DARBHANGA COLLEGE OF ENGINEERING

DARBHANGA

COURSE FILE

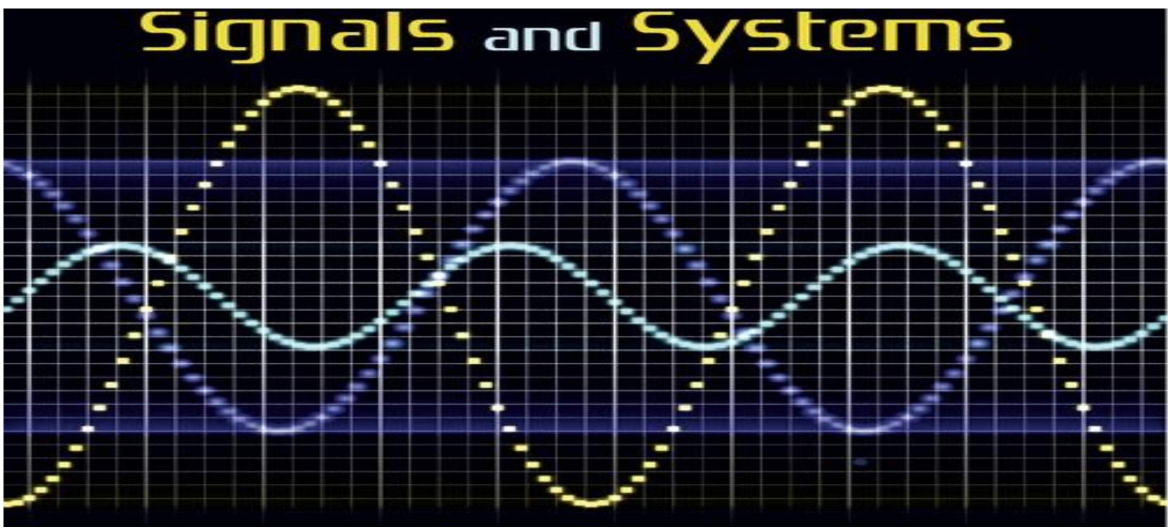
OF

SIGNAL AND SYSTEM

(EE 031510)

DEEPAK SINGH

ASSISTANT PROFESSOR, DEPARTMENT OF ELECTRICAL & ELECTRONICS ENGINEERING



विज्ञान एवं प्रावैधिकी विभाग

Department of Science and Technology

Government of Bihar

Vision of EEE: - To bring forth engineers with an emphasis on higher studies and a fervour to serve national and multinational organizations and, the society.

Mission of EEE: -

M1: - To provide domain knowledge with advanced pedagogical tools and applications.

M2: - To acquaint graduates to the latest technology and research through collaboration with industry and research institutes.

M3: - To instil skills related to professional growth and development.

M4: - To inculcate ethical valued in graduates through various social-cultural activities.

PEO of EEE

PEO 01 – The graduate will be able to apply the Electrical and Electrical Engineering concepts to excel in higher education and research and development.

PEO 02 – The graduate will be able to demonstrate the knowledge and skills to solve real life engineering problems and design electrical systems that are technically sound, economical and socially acceptable.

PEO 03 – The graduates will be able to showcase professional skills encapsulating team spirit, societal and ethical values.

Program Outcomes of B.Tech in Electrical and Electronics Engineering

1. Engineering knowledge: Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.

2. Problem analysis: Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3. Design/development of solutions: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4. Conduct investigations of complex problems: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5. Modern tool usage: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

6. The engineer and society: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. Environment and sustainability: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. Individual and team work: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. Communication: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. Project management and finance: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. Life-long learning: Recognize the need and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

PSO of EEE

PSO 01 Students will be able to identify, formulate and solve problems using various software and other tools in the areas of Automation, Control Systems, Power Engineering and PCB designing.

PSO 02 Students will be able to provide sustainable solutions to growing energy demands.

Course Description

The course presents and integrates the basic concepts for both continuous-time and discrete-time signals and systems. Signal and system representations are developed for both time and frequency domains. These representations are related through the Fourier transform and its generalizations, which are explored in detail. Neither energy nor power system application is analyze and solve by the z- transform method which are discussed and illustrated.

Course Objectives

- to develop good understanding about signals, systems and their classification;
- to provide with necessary tools and techniques to analyze electrical networks and systems
- to develop expertise in time-domain and frequency domain approaches
- to the analysis of continuous and discrete systems

After the completion of this course the students will be able to:

CO1: **Understand** Discrete-time systems, LTI systems & their realization using Z-transforms.

CO2: **Derive** Fourier series for continuous time signals and find Fourier transform for different signals

CO3: **Analyze** the Continuous Time systems by performing Convolution

CO4: **Design** different signals, Systems & identify LTI systems

Mapping of CO's with PO's

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2
CO1	3	1	2	2	3	-	2	1	-	-	2	3	3	1
CO2	2	3	2	2	3	-	-	-	1	-	1	2	1	3
CO3	2	1	3	3	2	-	2	-	-	-	2	1	2	3
CO4	3	2	3	1	3	1	2	2	-	-	3	2	3	2

B. Tech. V Semester (EEE)

EE- 31510 Signal and System

L T P/D Total	Max Marks: 100
3-1-0 4	Final Exam: 70 Marks
	Sessional: 20 Marks
	Internals: 10 Marks.

UNIT-I

System and Signal: Definition, classification of systems, standard test signal, properties of system, properties of liner system.

UNIT-II

Analogous System: Force voltage analogy, Force current analogy, Mechanical coupling devices, Electromechanical system

UNIT-III

Laplace transformation: Laplace transform of some important function, shift theorem and its application, Laplace transform of periodic functional, analysis of response, initial & final values theorem, response to periodic sinusoidal excitation

UNIT-IV

Analysis of Fourier Methods: Fourier series expansion of periodic functional symmetry condition, exponential form of Fourier series, Fourier integral & Fourier transform, Analysis by Fourier methods, Fast Fourier transform

UNIT-V

Z transformation: Z transform, Discrete time, LTI system, solution of difference equation, Application of Z transform to open loop system.

Books:

1. Analysis of Linear System by D.K Cheng, Narosa pub. House.

2. Modeling & Analysis of Liner System by J.P Tiwari. Dhanpat rai & Sons.

3. Signal & system by H.P Hus, Tata McGraw Hill.

4. Signal & system by I.J. et. at., Tata McGraw Hill.

DARBHANGA COLLEGE OF ENGINEERING, DARBHANGA

w.e.f. – 05-07-18

EEE Semester – 5th, Session (2018-19)

Day	Branch	1 (10am-10.50am)	2 (10.50am-11.40am)	3(11.40am-12.30pm)	4(12.30pm-1.20pm)	Lunch (1.20pm – 1.50pm)	5(1.50pm – 2.40pm)	6(2.40pm-3.30pm)	7(3.30pm-4.20pm)
Monday	E.E.E.	SS				LUNCH			
Tuesday	E.E.E.			SS					
Wednesday	E.E.E.			SS					
Thursday	E.E.E.								
Friday	E.E.E.	SS(T1)							
Saturday	E.E.E.	SS(T2)							

<u>Mechanical</u> – M1 - 1 to 30   <u>E.E.E.</u> - E1 - 1 to 30   <u>C. Sc.</u> - CS1 – 1 to30   <u>Civil</u> - C1 – 1 to 30

S-1 M2 –31 to All   S-2 E2 – 31 to All   S-3 CS2 – 31 to All   B.C.R. C2 – 31 to All

Prof . Incharge Routine
D.C.E. Darbhanga

Principal
D.C.E. Darbhanga

Institute/College Name:	Darbhanga College of Engineering
Program Name:	B.Tech (EEE, 5^{th} semester)
Course Code:	031510
Course Name:	Signal and System
Lecture/Tutorial(per week):	3/1
Course Credits:	4
Course Co-coordinator Name:	Mr. Deepak Singh

1. Scope and Objective of Course

This course is designed to introduce integrates the basic concepts for both continuous-time and discrete-time signals and systems. Signal and system representations are developed for both time and frequency domains. These representations are related through the Fourier transform and its generalizations, which are explored in detail. Filtering and filter design, modulation, and sampling for both analog and digital systems, as well as exposition and demonstration of the basic concepts of feedback systems for both analog and digital systems. There is also good scope in design of analog and digital system.

