm, whichever is less. (O.K.)

Check for deflection control: Maximum midspan steel in the end span

$$(A_{St})_{provided} = \frac{1000 \times 50.3}{110} = 457 \text{ mm}^2/\text{m}$$

providing a clear cover of 30 mm

$$d = 155 - 30 - \frac{8}{2} = 121 \text{ mm}$$

$\therefore$ percentage of steel $p_t = \dfrac{100 \times 457}{1000 \times 121} = 0.38$

and

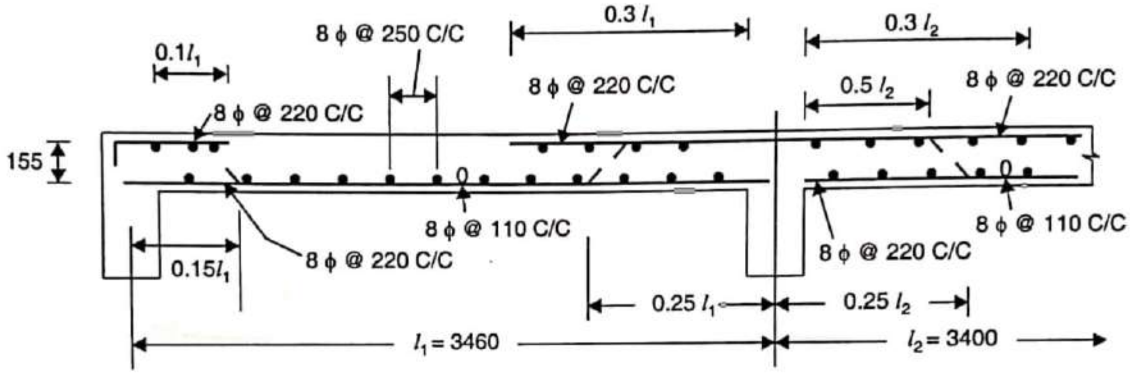
$$f_S = 0.58 \times 415 \times \frac{328}{457} = 172.8 \text{ N/mm}^2$$

∴ modification factor $K_t = 1.71$

$$\left(\frac{l}{d}\right)_{max} = \frac{1}{2}(20 + 26) \times 1.71 = 39.4$$

$$\left(\frac{l}{d}\right)_{provided} = \frac{3460}{121} = 28.6 < 39.4 \qquad \text{(O.K.)}$$

Detailing of reinforcement: The sectional details of the reinforcement are shown in Fig.



Details of one-way continuous slab.

Question - 10 : *Design a two way slab for a residential roof with internal dimension 4.5 m × 6.0 m and 230 mm thick brick wall all around. Assuming live load 2 kN/m² and a finish load of 1 kN/m². Use M20 and Fe 415.*

Edge conditions: Simply supported on all the sides of wall. Assume that the slab corners are free to lift up and exposure conditions are mild.

Solution. Assume overall thickness of slab as 160 mm with clear cover of 20 mm and say 10 ϕ bars

$$\therefore \quad d_x = 160 - 20 - 5 = 135 \text{ mm}$$

$$d_y = 135 - 10 = 125 \text{ mm}$$

Effective span is c/c distance or (clear span + d), whichever is smaller.

$$\therefore \quad l_x = 4500 + 135 = 4635 \text{ mm}$$

$$l_y = 6000 + 125 = 6125 \text{ mm}$$

$\therefore$ Effective span = 4.635 m, as this is less than centre-to-centre span between supports.

Also, $$\frac{l_y}{l_x} = \frac{6125}{4635} = 1.32$$

Loads on slab:

(*i*) Self weight of slab = 25 × 0.160 = 4.0 kN/m²

(*ii*) Live load (given) = 2 kN/m²

(*iii*) Finishes (given) = 1 kN/m²

Total = w = 7.0 kN/m²

$\therefore$ Factored load $w_u = 70 \times 1.5 = 10.5 \text{ kN/m}^2$

Design moments: Moments along short span M_x and along long span M_y are given by

$$M_x = \alpha_x \cdot w \cdot l_x^2$$
$$M_y = \alpha_y \cdot w \cdot l_x^2$$

For $\frac{l_y}{l_x} = 1.32,\ \alpha_x = 0.0942$ and $\alpha_y = 0.0534$ (Table 9.1)

$\therefore$ $M_x = 0.0942 \times 10.5 \times 4.635^2 = 21.25$ kN-m/m

$M_y = 0.0534 \times 10.5 \times 4.635^2 = 12.05$ kN-m/m

Check depth from B.M. Consideration:

Maximum B.M. $= 0.138 f_{CK}\, bd^2$

$$\therefore \quad d = \sqrt{\frac{21.25 \times 10^6}{0.138 \times 20 \times 1000}} = 87.75 \text{ mm} < 135 \text{ mm}$$

Design of Reinforcement: Area of steel $(A_{St})_x$ along short span

$$= \frac{0.5 f_{CK}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 M_x}{f_{CK}\, bd^2}}\right] bd$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 21.25 \times 10^6}{20 \times 1000 \times 135^2}}\right] \times 1000 \times 135$$

$= 470.2$ mm²/m

required spacing of 10 ϕ bars $= \frac{1000 \times 78.5}{470.2} = 167$ mm

Similarly, $$(A_{St})_y = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 12.05 \times 10^6}{20 \times 1000 \times 125^2}}\right] \times 1000 \times 125$$

$= 280.2$ mm²/m

required spacing of 10 ϕ bars $= \frac{1000 \times 78.5}{280.2} = 280$ mm

Maximum spacing for primary reinforcement = $3d$ or 300 mm

$$= \begin{cases} 3 \times 135 = 405 \text{ mm (short span)} \\ 3 \times 125 = 375 \text{ mm (long span)} \end{cases}$$

$$\therefore \quad \text{Provide} \begin{cases} 10\ \phi\ @\ 150 \text{ c/c (short span)} \Rightarrow (A_{St})_x = 524 \text{ mm}^2/\text{m} \\ 10\ \phi\ @\ 250 \text{ c/c (long span)} \Rightarrow (A_{St})_y = 314 \text{ mm}^2/\text{m} \end{cases}$$

The code requires that the minimum area of steel should be 0.12%

$$= \frac{0.12}{100} \times 1000 \times 160 = 192 \text{ mm}^2 < (A_{st})_x$$

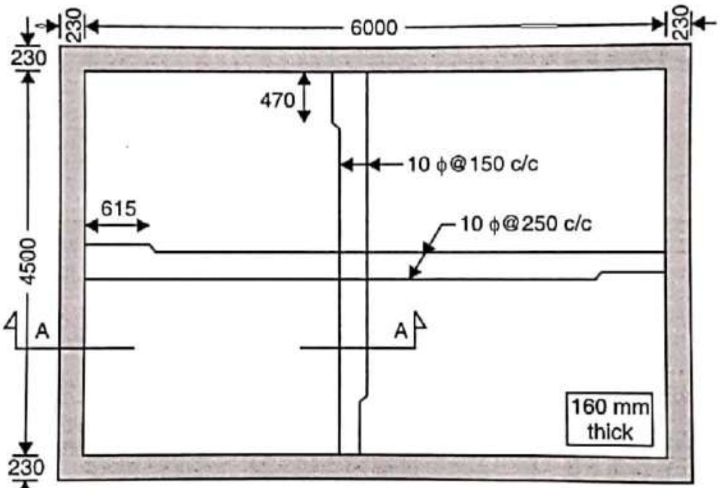
Curtail alternate bars at $\frac{1}{10}$th of effective span in each direction in accordance with clause D-2.1.1. of the code provide 50% of the maximum positive reinforcement at top near the supports to resist bending moment due to partial fixity. This reinforcement is provided in 0.1 l length from the face of supports.

The detailing of reinforcement is shown in Fig.

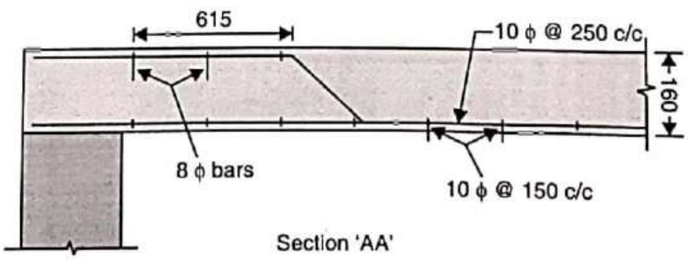
Shear strength in slabs $= \tau_c' = K\tau_c$

$$K = 1.28 \text{ for } D = 160 \text{ mm}$$

$$\therefore \quad \tau_c' = 1.28 \times 0.44 = 0.563 \text{ MPa} > \tau_v$$



Plan of roof



Section 'AA'

Check for Deflection: Percentage of steel along short span.

$$(p_t)_x = \frac{524}{10^3 \times 135} \times 100 = 0.39$$

$$\therefore \quad f_s = 0.58 \times 415 \times \frac{470.2}{524} = 216 \text{ MPa}$$

Modification factor $K_t = 1.75$

$$\therefore \quad (l/d)_{max} = 20 \times 1.75 = 35$$

$$(l/d)_{provided} = \frac{4635}{135} = 34.33 < 35$$

Check for shear:

average effective depth $d = (135 + 125)/2 = 130$ mm

$$V_u = w_u\left(\frac{l_x}{2} - d\right) = 10.5 \times \left(\frac{4.5}{2} - 0.130\right) = 22.26 \text{ kN/m}$$

Nominal shear stress $\tau_V = \dfrac{22.26 \times 10^3}{1000 \times 130} = 0.172$ MPa

percent tension steel $= 0.39\%$

Shear strength of M20 concrete for 0.39% steel

$$\tau_C = 0.44 \text{ MPa.}$$

Question - 11 : *Repeat* question 10 *, assuming the slab corners are prevented from lifting up.*

Solution. Assume overall thickness $D = 155$ mm

Assuming 8 ϕ bars $\Rightarrow d_x = 155 - 20 - 4 = 131$ mm

$$d_y = 131 - 8 = 123 \text{ mm}$$

$$\Rightarrow \begin{cases} l_x = 4500 + 131 = 4631 \text{ mm} \\ l_y = 6000 + 123 = 6123 \text{ mm} \end{cases} \Rightarrow \frac{l_y}{l_x} = 1.32$$

Loads on slab:

(*i*) Self weight of slab $= 25 \times 0.155 = 3.875$ kN/m²

(*ii*) live load (given) $= 2$ kN/m².

(*iii*) finishes (given) $= 1$ kN/m²

$w = 6.875$ kN/m²

$\therefore$ Factored load $w_u = 1.5 \times 6.875 = 10.32$ kN/m²

Design moments: As the slab corners are to be designed as torsionally restrained the moment coefficient given in Table may be applied for $\dfrac{l_y}{l_x} = 1.32$.

$\therefore$ Short span

$$\alpha_x = 0.079 + (0.085 - 0.079) \times \frac{1.32 - 1.3}{1.4 - 1.3} = 0.0802$$

$\therefore$ $$M_x = \alpha_x \cdot w_u \cdot l_x^2$$
$$= 0.0802 \times 10.32 \times 4.631^2 = 17.75 \text{ kN m/m.}$$

Long span

$$\alpha_y = 0.056$$
$$M_y = \alpha_y \, w_u \cdot l_x^2$$
$$= 0.056 \times 10.32 \times 4.631^2 = 12.40 \text{ kN m/m}$$

Check depth from B.M. consideration

$$\text{B.M.} = 0.138 f_{CK} \, bd^2$$

$\therefore$ $$d = \sqrt{\frac{17.75 \times 10^6}{0.138 \times 20 \times 1000}} = 80.2 \text{ mm} < 131 \text{ mm}$$

Design of Reinforcement:

Area of steel $(A_{St})_x$ along short span

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 17.75 \times 10^6}{20 \times 1000 \times 131^2}}\right] \times 1000 \times 131$$

$$= 401 \text{ mm}^2/\text{m}$$

required spacing of 8 ϕ bars $$= \frac{1000 \times 50}{401} = 124.7 \text{ mm}$$

Similarly, $$(A_{St})_y = \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 12.40 \times 10^6}{20 \times 1000 \times 123^2}}\right] \times 1000 \times 123$$

$$= 294 \text{ mm}^2/\text{m}$$

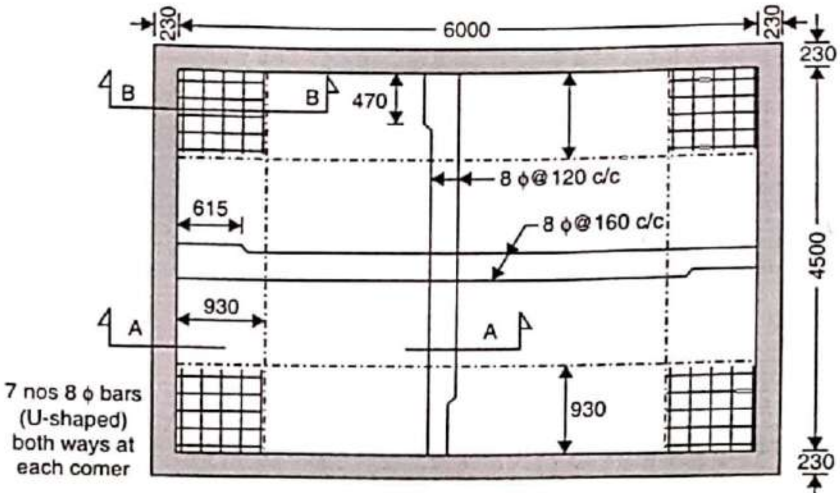
required spacing of 8 ϕ bars $$= \frac{1000 \times 50}{294} = 170 \text{ mm}$$

Maximum spacing permitted = $3d$ or 300 mm

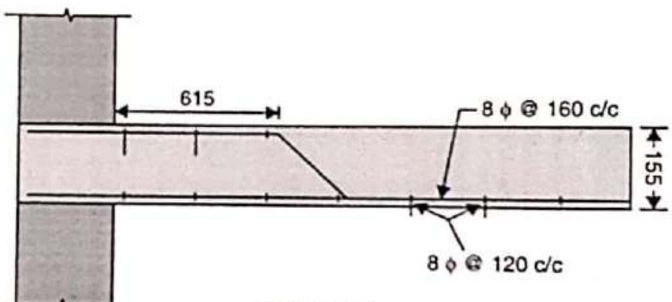
$$= \begin{cases} 3 \times 131 = 391 \text{ mm but} < 300 \text{ mm} \\ 3 \times 123 = 369 \text{ mm but} < 300 \text{ mm} \end{cases}$$

$\therefore$ Provide $$\begin{cases} 8\,\phi @ 120 \text{ mm c/c (short span)} \Rightarrow (A_{St})_x = 417 \text{ mm}^2/\text{m} \\ 8\,\phi @ 160 \text{ mm c/c (long span)} \Rightarrow (A_{St})_y = 313 \text{ mm}^2/\text{m} \end{cases}$$

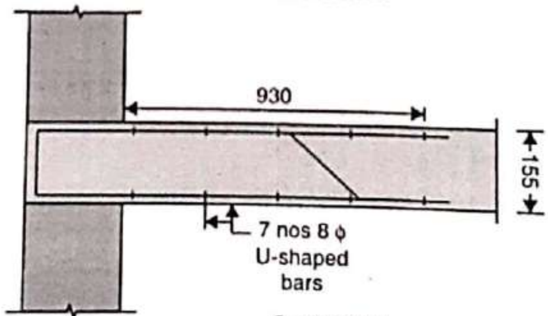
The detailing of reinforcement is shown in Fig.



Plan of roof



Section 'AA'



Section 'BB'

Check for deflection control

percentage of steel along short span

$$(p_t)_x = \frac{417 \times 100}{1000 \times 131} = 0.32$$

$\therefore$ $$f_s = 0.58 \times 415 \times \frac{401}{417} = 231.5 \text{ N/mm}^2$$

modification factor k_f $= 1.8$ (From Fig. 4 of code)

$$(l/d)_{max} = 20 \times 1.8 = 36$$

$$(l/d) \text{ provided} = \frac{4631}{131} = 35.4 < 36.$$

Corner Reinforcement: As the slab is designed as , torsionally restrained" at the corners. Corner reinforcement has to be provided (Cl. D–1.8 of the code) over a distance $\frac{l_x}{5} = 930$ mm in both directions in meshes at top and bottom (four layers) each layer comprising $0.75\,(A_{St})_x$

$$= 0.75 \times 417 = 313 \text{ mm}^2$$

$\therefore$ spacing of 8 ϕ bars $= \frac{120}{0.75} = 160$ c/c

Provide 8 ϕ @ 160 mm c/c bothways at top and bottom at each corner over an area 930 mm × 930 mm *i.e.*, 7 bars U-shaped in two directions.