The course outcomes are

- to develop good understanding about signals, systems and their classification;
- to provide with necessary tools and techniques to analyze electrical networks and systems
- to develop expertise in time-domain and frequency domain approaches
- to the analysis of continuous and discrete systems

1.

2. Textbooks

TB1: Simon & Haykins, Signals & Systems, John Wiley & Sons
TB2: Oppenheim, Willsky & Young; Signals and Systems PHI, EEE, New Delhi.
TB3: Signal & system by H.P Hus, Tata McGraw Hill

3. Reference Books

RB1: B.P.Lathi, Signals & Systems and Communications

Other readings and relevant websites

S. No.	Link of journals, Magazines, websites and Research papers
1.	https://nptel.ac.in/courses/108106075/
2.	https://nptel.ac.in/courses/108106075/5
3.	https://nptel.ac.in/courses/108106075/7
4.	https://www.youtube.com/watch?v=SNjcFH24UpI&list=PL27A1EECF55240D15
5	https://nptel.ac.in/courses/108106075/13
6	https://nptel.ac.in/courses/108106075/22
7	https://nptel.ac.in/courses/108106075/44
8	https://www.youtube.com/watch?v=ghz_puTV168&list=PL27A1EECF55240D15&index=20

Course plan

Lecture No.	Date of Lecture	Topics	Web Links for Videos Lecture	Text Books/Reference books/Reading Materials	Page No. of Text Books
1		**System and Signal** Definition, classification of systems, standard test signal	https://www.youtube.com/watch?v=SNjcFH24UpI&list=PL27A1EECF55240D15 https://www.youtube.com/watch?v=_DDLZjQIN1I&list=PL27A1EECF55240D15&index=4	TB2	13,27
		Tutorial-1			
2-3		**System and Signal** Properties of system, properties of liner system.	https://www.youtube.com/watch?v=_FQTrXfCYw8&index=6&list=PL27A1EECF55240D15	TB2	27-42
		Tutorial-2			

4-8		**Analogous System** Force voltage analogy, Force current analogy, Mechanical coupling devices, Electromechanical system	http://indianpowersector.com/power-station/thermal-power-plant/	TB2	13-26
		Tutorial-3			
9-13		**Laplace transformation** Laplace transform of some important function, shift theorem and its application	https://www.youtube.com/watch?v=ghz_puTV168&list=PL27A1EECF55240D15&index=20	TB2	1-12
		Tutorial-4, Assinment-1			
14-18		**Laplace transformation** Laplace transform of periodic functional, analysis of response, initial & final values theorem, response to periodic sinusoidal excitation	https://www.youtube.com/watch?v=-h0ZhCctSvw&list=PL27A1EECF55240D15&index=24	RB1	257-295
		Tutorial-5			
18-24		**Analysis of Fourier Methods** Fourier series expansion of periodic functional symmetry condition, exponential form of Fourier series	https://www.youtube.com/watch?v=_TqXBTKbK1E&index=12&list=PL27A1EECF55240D15	RB1	297-356
		Tutorial-6			
		Analysis of Fourier Methods			

25-33		Fourier integral & Fourier transform, Analysis by Fourier methods, Fast Fourier transform	https://www.youtube.com/watch?v=tLlXwMNbKrU&list=PL27A1EECF55240D15&index=16	TB2	501-524

		Z transformation			
34-38		Z transform, Discrete time, LTI system,	https://www.youtube.com/watch?v=gkC7cXa8ewk&list=PL27A1EECF55240D15&index=39	RB1	533-589
		Tutorial-8			
		Tutorial-7			
		Z transformation			
39-42		solution of difference equation, Application of Z transform to open loop system.	https://www.youtube.com/watch?v=VLVjf9dUQjc&list=PL27A1EECF55240D15&index=41	RB1	533-589
		Tutorial-9, Assinment-2			

Syllabus

Topics	No. of Lectures	Weightages
System and Signal: Definition, classification of systems, standard test signal, properties of system, properties of liner system.	3	8%
Analogous System: Force voltage analogy, Force current analogy, Mechanical coupling devices, Electromechanical system	5	12%
Laplace transformation: Laplace transform of some important function, shift theorem and its application, Laplace transform of periodic functional, analysis of response, initial & final values theorem, response to periodic sinusoidal excitation	10	24%
Analysis of Fourier Methods: Fourier series expansion of periodic functional symmetry condition, exponential form of Fourier series, Fourier integral & Fourier transform, Analysis by Fourier methods, Fast Fourier transform	15	35%
Z transformation: Z transform, Discrete time, LTI system, solution of difference equation, Application of Z transform to open loop system.	9	21%

This document is approved by

Designation	Name	Signature
Course Coordinator	Mr. Deepak Singh	
H.O.D	Mr. Santosh Kumar Gupta	
Principal		
Date		

Evaluation and Examination Blue Prints:

Internal assessment is done through quiz tests, presentations, assignments and project work. Two sets of question papers are asked from each faculty and out of these two, without the knowledge of faculty, one question paper is chosen for the concerned examination. The components of evaluations alongwith their weightage followed by the University is given below

Sessional Test	20%
Internals	10%
End term examination	70%

LECTURE PLAN

Institute / School Name :	DARBHANGA COLLEGE OF ENGINEEERING		
Program Name	**B.E, EEE**		
Course Code	031510		
Course Name	SIGNAL AND SYSTEM		
Lecture / Tutorial (per week):	3/1	**Course Credits**	4
Course Coordinator Name	Mr. Deepak Singh		

Topics	**Lecture Number**
System and Signal	
Definition, classification of systems	**1**
standard test signal	**2**
Properties of system, properties of linear system.	**3**
Analogous System	
Force voltage analogy	**4**
Force current analogy	**5**
Mechanical coupling devices,	**6**
Electromechanical system	**7,8**
Laplace transformation	
Laplace transform of some important function, shift theorem and its application,	**9-11**
Laplace transform of periodic functional, analysis of response, initial & final values theorem	**12-15**
response to periodic sinusoidal excitation	**16-18**
Analysis of Fourier Methods	
Fourier series expansion of periodic functional symmetry	**19-22**

DARBHANGA COLLEGE OF ENGINEERING, DARBHANGA

5th Sem. Branch:- Electrical & Electronics Engineering Batch (2016-20)

Subject :- Signal and System

Sl. No.	Name	Class Roll No.
1.	Rajbala Kumari	16-EE-01
2.	Sushmita Kumari	16-EE-03
3.	Shweta Rani	16-EE-04
4.	Nisha Raj	16-EE-05
5.	Priyanka Kumari	16-EE-07
6.	Rashmi Bharti	16-EE-08
7.	Sonika Kumari	16-EE-09
8.	Puja Prabhakar	16-EE-11
9.	Devashish Dubey	16-EE-12
10.	Alok Ranjan	16-EE-13
11.	Hemant Kumar patel	16-EE-14
12.	Kumar Satyam	16-EE-15
13.	Pratik Kumar	16-EE-17
14.	Prabhash Kumar Yadav	16-EE-18
15.	Ranjesh Kumar	16-EE-19
16.	Upendra Kumar	16-EE-20
17.	Amit Kumar	16-EE-21
18.	Wajahat Khalil	16-EE-22
19.	Prakash Ranjan	16-EE-24
20.	Rahul Kumar	16-EE-26
21.	Ankit Ranjan	16-EE-27
22.	Varinder Kumar	16-EE-29