Design of Compression Member

Question - 11 : *Calculate the area of steel required for a short R.C. column 400 mm × 450 mm in cross-section to carry an axial load of 1200 kN. Assume concrete grade M 20 and Fe 415 steel grade.*

Solution:

Given: $f_{ck} = 20\ \text{N/mm}^2, f_y = 415\ \text{N/mm}^2, b = 400\ \text{mm}$

$D = 450\ \text{mm}, P = 1200\ \text{kN}$

Find out: Area of steel.

Now, gross area of concrete section,

$$A_g = 400 \times 450 = 18 \times 10^4 \text{ mm}^2$$

Ultimate bad, $P_u = 1.5 \times 1200 = 1800$ kN

$$P_u = 0.4 f_{Ck} \cdot A_g + (0.67 f_y - 0.4 \times 20) A_{SC}$$

$$1800 \times 10^3 = 0.4 \times 20 \times 18 \times 10^4 + (0.67 \times 415 - 0.4 \times 20) A_{SC}$$

$$1800000 - 1440000 = 270.05 A_{SC}$$

$$\therefore \quad A_{SC} = 1333 \text{ mm}^2$$

Provide 6 Nos. $(4 - 20\,\phi + 2 - 12\,\phi)$ diameter bars in two rows

$$A_{SC} = 1482 \text{ mm}^2$$

Lateral ties:

$$\text{Tie diameter } \phi_t > \begin{cases} \dfrac{20}{4} \\ 6 \text{ mm} \end{cases} \quad \therefore \quad \text{Provide 6 mm ties.}$$

$$\text{Tie spacing } S_t < \begin{cases} 400 \text{ mm} \\ 16 \times 12 = 192 \text{ mm} \\ 300 \text{ mm} \end{cases} \quad \therefore \quad \text{Provide 192 mm.}$$

$\therefore$ Provide 6 mm diameter @ 192 mm c/c.

Question - 2 : *Design the reinforcement in a column of a 450 mm × 600 mm, subject to an axial load of 200 kN under service dead load and live loads. The column has an unsupported length of 3.0 m and is restrained in both directions. Use M 20 concrete and Fe 415 steel.*

Solution:

Given: $f_{Ck} = 20 \text{ N/mm}^2, f_y = 415 \text{ N/mm}^2$

$b = 450$ mm, $D = 600$ mm, $P = 2000$ kN

$l = 3.0$ m

and both ends are restrained.

Now, short column or long column ?

The effective length of column $= 0.65 \times l = 1.95$ m.

$$\text{Slenderness ratio} = \frac{l_{eff}}{\text{Least lateral dimension}} = \frac{1950}{450} = 4.33 < 12$$

Hence, the column may be designed as a short column.

To check minimum eccentricity:

We know,

$$e_{x,\,min} = \frac{3000}{500} + \frac{600}{30} = 26.0 \text{ mm } (> 20.0 \text{ mm})$$

$$e_{y,\,min} = \frac{3000}{500} + \frac{450}{30} = 21.0 \text{ mm } (> 20.0 \text{ mm})$$

but $\quad 0.05 \times D = 0.05 \times 600 = 30.0 \text{ mm} > e_{x, \min} = 26.0 \text{ mm}$

and $\quad 0.05 \times b = 0.05 \times 450 = 22.5 \text{ mm} > e_{y, \min} = 21.0 \text{ mm}.$

Design of longitudinal reinforcement:

We know, $\quad P_u = 0.4 f_{Ck} A_g + (0.67 f_y - 0.4 f_{Ck}) A_{SC}$

$\Rightarrow \quad 1.5 \times 2000 \times 10^3 = 0.4 \times 20 \times (450 \times 600) + (0.67 \times 415 - 0.4 \times 20) A_{SC}$

which gives

$$A_{SC} = 3111 \text{ mm}^2$$

Provide 4–25 ϕ at corner $4 \times 491 = 1964 \text{ mm}^2$

and 4–20 ϕ additional $4 \times 314 = 1256 \text{ mm}^2$

$$A_{SC} = 3220 \text{ mm}^2 > 3111 \text{ mm}^2$$

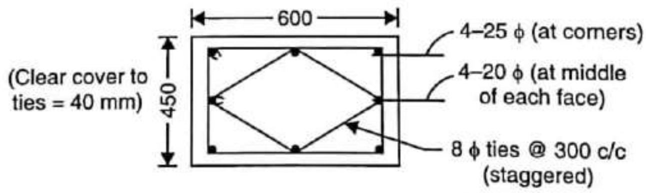
Check: Percentage of steel $= \dfrac{3220 \times 100}{450 \times 600} = 1.192\% > 0.8\%$ (minimum reinforcement)

Tie diameter, $\quad \phi_t > \begin{cases} 25/4 \\ 6 \text{ mm} \end{cases} \quad \therefore$ Provide 8 mm diameter

Tie spacing, $\quad S_t < \begin{cases} 450 \text{ mm} \\ 16 \times 20 = 320 \text{ mm} \\ 300 \text{ mm} \end{cases} \quad \therefore$ Provide 300 mm.

Therefore, provide 8 ϕ ties @ 300 mm c/c.

Detailing of reinforcement: The detailing of reinforcement is shown in Fig.



FIGURE

Question - 13 : *Design the reinforcement in a spiral column of 450 mm diameter subjected to service load of 1200 kN. The column has an unsupported length of 3.4 m. Use M 25 concrete and Fe 415 steel.*

Assume effective length to be equal to unsupported length.

Solution:

Given: $\quad l = 3.4 \text{ m}$

It is clear from equation, $l = l_{\text{eff}} = 3400 \text{ mm}$

$$\text{Slenderness ratio} = \frac{l_{\text{eff}}}{\text{Least dimension}} = \frac{3400}{450} = 7.5 < 12$$

The column may be designed as a short column.

Minimum eccentricity:

$$e_{min} = \frac{3400}{500} + \frac{450}{30} = 21.8 \text{ mm } (> 20.0 \text{ mm})$$

and

$$0.05\,D = 0.05 \times 450 = 22.5 \text{ mm} > 21.8 \text{ mm}$$

So the code formula for axially compressed short columns may be used.

Design of longitudinal reinforcement:

For spiral columns, 5% increase in strength is allowed.

Therefore, $P_u = 1.05[0.4 f_{ck} A_g + (0.67 f_y - 0.4 f_{ck}) A_{SC}]$

$$1.5 \times 1200 \times 10^3 = 1.05\left[0.4 \times 25 \times \frac{\pi}{4} \times (450)^2 + (0.67 \times 415 - 0.4 \times 25)\, A_{SC}\right]$$

$$1714.3 \times 10^3 = 1589.6 \times 10^3 + 268.05\, A_{SC}$$

$\therefore$ $A_{SC} = 465.2 \text{ mm}^2$

Check:

$$\text{Percentage of steel of gross area} = \frac{465.2 \times 100}{\frac{\pi}{2} \times (450)^2} = 0.30\%.$$