23.	Vikash Kumar Bharti	16-EE-30
24.	Navneet Kumar	16-EE-31
25.	Prem Raj	16-EE-32
26.	Juhi Kumari	16-EE-33
27.	Appu Kumar	16-EE-34
28.	Ashish Kumar	16-EE-35
29.	Babloo Kumar	16-EE-36
30.	Niraj Kumar Nirala	16-EE-37
31.	Poonam Priya	16-EE-38
32.	Kamal Nayan Jha	16-EE-40
33.	Rajneesh Kumar	16-EE-41
34.	Rohit Kumar	16-EE-42
35.	Amrita Kumari	16-EE-43
36.	Dimple Kumari	16-EE-44
37.	Khushbu Kumari	16-EE-45
38.	Alok Kumar	16-EE-46
39	Md. Ataur Rahman	16-EE-47
40	Shubhra Verma	16-EE-48
41	Khushboo Kumari	16-EE-49
42	Priti Kumari	16-EE-50
43	Md. Sharmajul Haque	16-EE-51
44	Saurav Kumar	16-EE-52
45	Anubhav Anand	16-EE-53
46	Shankar Suwan Kesri	16-EE-54
47	Tanuj Anand	16-EE-55
48	Ganesh Kumar	16-EE-56
49	Ashish Ranjan	16-EE-57
50	Shashi Kumar	16-EE-58

51	Vishnu Kumar	16-EE-59
52	Anku Rani	16-EE-60
53	Santosh Kumar Mahto	16-EE-61
54	Shubham Kumar	16-EE-62
55	Md. Seraj	16-EE-63
56	Amrendra Kishor	16-EE-64
57	Amit Kumar Jha	16-EE-65
58	Rohit Kumar Yadav	16-EE-66
59	Rahul Kumar Sahni	16-EE-67
60	Saurabh Kumar	16-EE-68
61	Pankaj Kumar	16-EE-69
62	Pooja Roy	16(LE)EE-01
63	Surya Rai	17(LE)EE-01
64	Santosh Kumar	17(LE)EE-02
65	Lalit Kumar Ram	17(LE)EE-03
66	Deepak Kumar Prabhat	17(LE)EE-04
67	Sumit Kumar	17(LE)EE-05
68	Avinash Choudhary	17(LE)EE-06
69	Ranjeet Kumar Sah	17(LE)EE-07
70	Baby Kumari	17(LE)EE-08
71	Virendra Kumar Sah	17(LE)EE-09
72	Abhinav Raj	17(LE)EE-10
73	Ejaz Ahmad Ansari	17(LE)EE-11

Darbhanga College of Engineering, Darbhanga

EEE Department

B.Tech [SEM V (EEE)]

Mid. Sem Exam

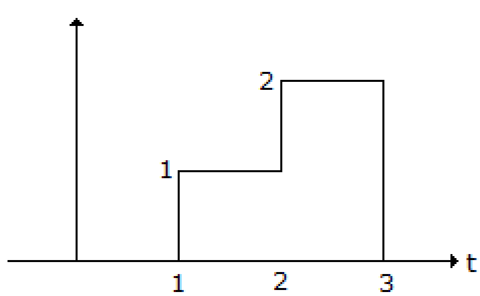
(Session: 2019-20)

Course Code–031510

Signal and System

Time: 2 Hours ***Max. Marks: 20***

Note: Attempt all questions. CO–Course Outcomes, BL–Bloom Level

S. No.	Questions	Marks	CO	BL
1.	(i). Plot the signal $x(t) = u(t+1) + 2u(t) - u(t-3) - 2u(t-5)$ (ii). Check the periodicity of the signal and determine the FTP if periodic? $x(t) = 10\sin 25\pi t + \cos 10\pi t$	4	CO5	L1
2.	1. Determine the integral $\int_{-10}^{10} cos\pi t.\delta(2t-10)dt$ 2. If I (*s*)= $\frac{5(s+250)}{s(s+100)}$, initial value of *i*(*t*) is ?	4	CO2	L2
3.	1. Determine energy and power signal (i) $e^{2t}u(-t)$ (ii) $e^{-3\|t\|}$ 2. If £[*f*(*t*)] = F(*s*), then £[*f*(*t* - *T*)] =	4	CO4	L3
4.	1. If $L[f(t)] = \frac{2(s+1)}{s^2+2s+5}$ then, $f(0^+)$ and $f(\infty)$ are given by 2.The Laplace transform of the waveform shown in the below figure is 	4	CO1	L5
5.	y(t) = x(t-2) + x(2-t) comment on linearity, causality and time variation?	4	CO3	L4

Signal and System
Assignment I

Question 1. Define Continuous-time Signal?

Question 2. Define Signal?

Question 3. What Are The Major Classifications Of The Signal?

Question 4. State Properties Of Convolution

Question 5. What Is Dft ?

Question 6. What Are The Applications Of Correlation?

Question 7. What Are The Classification Of Continuous Time Signals? Name Them?

Signal and System
Assignment II

Question 8. Give Some Examples Of Causal Signal?

Question 9. What Is Amplitude Scaling And Time Scaling?

Question 10. What Is Unit Delay Element ?

Question 11. Classify Discrete Time Signal?

Question 12. State Superposition Theorem?

Question 13. How The Analog To Digital Conversion Takes Place. Name All The Steps Involved?

Question 14. What Is Meant By Step Response Of The Dt System?

Signal and System
Assignment III

Question 15. Define Transfer Function Of The Dt System?

Question 16. Define Impulse Response Of A Dt System?

Question 17. State The Significance Of Difference Equations?

Question 18. Write The Difference Equation For Discrete Time System?

Question 19. What Are The Blocks Used For Block Diagram Representation?

Question 20. State The Significance Of Block Diagram Representation?

Question 21. What Are The Properties Of Convolution?

Signal and System
Assignment IV

Question 22. State The Commutative Properties Of Convolution?

Question 23. State The Associative Properties Of Convolution?

Question 24. Define Causal Lti Dt System?

Question 25. How The Discrete Time System Is Represented?

Question 26. What Are The Classification Of The System Based On Unit Sample Response?

Question 27. What Is Meant By Fir System?

Question 28. What Is Meant By IIR System?

Question 29. What Is Recursive System?

Signal and System
Assignment V

Question 30. What Is Non Recursive System?

Question 31. What Is The Difference Between Recursive And Non Recursive System?

Question 32. Define Realization Structure?

Question 33. What Are The Different Types Of Structure Realization?

Question 34. What Is Natural Response?

Question 35. What Is Zero Input Response?

Question 36. What Is Forced Response?

Question 37. What Is Complete Response?

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Code : 031510

B.Tech 5th Semester Examination, 2016

Signals and System

Time : 3 hours *Full Marks : 70*

Instructions :

(i) There are Nine Questions in this paper.

(ii) Attempt Five questions in all.

(iii) Question No. 1 is compulsory.

(iv) All the questions carry equal marks.

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1. Fill in the blanks of the following (any seven). 2×7=14

(a) The z-transform of the sequence $x[n]$ is given by $X(z)\frac{1}{(1-2z^{-1})^2}$, with the region of convergence $|z| > 2$. Then, x[2] is.........8.........

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(b) The input output relationship of a causal stable LTI system is given as $y[n] = \alpha\ y[n-1] + \beta \times [n]$ If the impulse response $h[n]$ of this system satisfies the condition $\sum_{n=0}^{\infty} h[n] = 2$, the relationship between α and β is ....

(c) Two discrete time systems with impulse responses $h_1[n] = \delta\ [n-1]$ and $h_2\ [n] = \delta\ [n-2]$ are connected in cascade. The overall Impulse response of the cascaded system is ....

(d) For a periodic signal $v(t) = 30 \sin 100t + 10 \cos 300t + 6 \sin (500t + \pi/4)$. the fundamental frequency in rad/s is ....

(e) A discrete time system has impulse response $h(n) = 2^n u(n-2)$, whether the system is stable or not ...not.....

(f) The impulse response of a system is $h(t) = t\,u(t)$. For an input of $u(t-1)$ the output is ....

(g) The average power in the signal $s(t) = 8 \cos (20\pi t - \pi / r) + 4 \sin (15\pi t)$ is.............

(h) Fourier series is preferred for ..periodic.. signal.

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(i) The lengths of two discrete time sequences $x_1(n)$ and $x_2(n)$ are 4 and 5, respectively. The maximum length of a sequence $x_1(n) * x2(n)$ is ....

(j) For a stable LTI system, bounded input always provide ..bounded.. output.