But minimum percentage of steel = 0.8% of gross area

3–20 ϕ : $3 \times 314 = 942 \text{ mm}^2$

3–12 ϕ : $3 \times 113 = 339 \text{ mm}^2$

$A_{SC} = 1381 \text{ mm}^2 > 1272 \text{ mm}^2$

Design of spiral reinforcement: Assuming a clear cover of 40 mm over spirals,

Core diameter = $450 - (2 \times 40) = 370$ mm

Assuming a bar diameter of 6 mm and pitch S_t,

$$\frac{\text{Volume of helical / spiral reinforcement}}{\text{Volume of core}} = \frac{\frac{\pi}{4} \times (6)^2 \times \pi \times (370 - 6)/S_t}{\frac{\pi}{4} \times (370)^2}$$

$$= \frac{0.3005}{S_t}$$

But as per code,

$$\frac{\text{Volume of spiral reinforcement}}{\text{Volume of core}} \geq 0.36\left(\frac{A_g}{A_{core}} - 1\right)\frac{f_{ck}}{f_y}$$

or,

$$\frac{0.3005}{S_l} \geq 0.36\left(\frac{\frac{\pi}{4}\times(450)^2}{\frac{\pi}{4}\times(370)^2}-1\right)\times\frac{25}{415}$$

$$\Rightarrow \quad S_l \leq 28.9 \text{ mm}$$

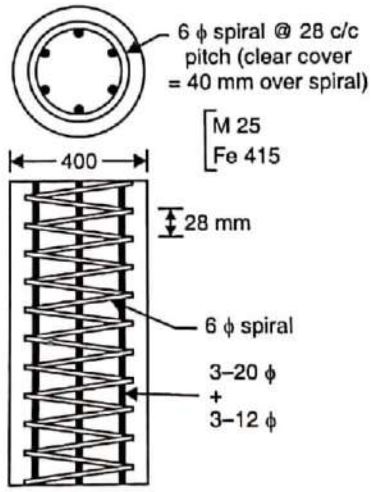
Code restriction on pitch,

$$S_l < \begin{cases} 75 \text{ mm} \\ \dfrac{\text{core diameter}}{6} = \dfrac{370}{6} = 61.67 \text{ mm} \end{cases}$$

$$S_l > \begin{cases} 25 \text{ mm} \\ 3\,\phi_l = 3\times 6 = 18 \text{ mm} \end{cases}$$

Provide 6 ϕ spiral @ 28 mm c/c pitch.

Detailed of reinforcement: The.detailing of reinforcement is shown in Fig.



FIGURE

Design of Isolated Footing

Question - 7 : *Design a footing for an axially loaded square column of 450 mm side, transmitting a load of P_u = 1000 kN and safe bearing capacity of soil is 300 kN/m². Use M 20 grade of concrete and Fe 415. Draw sectional elevation and plan showing reinforcement details.*

Solution. $P_u = 1000$ kN, S.B.C. = 300 kN/m², size of column = 450 × 450, M 20 and Fe 415.

Design constants: M 20, Fe 415, for Fe 415 – $k_{u\,max} = 0.48$

$$R_u = 0.36 f_{ck}\, k_{u\,max}\,(1 - 0.42\, k_{u\,max})$$
$$= 0.36 \times 20 \times 0.48\,(1 - 0.42 \times 0.48)$$
$$= 2.76\ \text{N/mm}^2$$

Step 1: Load on column = 1000 kN

$$\begin{bmatrix}\text{Self weight of footing} \\ \text{(about 10\%)}\end{bmatrix} = \underline{100\ \text{kN}}$$

Total load = 1100 kN

Step 2: Area of footing required:

$$\text{Area of footing required} = \frac{\text{Load}}{\text{S.B.C.}} = \frac{1100}{300}$$
$$= 3.67\ \text{m}^2$$

Step 3: Size of footing

As column size is equal, so for equal cantilever projection, square footing is provided.

∴ Side of square footing = 1.915 m

So provide footing as 2 m × 2 m.

∴ Area provided = 4 m^2

∴ Cantilever projection (along x and y direction)

$$= \left(\frac{2000 - 450}{2}\right) = 775 \text{ mm}$$

$$\text{Net upward soil pressure intensity} = \frac{\text{Load}}{\text{Area provided}}$$

$$= \frac{1000}{4} = 250 \text{ kN/m}^2$$

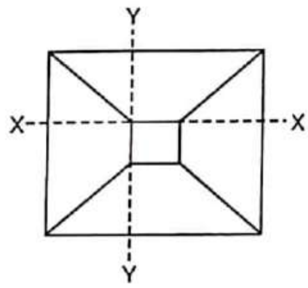
Step 4: Depth of footing from B.M. consideration.

Bending moment at X-X

$$M_{xx} = p \times B_f \times \frac{C^2}{2}$$

$$= 250 \times 2 \times \frac{0.775^2}{2}$$

$$= 150 \text{ kN-m}$$



FIGURE

Width at top = width of column + offset

$$B = 450 + 2 \times 50 = 550 \text{ mm}$$

$$\therefore \quad M = R_u . Bd^2$$

$$150 \times 10^6 = 2.76 \times 550 \times d^2$$

$$\therefore \quad d = 314.66 \text{ mm}$$

Considering 60 mm as clear cover and 8 mm diameter bars:

$$\text{Overall depth} = 314.66 + 60 + \frac{8}{2} = 438 \text{ mm}$$

∴ Provide overall depth = 500 mm

$$d_x = \text{overall depth} - \text{effective cover}$$

$$= (500 - 60 - 8/2) = 436 \text{ mm}$$

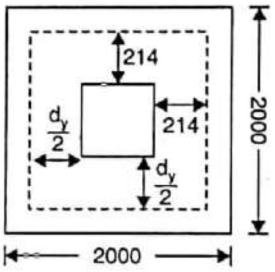
$$\therefore \quad d_y = (d_x - \phi) = 436 - 8 = 428 \text{ mm}$$

∴ Dimension of footing = 2 m × 2 m × 0.5 m

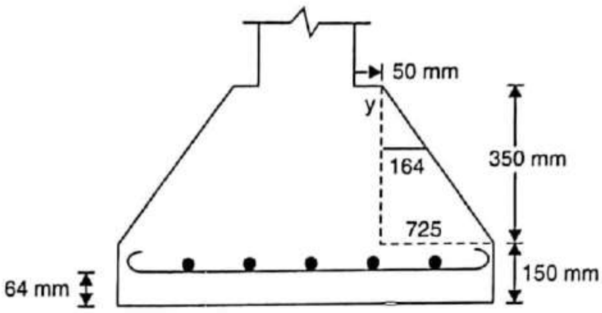
Step 5: Check depth of footing for two-way shear: Critical section of two-way shear is at $\frac{d_y}{2}$ from the face of the column.

Perimeter at critical section = 2 [878 + 878] = 3512 mm

Sheared area = 878 × 878 = 770884 mm^2



FIGURE



FIGURE

Effective depth at peripheral section:

$$\frac{y}{164} = \frac{350}{725}$$

$\therefore$ $$y = 79.17 \text{ mm}$$

$\therefore$ $$\text{Effective depth} = 500 - 64 - 8 - 79.17 = 348.83 \text{ mm}$$

$$\begin{bmatrix}\text{Area resisting at peripheral} \\ \text{section}\end{bmatrix} = \text{Perimeter} \times \text{Effective depth}$$

$$= 3512 \times 348.83 = 1225090.96 \text{ mm}^2$$

$\therefore$ $$\text{Design shear force} = \text{Pressure intensity} \times [\text{Total area} - \text{sheared area}]$$

$$= 250 \times [4 \times 10^6 - 770884] \times 10^{-3}$$

$$= 807.28 \times 10^3 \text{ N} = 807.28 \text{ kN}$$

$$\text{Shear resisted by concrete} = k_s\, \tau_{uC}$$

$$\tau'_{uC} = 0.25\sqrt{f_{ck}} = 1.118 \text{ N/mm}^2$$

$$k_s = 0.5 + \left(\frac{b}{D}\right) = 1.5 > 1 \quad \therefore k_s = 1.0$$

$\therefore$ $$\tau_{uC} = 1.0 \times 1.118 = 1.118 \text{ N/mm}^2$$

$$\text{Shear resisted by concrete} = \tau_{uC} \times \text{Area resisted at peripheral section}$$

$$= 1.118 \times 1225090.96$$

$$V_{uC} = 1369.65 \times 10^3 \text{ N} = 1369.65 \text{ kN}$$

$\therefore$ $$V_{uC} > V_{uD}$$

$\therefore$ Footing is safe for two-way shear.