2. Define the following : 2×7=14

(a) Stability

(b) Causality

(c) Random Signal

(d) Time variant system

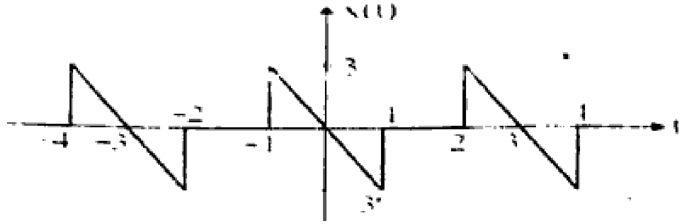
(e) Linear system

(f) Delta function

(g) Memory less system

3. (a) Consider the system $y(t) = 2 \times (t) + 3$. Determine whether it is Memory less, Causal, Linear and Time invariant. 7

(b) Determine the average power of signal $x(t)$. 7



4. A continuous time signal $x(t)$ is shown in figure. Sketch the following signals. $2\times7=14$

(a) $x(3-t)$

(b) $x(4t+1)$

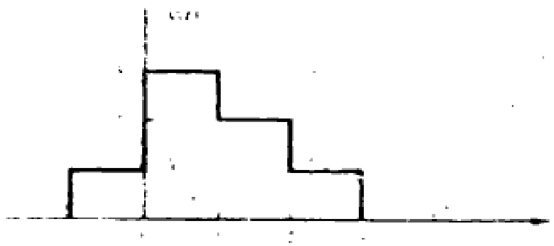
(c) $[x(t) \div x(-t)]\, u(t)$

(d) $[\delta(t+1)+\delta(t-1)]x(t)$

(e) $x(t)\, x(t-5)$

(f) $x(t)\delta(t-3)$

(g) $x(2t-5)$



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(a) $x[n] = \sin(\pi^2 n)$

(b) $x(t) = \cos t + \sin 3t$

(c) $x[n] = \cos \dfrac{n}{4}$

(d) $x[n] = \cos^2 \dfrac{\pi}{8} n$

(e) $x(t) = \sin t + \sin 2t$

(f) $x(n) = \sin(5\pi n)$

(g) $x(t) = e^{-j3\pi\ 4}$

8\. (a) Calculate the Fourier series coefficient a_k for the continuous time periodic signal. 7

$$x(t) = \begin{cases} 1.5, & 0 \le t < 1 \\ -1.5, & 1 \le t < 2 \end{cases}$$

With fundamental frequency $\omega_0 = \pi$.

(b) Given 7

$$X(z) = \frac{z(z-4)}{(z-1)(z-2)(z-3)}$$

(a) State all possible regions of convergence.

(b) For which ROC the X (z) is z-transform of a causal sequence.

9\. Write short notes on any two: $7 \times 2 = 14$

(a) Properties of convolution

(b) Initial and Final value theorem of Laplace transform

(c) Energy and power signals

(d) Force voltage analogy

***

CHAPTER 3

SIGNALS & SYSTEMS

YEAR 2012 **ONE MARK**

MCQ 3.1 If $x[n] = (1/3)^{|n|} - (1/2)^n u[n]$, then the region of convergence (ROC) of its z-transform in the z-plane will be

(A) $\frac{1}{3} < |z| < 3$ (B) $\frac{1}{3} < |z| < \frac{1}{2}$

(C) $\frac{1}{2} < |z| < 3$ (D) $\frac{1}{3} < |z|$

MCQ 3.2 The unilateral Laplace transform of $f(t)$ is $\frac{1}{s^2 + s + 1}$. The unilateral Laplace transform of $tf(t)$ is

(A) $-\frac{s}{(s^2 + s + 1)^2}$ (B) $-\frac{2s + 1}{(s^2 + s + 1)^2}$

(C) $\frac{s}{(s^2 + s + 1)^2}$ (D) $\frac{2s + 1}{(s^2 + s + 1)^2}$

YEAR 2012 **TWO MARKS**

MCQ 3.3 Let $y[n]$ denote the convolution of $h[n]$ and $g[n]$, where $h[n] = (1/2)^n u[n]$ and $g[n]$ is a causal sequence. If $y[0] = 1$ and $y[1] = 1/2$, then $g[1]$ equals

(A) 0 (B) 1/2

(C) 1 (D) 3/2

MCQ 3.4 The Fourier transform of a signal $h(t)$ is $H(j\omega) = (2\cos\omega)(\sin 2\omega)/\omega$. The value of $h(0)$ is

(A) 1/4 (B) 1/2

(C) 1 (D) 2

MCQ 3.5 The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\, d\tau$. The system is

(A) time-invariant and stable

(B) stable and not time-invariant

(C) time-invariant and not stable

(D) not time-invariant and not stable

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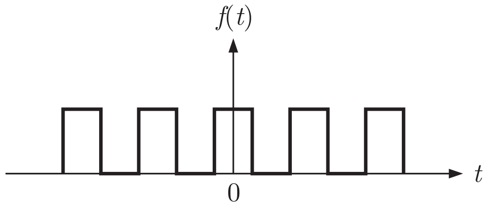
YEAR 2011 **ONE MARK**

MCQ 3.6 The Fourier series expansion $f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$ of the periodic signal shown below will contain the following nonzero terms



(A) a_0 and $b_n, n = 1,3,5,...\infty$
(B) a_0 and $a_n, n = 1,2,3,...\infty$
(C) $a_0\, a_n$ and $b_n, n = 1,2,3,...\infty$
(D) a_0 and $a_n\, n = 1,3,5,...\infty$

MCQ 3.7 Given two continuous time signals $x(t) = e^{-t}$ and $y(t) = e^{-2t}$ which exist for $t > 0$, the convolution $z(t) = x(t)^* y(t)$ is
(A) $e^{-t} - e^{-2t}$
(B) e^{-3t}
(C) e^{+t}
(D) $e^{-t} + e^{-2t}$

YEAR 2011 **TWO MARKS**

MCQ 3.8 Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t - \tau)$ be $F_2(s)$. Let $F_1{}^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set $s = \sigma + j\omega$. If $G(s) = \frac{F_2(s)\,F_1{}^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is an ideal
(A) impulse $\delta(t)$
(B) delayed impulse $\delta(t - \tau)$
(C) step function $u(t)$
(D) delayed step function $u(t - \tau)$

MCQ 3.9 The response $h(t)$ of a linear time invariant system to an impulse $\delta(t)$, under initially relaxed condition is $h(t) = e^{-t} + e^{-2t}$. The response of this system for a unit step input $u(t)$ is
(A) $u(t) + e^{-t} + e^{-2t}$
(B) $(e^{-t} + e^{-2t})\, u(t)$
(C) $(1.5 - e^{-t} - 0.5e^{-2t})\, u(t)$
(D) $e^{-t}\delta(t) + e^{-2t} u(t)$

YEAR 2010 **ONE MARK**

MCQ 3.10 For the system $2/(s + 1)$, the approximate time taken for a step response to reach 98% of the final value is
(A) 1 s
(B) 2 s
(C) 4 s
(D) 8 s

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MCQ 3.11 The period of the signal $x(t) = 8\sin\left(0.8\pi t + \frac{\pi}{4}\right)$ is

(A) 0.4π s (B) 0.8π s

(C) 1.25 s (D) 2.5 s

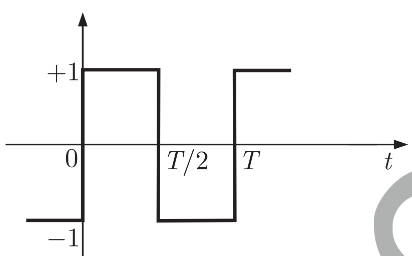
MCQ 3.12 The system represented by the input-output relationship

$$y(t) = \int_{-\infty}^{5t} x(\tau)\, d\tau, t > 0$$

(A) Linear and causal (B) Linear but not causal

(C) Causal but not linear (D) Neither liner nor causal

MCQ 3.13 The second harmonic component of the periodic waveform given in the figure has an amplitude of

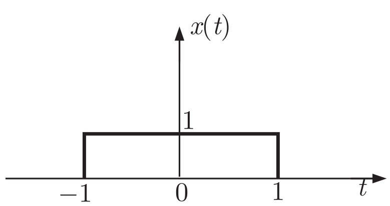


(A) 0 (B) 1

(C) $2/\pi$ (D) $\sqrt{5}$

YEAR 2010 TWO MARKS

MCQ 3.14 $x(t)$ is a positive rectangular pulse from $t = -1$ to $t = +1$ with unit height as shown in the figure. The value of $\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$ {where $X(\omega)$ is the Fourier transform of $x(t)$} is.



(A) 2 (B) 2π

(C) 4 (D) 4π

MCQ 3.15 Given the finite length input $x[n]$ and the corresponding finite length output $y[n]$ of an LTI system as shown below, the impulse response $h[n]$ of the system is

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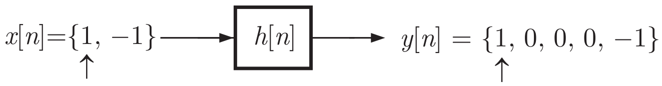
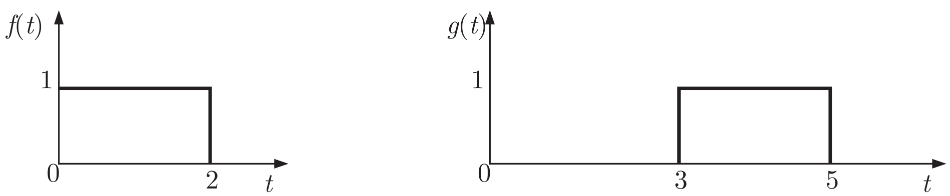


(A) $h[n]=\{\underset{\uparrow}{1},0,0,1\}$
(B) $h[n]=\{\underset{\uparrow}{1},0,1\}$
(C) $h[n]=\{\underset{\uparrow}{1},1,1,1\}$
(D) $h[n]=\{\underset{\uparrow}{1},1,1\}$

Common Data Questions Q.6-7.

Given $f(t)$ and $g(t)$ as show below



MCQ 3.16 $g(t)$ can be expressed as

(A) $g(t)=f(2t-3)$
(B) $g(t)=f\left(\frac{t}{2}-3\right)$
(C) $g(t)=f\left(2t-\frac{3}{2}\right)$
(D) $g(t)=f\left(\frac{t}{2}-\frac{3}{2}\right)$

MCQ 3.17 The Laplace transform of $g(t)$ is

(A) $\frac{1}{s}(e^{3s}-e^{5s})$
(B) $\frac{1}{s}(e^{-5s}-e^{-3s})$
(C) $\frac{e^{-3s}}{s}(1-e^{-2s})$
(D) $\frac{1}{s}(e^{5s}-e^{3s})$

YEAR 2009 ONE MARK

MCQ 3.18 A Linear Time Invariant system with an impulse response $h(t)$ produces output $y(t)$ when input $x(t)$ is applied. When the input $x(t-\tau)$ is applied to a system with impulse response $h(t-\tau)$, the output will be

(A) $y(\tau)$
(B) $y(2(t-\tau))$
(C) $y(t-\tau)$
(D) $y(t-2\tau)$

YEAR 2009 TWO MARKS

MCQ 3.19 A cascade of three Linear Time Invariant systems is causal and unstable. From this, we conclude that

(A) each system in the cascade is individually causal and unstable
(B) at least on system is unstable and at least one system is causal
(C) at least one system is causal and all systems are unstable

(D) the majority are unstable and the majority are causal

MCQ 3.20 The Fourier Series coefficients of a periodic signal $x(t)$ expressed as $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$ are given by $a_{-2} = 2 - j1$, $a_{-1} = 0.5 + j0.2$, $a_0 = j2$, $a_1 = 0.5 - j0.2$, $a_2 = 2 + j1$ and $a_k = 0$ for $|k| > 2$
Which of the following is true ?

(A) $x(t)$ has finite energy because only finitely many coefficients are non-zero

(B) $x(t)$ has zero average value because it is periodic

(C) The imaginary part of $x(t)$ is constant

(D) The real part of $x(t)$ is even

MCQ 3.21 The z-transform of a signal $x[n]$ is given by $4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$
It is applied to a system, with a transfer function $H(z) = 3z^{-1} - 2$
Let the output be $y[n]$. Which of the following is true ?

(A) $y[n]$ is non causal with finite support

(B) $y[n]$ is causal with infinite support

(C) $y[n] = 0; |n| > 3$

(D) $\text{Re}[Y(z)]_{z=e^{j\theta}} = -\text{Re}[Y(z)]_{z=e^{-j\theta}}$
$\text{Im}[Y(z)]_{z=e^{j\theta}} = \text{Im}[Y(z)]_{z=e^{-j\theta}}; \; -\pi \le \theta < \pi$

YEAR 2008 ONE MARK

MCQ 3.22 The impulse response of a causal linear time-invariant system is given as $h(t)$. Now consider the following two statements :
Statement (I): Principle of superposition holds
Statement (II): $h(t) = 0$ for $t < 0$
Which one of the following statements is correct ?

(A) Statements (I) is correct and statement (II) is wrong

(B) Statements (II) is correct and statement (I) is wrong

(C) Both Statement (I) and Statement (II) are wrong

(D) Both Statement (I) and Statement (II) are correct

MCQ 3.23 A signal $e^{-\alpha t}\sin(\omega t)$ is the input to a real Linear Time Invariant system. Given K and ϕ are constants, the output of the system will be of the form $Ke^{-\beta t}\sin(vt + \phi)$ where

(A) β need not be equal to α but v equal to ω

(B) v need not be equal to ω but β equal to α

(C) β equal to α and v equal to ω

(D) β need not be equal to α and v need not be equal to ω

YEAR 2008 **TWO MARKS**

MCQ 3.24 A system with $x(t)$ and output $y(t)$ is defined by the input-output relation :

$$y(t) = \int_{-\infty}^{-2t} x(t)\, d\tau$$

The system will be

(A) Casual, time-invariant and unstable

(B) Casual, time-invariant and stable

(C) non-casual, time-invariant and unstable

(D) non-casual, time-variant and unstable

MCQ 3.25 A signal $x(t) = \text{sinc}(\alpha t)$ where α is a real constant $(\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x})$ is the input to a Linear Time Invariant system whose impulse response $h(t) = \text{sinc}(\beta t)$, where β is a real constant. If min (α, β) denotes the minimum of α and β and similarly, max (α,β) denotes the maximum of α and β, and K is a constant, which one of the following statements is true about the output of the system ?

(A) It will be of the form $K\text{sinc}(\gamma t)$ where $\gamma = \min(\alpha, \beta)$

(B) It will be of the form $K\text{sinc}(\gamma t)$ where $\gamma = \max(\alpha, \beta)$

(C) It will be of the form $K\text{sinc}(\alpha t)$

(D) It can not be a sinc type of signal

MCQ 3.26 Let $x(t)$ be a periodic signal with time period T, Let $y(t) = x(t - t_0) + x(t + t_0)$ for some t_0. The Fourier Series coefficients of $y(t)$ are denoted by b_k. If $b_k = 0$ for all odd k, then t_0 can be equal to

(A) $T/8$ (B) $T/4$

(C) $T/2$ (D) $2T$

MCQ 3.27 $H(z)$ is a transfer function of a real system. When a signal $x[n] = (1 + j)^n$ is the input to such a system, the output is zero. Further, the Region of convergence (ROC) of $(1 - \frac{1}{2}z^{-1})$ H(z) is the entire Z-plane (except $z = 0$). It can then be inferred that $H(z)$ can have a minimum of

(A) one pole and one zero

(B) one pole and two zeros

(C) two poles and one zero

D) two poles and two zeros

MCQ 3.28 Given $X(z) = \frac{z}{(z - a)^2}$ with $|z| > a$, the residue of $X(z)\, z^{n-1}$ at $z = a$ for $n \geq 0$ will be

(A) a^{n-1} (B) a^n

(C) na^n (D) na^{n-1}

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MCQ 3.29 Let $x(t) = \text{rect}(t - \frac{1}{2})$ (where $rect(x) = 1$ for $-\frac{1}{2} \le x \le \frac{1}{2}$ and zero otherwise. If $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$, then the FTof $x(t) + x(-t)$ will be given by

(A) $\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(B) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)$

(C) $2\text{sinc}\left(\frac{\omega}{2\pi}\right)\cos\left(\frac{\omega}{2}\right)$

(D) $\text{sinc}\left(\frac{\omega}{2\pi}\right)\sin\left(\frac{\omega}{2}\right)$

MCQ 3.30 Given a sequence $x[n]$, to generate the sequence $y[n] = x[3 - 4n]$, which one of the following procedures would be correct ?