Step 6: Steel reinforcement:

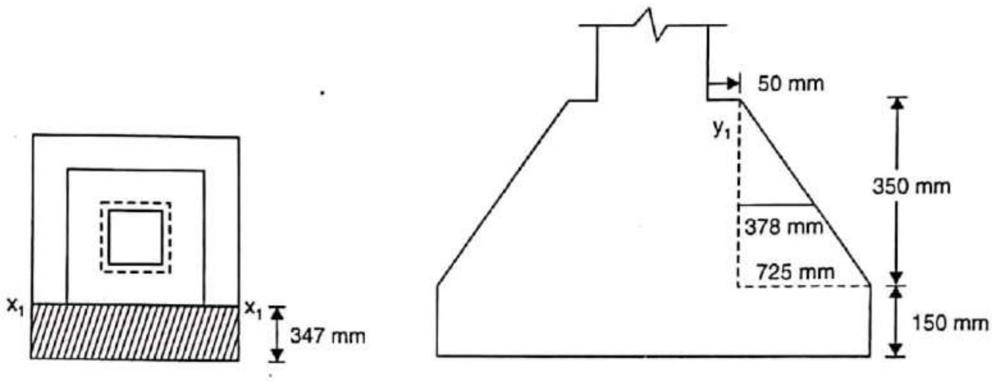
$$A_{st} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6\, M_u}{f_{ck}\, Bd^2}}\right) bd$$

$$= \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 160 \times 10^6}{20 \times 550 \times (436)^2}}\right) 550 \times 436$$

$$= 1126.77 \text{ mm}^2$$

Provide 24 Nos., # 8 bars along both directions.

$\therefore$ Area provided = 1206.37 mm².



(a) (b)

FIGURE

Step 7: Check for one-way shear: Critical section for one-way shear is at d_y from face of column.

Width of footing at $x_1 - x_1$

$$= 550 + \frac{(2000 - 550)}{(775 - 50)} \times (428 - 50)$$

$$= 1306 \text{ mm}$$

Depth of footing at critical section:

$$\frac{y_1}{378} = \frac{350}{725}$$

$$\therefore \quad y_1 = 182.48 \text{ mm}$$

Effective depth of critical section = $d_y - y_1$

$$= 428 - 182.48 = 245.52 \text{ mm}$$

At critical section, we have trapezoidal area upto $D_{f\,\min}$ and rectangular section below it.

$$\therefore \quad \text{Area at critical section} = (245.52 - 78) \times \left(\frac{1306 + 2000}{2}\right) + 2000 \times 78$$

$$= 432910.56 \text{ mm}^2$$

Shear force at critical section = Soil pressure intensity × Area shown by lines

$$V_u = 250 \times 2 \times 0.347 = 173.5 \text{ kN}$$

$$\therefore \text{ Nominal shear stress} = \frac{\text{Shear force}}{\text{Area at critical section}}$$

$$= \frac{173.5 \times 10^3}{432910.56} = 0.4 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{100 A_{st}}{\text{Width of critical section} \times \text{Depth of critical section}}$$

$$= \frac{100 \times 1206.37}{1306 \times 245.52}$$

$$= 0.38$$

$\therefore$ Shear resisted by concrete = 0.42 N/mm² > 0.4 N/mm².

$\therefore$ Section is safe against one-way shear.

Step 8: Check for bearing pressure:

$$\text{Actual bearing pressure} = \frac{P_u}{b \times D} = \frac{1000 \times 10^3}{450 \times 450} = 4.94 \text{ N/mm}^2$$

$$A_1 = L_f \times B_f \text{ or } (b + 4D_f) \times (D + 4D_f)$$

$$= 2000 \times 2000 \text{ or } (450 + 4 \times 500) \times (450 + 4 \times 500)$$

$$= 4 \times 10^6 \text{ or } 6002500 \text{ whichever is less.}$$

$$\therefore \quad A_1 = 4 \times 10^6 \text{ mm}^2$$

$$A_2 = b \times D = 450 \times 450 = 202500 \text{ mm}^2$$

$$\therefore \quad \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{4 \times 10^6}{202500}} = 4.44 > 2 \quad \therefore \quad \sqrt{\frac{A_1}{A_2}} = 2$$

$$\text{Permissible bearing stress} = 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0.45 \times 20 \times 2$$

$$= 18.00 \text{ N/mm}^2 > 4.94 \text{ N/mm}^2$$

$\therefore$ Safe.

Step 9: Check for development length:

$$L_d = \frac{0.87 f_y}{4 \tau_{bd}} \phi$$

$$= \frac{0.87 \times 415}{4 \times 1.9} \times 8 = 380.05 \text{ mm}$$

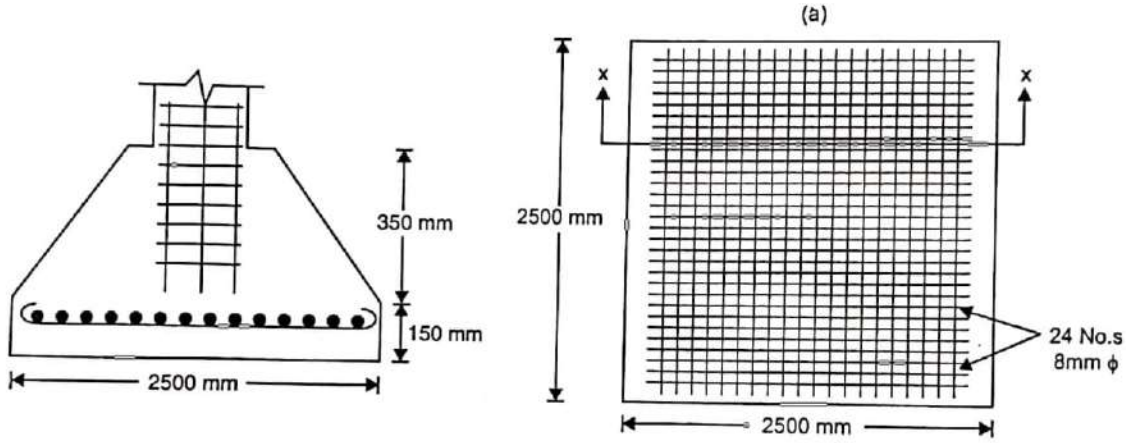
Provide 60 mm side cover.

$\therefore$ Length of bar available $= \frac{1}{2}(B-b) - 60$

$$= \frac{1}{2}(2000-450) - 60$$

$$= 715 \text{ mm} > L_d$$

Hence safe.



(*a*) Section *x-x*

(*b*) Plan showing reinforcement

FIGURE

Question - 8 : *A column carries axial load P_u = 1200 kN. Design an isolated rectangular footing for the column. Safe bearing capacity of soil is 250 kN/m². The column size is 300 mm × 500 mm. Use M 20 grade of concrete and Fe 415 steel. Draw sectional elevation and plan showing reinforcement details.*

Solution. P_u = 1200 kN, S.B.C. = 250 kN/m², size of column = 300 mm × 500 mm, M 20, Fe 415.

Design constants: M 20, Fe 415, For Fe 415 – $k_{u\,max} = 0.48$

$$R_u = 0.36 f_{ck}\, k_{u\,max}(1 - 0.42\, k_{u\,max})$$

$$= 0.36 \times 20 \times 0.48\,(1 - 0.42 \times 0.48) = 2.76 \text{ N/mm}^2$$

Step 1: Load on column = 1200 kN

Self weight of footing (@ 10%) = 120 kN

Total load = 1320 kN

Step 2: Area required,

$$A = \frac{\text{Total load}}{\text{S.B.C.}}$$

$$A = \frac{1320}{250} = 5.28 \text{ m}^2$$

Step 3: Size of footing: Considering equal cantilever projection on both sides,

$$\therefore \quad L_f = \left(\frac{D-b}{2}\right) + \sqrt{\frac{(D-b)^2}{4} + A_f}$$

$$= \left(\frac{500-300}{2}\right) + \sqrt{\frac{(500-300)^2}{4} + 5.28 \times 10^6}$$

$$= 2400 \text{ mm} \therefore \text{ Provide, } L_f = 2500 \text{ mm}$$

$$\therefore \text{Cantilever projection} = \left(\frac{L_f - D}{2}\right) = \left(\frac{2500-500}{2}\right)$$

$$\therefore \quad C_{xx} = 1000 \text{ mm}$$

$$\therefore \quad B_f = b + 2\,C_{xx} = 300 + (2 \times 1000) = 2300 \text{ mm}$$

$\therefore$ Size of footing = 2.5 m × 2.3 m

$\therefore$ Area provided = 2.5 × 2.3

$= 5.75 \text{ m}^2 > 5.68 \text{ m}^2 \quad \therefore$ Safe

Net upward soil pressure intensity

$$= \frac{\text{Load}}{\text{Area provided}} = \frac{1200}{5.75} = 208.7 \text{ kN/m}^2.$$

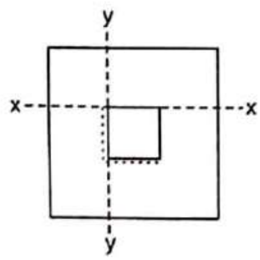
Step 4: Depth of footing from B.M.: Bending moment along x-x axis,

$$\text{B.M.}_{xx} = p \times B_f \times \frac{C_{xx}^2}{2} = 208.7 \times 2.3 \times \frac{(1)^2}{2}$$

$$= 240 \text{ kN-m}$$

$$B = (b + 2e) = 300 + 2 \times 75$$

$$= 450 \text{ mm}$$



FIGURE

$$\therefore \quad d_x = \sqrt{\frac{\text{B.M.}_{xx}}{R_u . B}}$$

$$= \sqrt{\frac{240 \times 10^6}{2.76 \times 450}} = 439.59 \text{ mm}$$

Bending moment along y-y axis,

$$\text{B.M.}_{yy} = p \times L_f \times \frac{C_{yy}^2}{2}$$

$$= 208.7 \times 2.5 \times \frac{(1)^2}{2}$$

$$= 260.86 \text{ kN-m}$$

$$D_1 = D + 2e = 500 + 2 \times 75 = 650 \text{ mm}$$

$$\therefore \qquad d_y = \sqrt{\frac{260.86 \times 10^6}{2.76 \times 650}} = 381.32 \text{ mm}$$

So provide overall depth 550 mm.

Assume effective cover in x direction = 60 mm.

Bar diameter = 10 mm

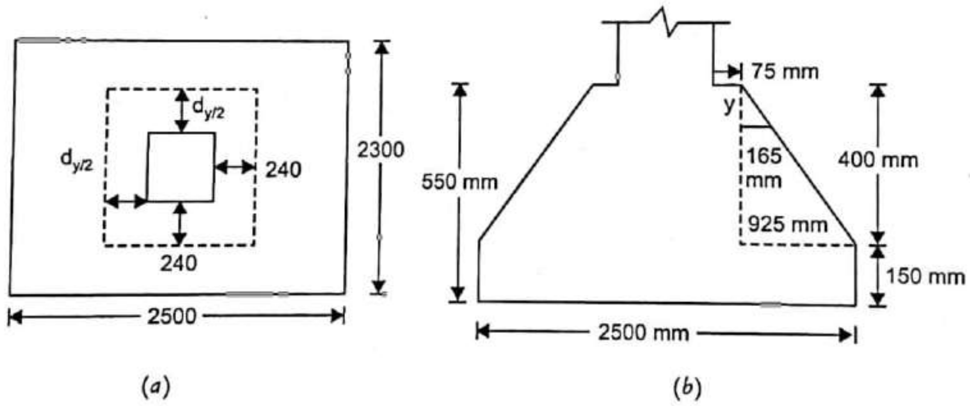
$$\therefore \qquad d_x = \text{Overall depth} - \text{Effective cover}$$
$$= 500 - 60 = 490 \text{ mm}$$
$$\therefore \qquad d_y = (d_x - \phi) = 490 - 10 = 480 \text{ mm}$$

Step 5: Check depth of footing for two-way shear: Critical section for two-way shear is at $(d_y/2)$ from faces of column.

$\therefore$ Perimeter of critical section

$$= 2\,[300 + (2 \times 240) + 500 + (2 \times 240)]$$
$$= 3520 \text{ mm}$$

$$\therefore \qquad \text{Sheared area} = [300 + (2 \times 240)] \times [500 + (2 \times 240)]$$
$$= 780 \times 980 = 764400 \text{ mm}^2$$



FIGURE

Effective depth at peripheral section,

$$\frac{y}{165} = \frac{400}{925}$$
$$\therefore \qquad y = 71.35 \text{ m.}$$

$\therefore$ Effective depth at critical section

$$= d_y - y = 480 - 71.35$$
$$= 408.65 \text{ mm}$$

$\therefore$ Area resisting at peripheral section

$$= \text{Perimeter} \times \text{Effective depth}$$
$$= 3520 \times 408.65 = 1438448 \text{ mm}^2$$

Shear resisted by concrete:

$$\tau'_{uC} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20}$$

$$= 1.118 \text{ N/mm}^2$$

$$k_s = 0.5 + \left(\frac{b}{D}\right) = 0.5 + \frac{300}{500}$$

$$= 1.1 > 1 \text{ (maximum value)}$$

$\therefore$ $$k_s = 1.0$$

$\therefore$ $$\tau_{uC} = k_s\,\tau_{uC} = 1 \times 1.118 = 1.118 \text{ N/mm}^2$$

$\therefore$ Shear resisted by concrete = τ_{uC} × Area

$$V_{uC} = 1.118 \times 1438448$$

$$V_{uC} = 1608.18 \times 10^3 \text{ N} = 1608.18 \text{ kN}$$

Design shear force = Pressure intensity × Area

$$= 208.7\,(5.75 \times 10^6 - 764400) \times 10^{-3}$$

$$= 1040.5 \times 10^3 \text{ N}$$

$$= 1040.5 \text{ kN}$$

As $V_{uC} > V_{uD}$ $\therefore$ Section is safe for two-way shear.

Step 6: Steel reinforcement:

$$A_{st\,x} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 \times \text{B.M.}_{xx}}{f_{ck}\, B\, d_x^2}}\right] B d_x$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 240 \times 10^6}{20 \times 450 \times (490)^2}}\right] \times 450 \times 490$$

$$= 1597.39 \text{ mm}^2$$

$\therefore$ Number of bars provided: 22 nos, # 10 mm.

$\therefore$ $$A_{st\,x\,\text{provided}} = 1727.89 \text{ mm}^2$$

$$A_{st\,y} = \frac{0.5 f_{ck}}{f_y}\left[1 - \sqrt{1 - \frac{4.6 \times \text{B.M.}_{yy}}{f_{ck} \times D_1 \times d_y^2}}\right] D_1 \times d_y$$

$$= \frac{0.5 \times 20}{415}\left[1 - \sqrt{1 - \frac{4.6 \times 260.86 \times 10^6}{20 \times 650 \times (480)^2}}\right] \times 650 \times 480$$

$$= 1697.39 \text{ mm}^2$$

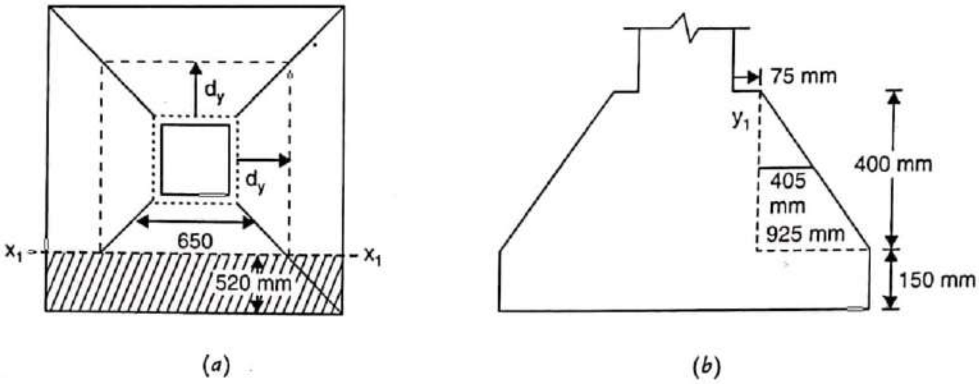
Provide 22 nos, # 10 mm.

Step 7: Check for one-way shear about *y-y* axis: Critical section for one-way shear is at d_y from face of column.

Width of footing at section x_1-x_1

$$= 650 + \frac{(2500 - 650)}{(1000 - 75)} \times (480 - 75)$$

$$= 1460 \text{ mm}$$



FIGURE

Depth of footing at critical section:

$$\frac{y_1}{405} = \frac{400}{925}$$

$\therefore$ $$y_1 = 175.14 \text{ mm}$$

$\therefore$ Effective depth at critical section

$$= d_y - y_1 = 480 - 175.14$$

$$= 304.86 \text{ mm}$$

At critical section we have trapezoidal section and rectangular section.