(A) First delay $x(n)$ by 3 samples to generate $z_1[n]$, then pick every 4^{th} sample of $z_1[n]$ to generate $z_2[n]$, and than finally time reverse $z_2[n]$ to obtain $y[n]$.

(B) First advance $x[n]$ by 3 samples to generate $z_1[n]$, then pick every 4^{th} sample of $z_1[n]$ to generate $z_2[n]$, and then finally time reverse $z_2[n]$ to obtain $y[n]$

(C) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally advance $v_2[n]$ by 3 samples to obtain $y[n]$

(D) First pick every fourth sample of $x[n]$ to generate $v_1[n]$, time-reverse $v_1[n]$ to obtain $v_2[n]$, and finally delay $v_2[n]$ by 3 samples to obtain $y[n]$

YEAR 2007 ONE MARK

MCQ 3.31 Let a signal $a_1 \sin(\omega_1 t + \phi)$ be applied to a stable linear time variant system. Let the corresponding steady state output be represented as $a_2 F(\omega_2 t + \phi_2)$. Then which of the following statement is true?

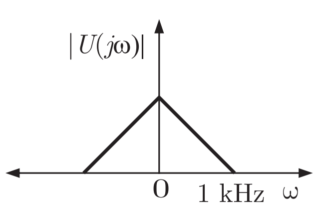
(A) F is not necessarily a "Sine" or "Cosine" function but must be periodic with $\omega_1 = \omega_2$.

(B) F must be a "Sine" or "Cosine" function with $a_1 = a_2$

(C) F must be a "Sine" function with $\omega_1 = \omega_2$ and $\phi_1 = \phi_2$

(D) F must be a "Sine" or "Cosine" function with $\omega_1 = \omega_2$

MCQ 3.32 The frequency spectrum of a signal is shown in the figure. If this is ideally sampled at intervals of 1 ms, then the frequency spectrum of the sampled signal will be



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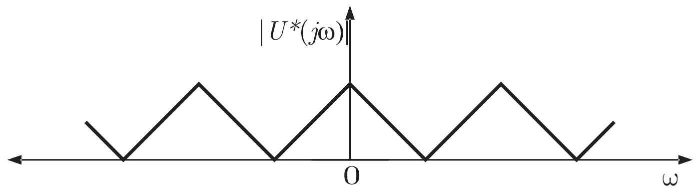
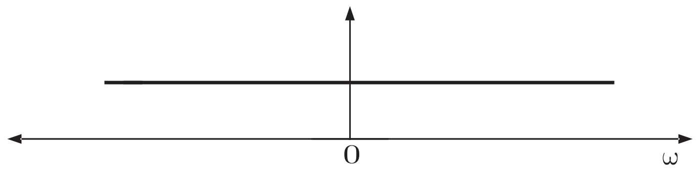
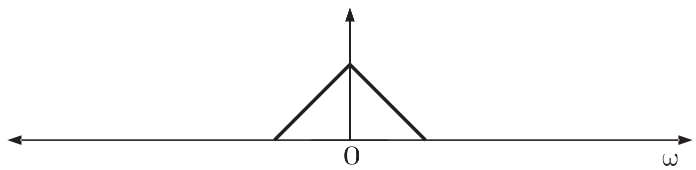
(A)



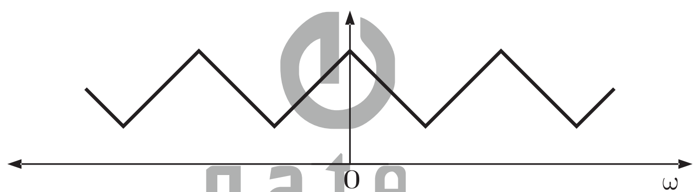
(B)



(C)



(D)



YEAR 2007 **TWO MARKS**

MCQ 3.33 A signal $x(t)$ is given by

$$x(t) = \begin{cases} 1, -T/4 < t \leq 3T/4 \\ -1, 3T/4 < t \leq 7T/4 \\ -x(t+T) \end{cases}$$

Which among the following gives the fundamental fourier term of $x(t)$?

(A) $\frac{4}{\pi}\cos\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

(B) $\frac{\pi}{4}\cos\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

(C) $\frac{4}{\pi}\sin\left(\frac{\pi t}{T} - \frac{\pi}{4}\right)$

(D) $\frac{\pi}{4}\sin\left(\frac{\pi t}{2T} + \frac{\pi}{4}\right)$

Statement for Linked Answer Question 34 and 35 :

MCQ 3.34 A signal is processed by a causal filter with transfer function $G(s)$

For a distortion free output signal wave form, $G(s)$ must

(A) provides zero phase shift for all frequency

(B) provides constant phase shift for all frequency

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(C) provides linear phase shift that is proportional to frequency

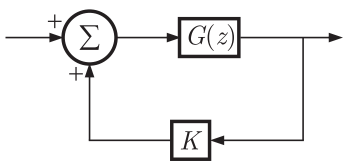
(D) provides a phase shift that is inversely proportional to frequency

MCQ 3.35 $G(z) = \alpha z^{-1} + \beta z^{-3}$ is a low pass digital filter with a phase characteristics same as that of the above question if

(A) $\alpha = \beta$ (B) $\alpha = -\beta$

(C) $\alpha = \beta^{(1/3)}$ (D) $\alpha = \beta^{(-1/3)}$

MCQ 3.36 Consider the discrete-time system shown in the figure where the impulse response of $G(z)$ is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \cdots = 0$



This system is stable for range of values of K

(A) $[-1, \frac{1}{2}]$ (B) $[-1, 1]$

(C) $[-\frac{1}{2}, 1]$ (D) $[-\frac{1}{2}, 2]$

MCQ 3.37 If $u(t), r(t)$ denote the unit step and unit ramp functions respectively and $u(t) * r(t)$ their convolution, then the function $u(t+1) * r(t-2)$ is given by

(A) $\frac{1}{2}(t-1)u(t-1)$ (B) $\frac{1}{2}(t-1)u(t-2)$

(C) $\frac{1}{2}(t-1)^2 u(t-1)$ (D) None of the above

MCQ 3.38 $X(z) = 1 - 3z^{-1}$, $Y(z) = 1 + 2z^{-2}$ are Z transforms of two signals $x[n], y[n]$ respectively. A linear time invariant system has the impulse response $h[n]$ defined by these two signals as $h[n] = x[n-1] * y[n]$ where * denotes discrete time convolution. Then the output of the system for the input $\delta[n-1]$

(A) has Z-transform $z^{-1}X(z)\,Y(z)$

(B) equals $\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$

(C) has Z-transform $1 - 3z^{-1} + 2z^{-2} - 6z^{-3}$

(D) does not satisfy any of the above three

YEAR 2006 ONE MARK

MCQ 3.39 The following is true

(A) A finite signal is always bounded

(B) A bounded signal always possesses finite energy

(C) A bounded signal is always zero outside the interval $[-t_0, t_0]$ for some t_0

(D) A bounded signal is always finite

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MCQ 3.40 $x(t)$ is a real valued function of a real variable with period T. Its trigonometric Fourier Series expansion contains no terms of frequency $\omega = 2\pi(2k)/T; k = 1, 2\cdots$ Also, no sine terms are present. Then $x(t)$ satisfies the equation

(A) $x(t) = -x(t-T)$

(B) $x(t) = x(T-t) = -x(-t)$

(C) $x(t) = x(T-t) = -x(t-T/2)$

(D) $x(t) = x(t-T) = x(t-T/2)$

MCQ 3.41 A discrete real all pass system has a pole at $z = 2\angle 30^\circ$: it, therefore

(A) also has a pole at $\frac{1}{2}\angle 30^\circ$

(B) has a constant phase response over the z-plane: $\arg|H(z)| =$ constant constant

(C) is stable only if it is anti-causal

(D) has a constant phase response over the unit circle: $\arg|H(e^{i\Omega})| =$ constant