$\therefore$ Area at critical section

$$= (304.86 - 80) \times \left(\frac{1460 + 2500}{2}\right) + 80 \times 2500$$

$$= 645222.8 \text{ mm}^2$$

Shear force at critical section

= Soil pressure intensity × Area shown by lines

$$= 208.7 \times 2.5 \times 0.52$$

$$= 271.31 \text{ kN}$$

$\therefore$ Nominal shear stress

$$= \frac{V_u}{\text{Area at critical section}} = \frac{271.31 \times 10^3}{645222.8} = 0.42 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{100\, A_{st}}{\text{Width of critical section} \times \text{Depth, of critical section}}$$

$$= \frac{100 \times 1727.89}{1460 \times 304.86} = 0.39$$

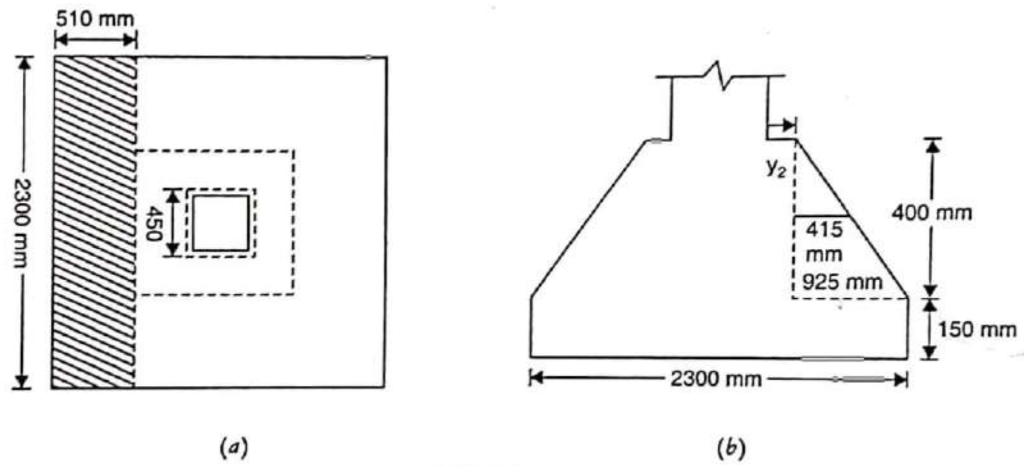
$\therefore$ Shear stress resisted by concrete = 0.43 N/mm^2 > 0.42 N.

$\therefore$ Safe against one-way shear.

Step 8: Check for one-way shear about x-x: Critical section is at d_x from face of column.

Width of footing at critical section

$$= 300 + \frac{(2300-450)}{(1000-75)} \times (490-75) = 1130 \text{ mm}$$



FIGURE

Depth of footing at critical section

$$\frac{y_2}{415} = \frac{400}{925}$$

$\therefore$ $y_2 = 179.46$ mm

$\therefore$ Effective depth of critical section

$$= 490 - 179.46 = 310.54 \text{ mm}$$

$\therefore$ Area at critical section

$$= \left(\frac{2300+1130}{2}\right)(310.54-90) + 2300 \times 90$$

$$= 585226.1 \text{ mm}^2$$

Shear force at critical section

= Soil pressure intensity × Area shown by lines

$$= 208.7 \times 2.3 \times 0.51$$

$$= 244.81 \text{ kN}$$

$\therefore$ Nominal shear stress $= \dfrac{\text{S.F.}}{\text{Area}} = \dfrac{244.81 \times 10^3}{585226.1} = 0.42 \text{ N/mm}^2$.

$$\% \text{ of steel} = \frac{100\, A_{st}}{\text{Width of critical section} \times \text{Depth of critical section}}$$

$$= \frac{100 \times 1727.89}{1130 \times 310.54} = 0.49$$

$\therefore$ Shear resisted by concrete $\approx 0.46 \text{ N/mm}^2 > 0.42 \text{ N/mm}^2$.

$\therefore$ Section is safe against one-way shear.

Step 9: Check for bearing pressure:

$$\text{Actual bearing stress} = \frac{P_u}{b \times D} = \frac{1200 \times 10^3}{300 \times 500} = 8 \text{ N/mm}^2$$

$$A_1 = (L_f \times B_f) \text{ or } (b + 4D_f) \times (D + 4D_f)$$

$$= (2500 \times 2300) \text{ or } (300 + 4 \times 500) \times (500 + 4 \times 500)$$

$$= 5750000 \text{ or } 6750000 \text{ whichever is less.}$$

$$A_2 = b \times D = 300 \times 500 = 150000 \text{ mm}^2$$

$$\therefore \quad \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{5750000}{150000}} = 6.19 > 2 \quad \therefore \sqrt{\frac{A_1}{A_2}} = 2$$

$\therefore$ Permissible bearing stress

$$= 0.45 f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0.45 \times 20 \sqrt{2}$$

$$= 12.72 \text{ N/mm}^2 > 8 \text{ N/mm}^2$$

$\therefore$ Safe.

Step 10: Check for development length:

$$L_d = \frac{0.87 f_y}{4\, \tau_{bd}} \phi$$

$$L_d = \frac{0.87 \times 415}{4 \times 1.92} \times 10 = 470.12 \text{ mm}$$

Providing 60 mm side cover,

Length of bar available $= \dfrac{1}{2} (B - d) - 60$

$$= \frac{1}{2}(2300 - 300) - 60$$

$$= 940 \text{ mm} > L_d$$

Hence, safe.

Question - 9 : *Design a square footing for a 400 mm × 400 mm size column, carrying a direct load of 800 kN and subjected to a moment of 70 kN-m. The safe bearing capacity of soil is 150 kN/m². Use M 20 grade concrete and Fe 415.*

Solution. $P_u = 800$ kN, S.B.C. = 150 kN/m²,

Size of column = 400 mm × 400 mm, M 20, Fe 415.

Design constants: M 20, Fe 415, For Fe 415 – $k_{u\,max} = 0.48$.

$$R_u = 0.36 f_{ck}\, k_{u\,max} (1 - 0.42 \times k_{u\,max})$$

$$= 0.36 \times 20 \times 0.48 (1 - 0.42 \times 0.48)$$

$$\approx 2.76 \text{ N/mm}^2$$

Step 1: Load from column = 800 kN

Self weight @ 10% = 80 kN

Total load = 8000 kN

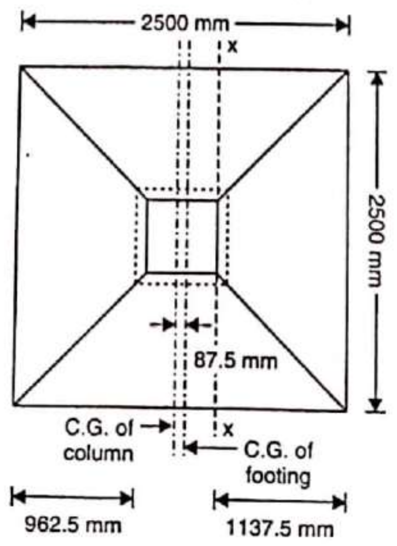
Step 2: Area required $= \frac{\text{Load}}{\text{S.B.C.}} = \frac{880}{150} = 5.87 \text{ m}^2$

Step 3: Size of footing:

Provide side of square footing = 2.5 m

∴ Eccentricity of load from the centre of footing

$$= \frac{M}{P} = \frac{70 \times 10^6}{800 \times 10^3} = 87.5 \text{ mm}$$



FIGURE

If the footing is so placed so that C.G. of column coincides with the C.G. of the footing, the soil pressure distribution is non-uniform. So place the footing eccentric by 87.5 mm from face of column so that soil pressure distribution is uniform.

$$\text{Net upward soil pressure intensity} = \frac{\text{Load}}{\text{Area provided}}$$

$$= \frac{800}{2.5 \times 2.5} = 128 \text{ kN/m}^2$$

Cantilever projection on right side of column

$$C_{xx} = \left(\frac{2500 - 400}{2}\right) + 87.5 = 1137.5 \text{ mm}$$

Step 4: Depth of footing from B.M. consideration:

$$\text{Bending moment about } x\text{-}x \text{ axis} = p \times B \times \frac{C_{xx}^2}{2}$$

$$= 128 \times 2.5 \times \frac{(1.137)^2}{2}$$

$$= 206.84 \text{ kN-m}$$

For M 20, Fe 415, $R_u = 2.76 \text{ N/mm}^2$

$\therefore$ $\text{B.M.} = R_u . b . D^2$

Considering 100 mm offset (e),

$$B = (b + 2e) = 400 + 200 = 600 \text{ mm}$$

$$\therefore \quad d = \sqrt{\frac{B.M.}{R_u \cdot b}} = \sqrt{\frac{206.84 \times 10^6}{2.76 \times 600}}$$

$$= 353.42 \text{ mm}$$

Provide overall depth of footing as 500 mm

$$\therefore \quad d_x = 500 - 55 = 445 \text{ mm}$$

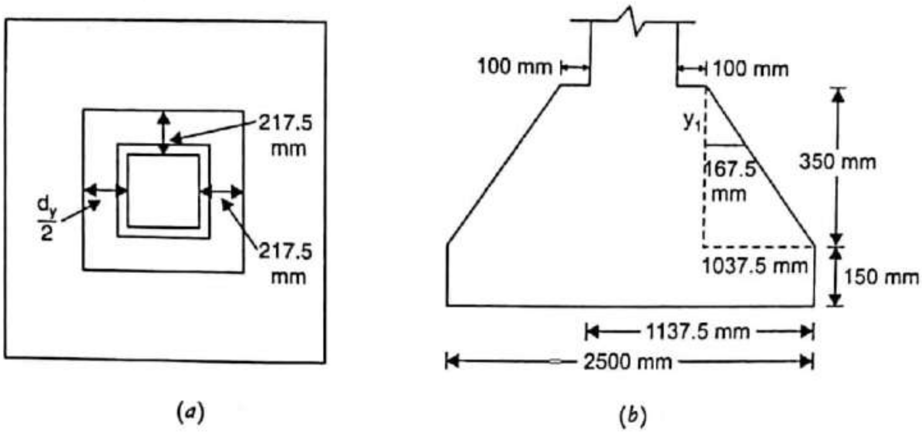
$$d_y = 445 - 10 = 435 \text{ mm}$$

Step 5: Depth of footing:

Check for two-way shear: Critical section of two-way shear is at $\left(\frac{d_y}{2}\right)$ from the face of column.

Perimeter at critical section = $(400 + 435) \times 4 = 3340$ mm

Area sheared at critical section = $(835 \times 835) = 697225 \text{ mm}^2$



FIGURE

Effective depth at critical section

$$\frac{y_1}{167.5} = \frac{350}{1037.5}$$

$\therefore \qquad y_1 = 56.5 \text{ mm}$

$\therefore$ Effective depth at critical section

$$= 425 - y_1 = 425 - 56.5$$
$$= 368.5$$

$\therefore$ Area resisting at peripheral section

$$= \text{Perimeter} \times \text{Effective depth}$$
$$= 3340 \times 368.5 = 1230790 \text{ mm}^2$$

Shear resisted by concrete

$$\tau'_{uC} = 0.25\sqrt{f_{ck}} = 0.25\sqrt{20} = 1.118 \text{ N/mm}^2$$

$$k_s = 0.5 + \left(\frac{b}{D}\right) \leq 1$$

$$= 0.5 + \left(\frac{400}{400}\right) = 1.5 > 1$$

$\therefore \qquad k_s = 1.0$

$\therefore \qquad \tau_{uC} = 1 \times 1.118 = 1.118 \text{ N/mm}^2$

Shear resisted by concrete = τ_{uC} × Area resisting

$$V_{uC} = 1.118 \times 1230790 = 1376.02 \times 10^3 \text{ N}$$
$$V_{uC} = 1376.02 \text{ kN}$$

Design shear force = Pressure intensity × Area

$$V_{uD} = 128 \times [(2500)^2 - 697225] \times 10^{-3}$$

$$V_{uD} = 710.76 \times 10^3 \text{ N} = 710.76 \text{ kN}$$

$\therefore$ $V_{uC} > V_{uD}$ $\therefore$ Section is safe against two-way shear.

Step 6: Steel reinforcement:

$$A_{st\,x} = \frac{0.5 f_{ck}}{f_y}\left(1 - \sqrt{1 - \frac{4.6 M_u}{f_{ck}\, B\, d_x^2}}\right) B\, d_x$$

$$= \frac{0.5 \times 20}{415}\left(1 - \sqrt{1 - \frac{4.6 \times 206.84 \times 10^6}{20 \times 600 \times (445)^2}}\right) \times 600 \times 445$$

$$= 1451.84 \text{ mm}^2$$

Provide 20 nos., # 10 mm.

$\therefore$ $A_{st\ provided} = 1570.8 \text{ mm}^2$

Step 7: Critical section for one-way shear about *x-x* axis:

Width of footing at $x_1 - x_1$

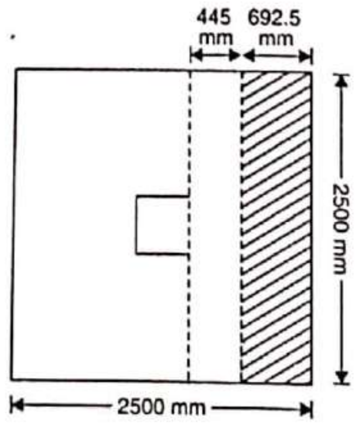
$$= 600 \times \frac{(2500 - 600)}{1037.5} \times (445 - 100)$$

$$= 1231.81 \text{ mm}$$

Depth of footing at critical section

$$\frac{y_1}{335} = \frac{350}{1037.5}$$

$\therefore$ $y_1 = 113.01 \text{ mm}$



FIGURE

$\therefore$ Effective depth at critical section

$$= d_y - y_1 = 435 - 113.01 = 321.99 \text{ mm}$$

Area at critical section

$$= (321.99 - 85)\left(\frac{600 + 2500}{2}\right) + 2500 \times 85$$

$$= 579834.5 \text{ mm}^2$$

Shea force at critical section

= Soil pressure intensity × Area shown by lines

$$= 128 \times 2.5 \times (0.6925) = 221.6 \text{ kN}$$

$$\text{Nominal shear stress} = \frac{\text{Shear force}}{\text{Area}}$$

$$= \frac{221.6 \times 10^3}{579834.5} = 0.38 \text{ N/mm}^2$$

$$\% \text{ of steel} = \frac{100\, A_{st}}{\text{Area}}$$

$$= \frac{100 \times 1570.8}{1231.81 \times 321.99} = 0.4$$

∴ Shear resisted by concrete = $0.43 \text{ N/mm}^2 > 0.4 \text{ N/mm}^2$

∴ Section is safe against one-way shear.

Step 8: Check for bearing pressure:

$$\text{Actual bearing pressure} = \frac{P}{b \times D} = \frac{800 \times 10^3}{400 \times 400}$$

$$= 5 \text{ N/mm}^2$$

$$A_1 = (\text{Side})^2 \text{ or } (b + 4\,D_f) \times (D + 4\,D_f)$$

$$= (2500)^2 \text{ or } (400 + 4 \times 500) \times (400 + 4 \times 500)$$

$$= 625 \times 10^4 \text{ or } 576 \times 10^4 \text{ mm}^2 \text{ whichever is less}$$

$$A_2 = b \times D = 400 \times 400 = 16 \times 10^4 \text{ mm}^2$$

$$\therefore \quad \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{576 \times 10^4}{16 \times 10^4}} = 6 > 2 \quad \therefore \sqrt{\frac{A_1}{A_2}} = 2$$

$$\text{Permissible bearing stress} = 0.45\, f_{ck} \sqrt{\frac{A_1}{A_2}}$$

$$= 0.45 \times 20 \times 2$$

$$= 18 \text{ N/mm}^2 > 5 \text{ N/mm}^2$$

∴ Safe.

Step 9: Check for development length:

$$L_d = \frac{0.87\, f_y}{4\, \tau_{bd}}\, \phi = \frac{0.87 \times 415}{4 \times 1.9} \times 10$$

$$= 475.07 \text{ mm}$$

Provid 60 mm side cover.

$$\therefore \quad \text{Length of bar available} = \frac{1}{2}(B - b) - 60 = \frac{1}{2}(2500 - 400) - 60$$

$$= 990 \text{ mm} > 475.07 \text{ mm}$$

Hence, safe.