YEAR 2006 TWO MARKS

MCQ 3.42 $x[n] = 0; n < -1, n > 0, x[-1] = -1, x[0] = 2$ is the input and $y[n] = 0; n < -1, n > 2, y[-1] = -1 = y[1], y[0] = 3, y[2] = -2$ is the output of a discrete-time LTI system. The system impulse response $h[n]$ will be

(A) $h[n] = 0; n < 0, n > 2, h[0] = 1, h[1] = h[2] = -1$

(B) $h[n] = 0; n < -1, n > 1, h[-1] = 1, h[0] = h[1] = 2$

(C) $h[n] = 0; n < 0, n > 3, h[0] = -1, h[1] = 2, h[2] = 1$

(D) $h[n] = 0; n < -2, n > 1, h[-2] = h[1] = h[-1] = -h[0] = 3$

MCQ 3.43 The discrete-time signal $x[n] \longleftrightarrow X(z) = \sum_{n=0}^{\infty} \frac{3^n}{2+n} z^{2n}$, where $\longleftrightarrow$ denotes a transform-pair relationship, is orthogonal to the signal

(A) $y_1[n] \leftrightarrow Y_1(z) = \sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^n z^{-n}$

(B) $y_2[n] \leftrightarrow Y_2(z) = \sum_{n=0}^{\infty} (5^n - n) z^{-(2n+1)}$

(C) $y_3[n] \leftrightarrow Y_3(z) = \sum_{n=-\infty}^{\infty} 2^{-|n|} z^{-n}$

(D) $y_4[n] \leftrightarrow Y_4(z) = 2z^{-4} + 3z^{-2} + 1$

MCQ 3.44 A continuous-time system is described by $y(t) = e^{-|x(t)|}$, where $y(t)$ is the output and $x(t)$ is the input. $y(t)$ is bounded

(A) only when $x(t)$ is bounded

(B) only when $x(t)$ is non-negative

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(C) only for $t \leq 0$ if $x(t)$ is bounded for $t \geq 0$

(D) even when $x(t)$ is not bounded

MCQ 3.45 The running integration, given by $y(t) = \int_{-\infty}^{t} x(t') \, dt'$

(A) has no finite singularities in its double sided Laplace Transform $Y(s)$

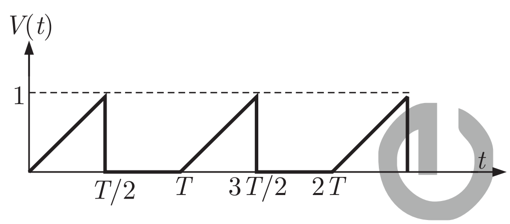
(B) produces a bounded output for every causal bounded input

(C) produces a bounded output for every anticausal bounded input

(D) has no finite zeroes in its double sided Laplace Transform $Y(s)$

YEAR 2005 TWO MARKS

MCQ 3.46 For the triangular wave from shown in the figure, the RMS value of the voltage is equal to



(A) $\sqrt{\frac{1}{6}}$ (B) $\sqrt{\frac{1}{3}}$

(C) $\frac{1}{3}$ (D) $\sqrt{\frac{2}{3}}$

MCQ 3.47 The Laplace transform of a function $f(t)$ is $F(s) = \frac{5s^2 + 23s + 6}{s(s^2 + 2s + 2)}$ as $t \rightarrow \infty$, $f(t)$ approaches

(A) 3 (B) 5

(C) $\frac{17}{2}$ (D) ∞

MCQ 3.48 The Fourier series for the function $f(x) = \sin^2 x$ is

(A) $\sin x + \sin 2x$

(B) $1 - \cos 2x$

(C) $\sin 2x + \cos 2x$

(D) $0.5 - 0.5 \cos 2x$

MCQ 3.49 If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse z-transform of $F(z) = \frac{1}{z+1}$ for $k > 0$ is

(A) $(-1)^k \delta(k)$ (B) $\delta(k) - (-1)^k$

(C) $(-1)^k u(k)$ (D) $u(k) - (-1)^k$

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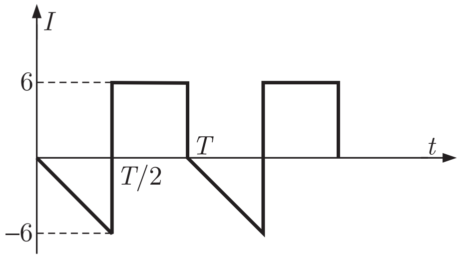
YEAR 2004 **TWO MARKS**

MCQ 3.50 The rms value of the periodic waveform given in figure is



(A) $2\sqrt{6}$ A (B) $6\sqrt{2}$ A

(C) $\sqrt{4/3}$ A (D) 1.5 A

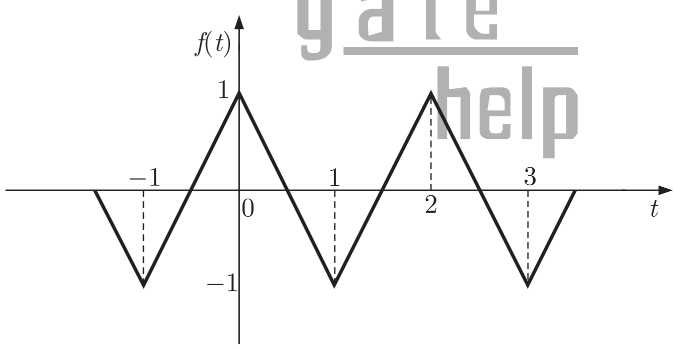
MCQ 3.51 The rms value of the resultant current in a wire which carries a dc current of 10 A and a sinusoidal alternating current of peak value 20 is

(A) 14.1 A (B) 17.3 A

(C) 22.4 A (D) 30.0 A

YEAR 2002 **ONE MARK**

MCQ 3.52 Fourier Series for the waveform, $f(t)$ shown in Figure is



(A) $\frac{8}{\pi^2}\left[\sin(\pi t) + \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$

(B) $\frac{8}{\pi^2}\left[\sin(\pi t) - \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$

(C) $\frac{8}{\pi^2}\left[\cos(\pi t) + \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + ...\right]$

(D) $\frac{8}{\pi^2}\left[\cos(\pi t) - \frac{1}{9}\sin(3\pi t) + \frac{1}{25}\sin(5\pi t) + ...\right]$

MCQ 3.53 Let $s(t)$ be the step response of a linear system with zero initial conditions; then the response of this system to an an input $u(t)$ is

(A) $\int_0^t s(t-\tau)\,u(\tau)\,d\tau$ (B) $\frac{d}{dt}\left[\int_0^t s(t-\tau)\,u(\tau)\,d\tau\right]$

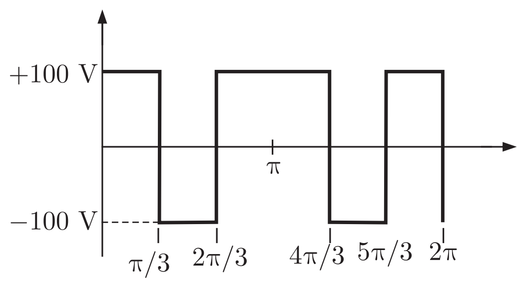
(C) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1)\, d\tau_1\right] d\tau$ (D) $\int_0^1 [s(t-\tau)]^2 u(\tau)\, d\tau$

MCQ 3.54 Let $Y(s)$ be the Laplace transformation of the function $y(t)$, then the final value of the function is

(A) $\lim_{s \to 0} Y(s)$ (B) $\lim_{s \to \infty} Y(s)$

(C) $\lim_{s \to 0} sY(s)$ (D) $\lim_{s \to \infty} sY(s)$

MCQ 3.55 What is the rms value of the voltage waveform shown in Figure ?



(A) $(200/\pi)$ V (B) $(100/\pi)$ V

(C) 200 V (D) 100 V

YEAR 2001 ONE MARK

MCQ 3.56 Given the relationship between the input $u(t)$ and the output $y(t)$ to be

$$y(t) = \int_0^t (2 + t - \tau)\, e^{-3(t-\tau)} u(\tau)\, d\tau,$$

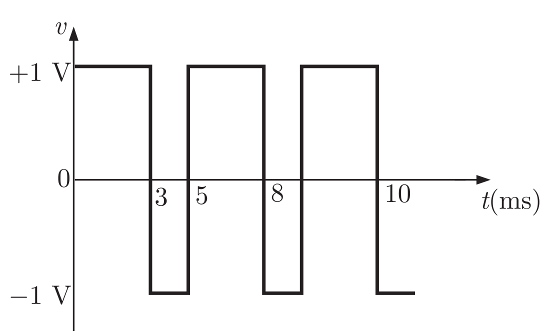
The transfer function $Y(s)/U(s)$ is

(A) $\frac{2e^{-2s}}{s+3}$ (B) $\frac{s+2}{(s+3)^2}$

(C) $\frac{2s+5}{s+3}$ (D) $\frac{2s+7}{(s+3)^2}$

Common data Questions Q.57-58*

Consider the voltage waveform v as shown in figure



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MCQ 3.57 The DC component of v is

(A) 0.4 (B) 0.2

(C) 0.8 (D) 0.1

MCQ 3.58 The amplitude of fundamental component of v is

(A) 1.20 V (B) 2.40 V

(C) 2 V (D) 1 V

****

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SOLUTION

SOL 3.1 Option (C) is correct.

$$x[n] = \left(\frac{1}{3}\right)^{|n|} - \left(\frac{1}{2}\right)^n u[n]$$
$$= \left(\frac{1}{3}\right)^n u[n] + \left(\frac{1}{3}\right)^{-n} u[-n-1] - \left(\frac{1}{2}\right)^n u(n)$$

Taking z-transform

$$X[z] = \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} u[n] + \sum_{n=-\infty}^{\infty}\left(\frac{1}{3}\right)^{-n} z^{-n} u[-n-1]$$
$$-\sum_{n=-\infty}^{\infty}\left(\frac{1}{2}\right)^n z^{-n} u[n] = \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n z^{-n} + \sum_{n=-\infty}^{-1}\left(\frac{1}{3}\right)^{-n} z^{-n} - \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n z^{-n}$$
$$= \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{3z}\right)^n}_{\text{I}} + \underbrace{\sum_{m=1}^{\infty}\left(\frac{1}{3}z\right)^m}_{\text{II}} - \underbrace{\sum_{n=0}^{\infty}\left(\frac{1}{2z}\right)^n}_{\text{III}} \qquad \text{Taking } m = -n$$

Series I converges if $\left|\frac{1}{3z}\right| < 1$ or $|z| > \frac{1}{3}$

Series II converges if $\left|\frac{1}{3}z\right| < 1$ or $|z| < 3$

Series III converges if $\left|\frac{1}{2z}\right| < 1$ or $|z| > \frac{1}{2}$

Region of convergence of $X(z)$ will be intersection of above three

So, $\quad$ ROC : $\frac{1}{2} < |z| < 3$

SOL 3.2 Option (D) is correct.

Using s-domain differentiation property of Laplace transform.

If $\quad f(t) \overset{\mathcal{L}}{\longleftrightarrow} F(s)$

$$tf(t) \overset{\mathcal{L}}{\longleftrightarrow} -\frac{dF(s)}{ds}$$

So, $$\mathcal{L}[tf(t)] = \frac{-d}{ds}\left[\frac{1}{s^2+s+1}\right] = \frac{2s+1}{(s^2+s+1)^2}$$

SOL 3.3 Option (A) is correct.

Convolution sum is defined as

$$y[n] = h[n] * g[n] = \sum_{k=-\infty}^{\infty} h[n]\, g[n-k]$$

For causal sequence, $$y[n] = \sum_{k=0}^{\infty} h[n]\, g[n-k]$$
$$y[n] = h[n]\, g[n] + h[n]\, g[n-1] + h[n]\, g[n-2] + \ldots..$$

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For $n = 0$,
$$y[0] = h[0]\,g[0] + h[1]\,g[-1] + ...$$
$$= h[0]\,g[0] \qquad g[-1] = g[-2] = ...0$$
$$= h[0]\,g[0] \qquad ...(i)$$

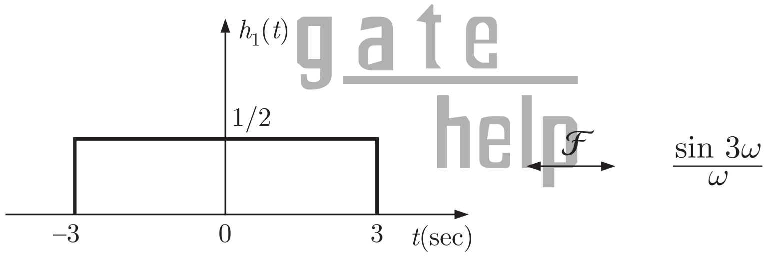
For $n = 1$,
$$y[1] = h[1]\,g[1] + h[1]\,g[0] + h[1]\,g[-1] + ...$$
$$= h[1]\,g[1] + h[1]\,g[0]$$
$$\frac{1}{2} = \frac{1}{2}g[1] + \frac{1}{2}g[0] \qquad h[1] = \left(\frac{1}{2}\right)^1 = \frac{1}{2}$$
$$1 = g[1] + g[0]$$
$$g[1] = 1 - g[0]$$

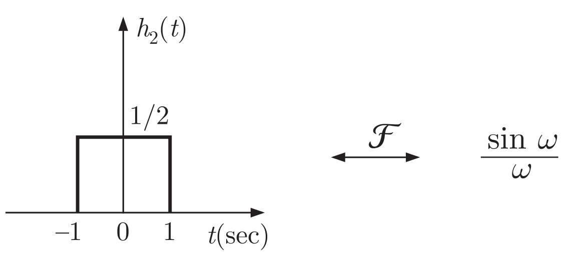
From equation (i),
$$g[0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$$

So,
$$g[1] = 1 - 1 = 0$$

SOL 3.4 Option (C) is correct.
$$H(j\omega) = \frac{(2\cos\omega)(\sin 2\omega)}{\omega} = \frac{\sin 3\omega}{\omega} + \frac{\sin\omega}{\omega}$$

We know that inverse Fourier transform of $\sin c$ function is a rectangular function.





So, inverse Fourier transform of $H(j\omega)$
$$h(t) = h_1(t) + h_2(t)$$
$$h(0) = h_1(0) + h_2(0) = \frac{1}{2} + \frac{1}{2} = 1$$

SOL 3.5 Option (D) is correct.
$$y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)\,d\tau$$

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Time invariance :

Let, $x(t) = \delta(t)$

$$y(t) = \int_{-\infty}^{t} \delta(t)\cos(3\tau)\,d\tau = u(t)\cos(0) = u(t)$$

For a delayed input $(t - t_0)$ output is

$$y(t, t_0) = \int_{-\infty}^{t} \delta(t - t_0)\cos(3\tau)\,d\tau = u(t)\cos(3t_0)$$

Delayed output

$$y(t - t_0) = u(t - t_0)$$

$$y(t, t_0) \neq y(t - t_0)$$

System is not time invariant.

Stability :

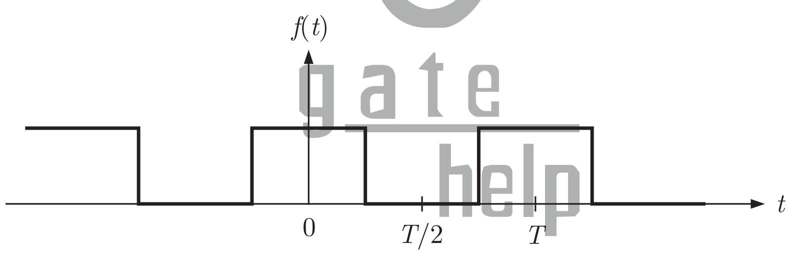
Consider a bounded input $x(t) = \cos 3t$

$$y(t) = \int_{-\infty}^{t} \cos^2 3t = \int_{-\infty}^{t} \frac{1 - \cos 6t}{2} = \frac{1}{2}\int_{-\infty}^{t} 1\,dt - \frac{1}{2}\int_{-\infty}^{t} \cos 6t\,dt$$

As $t \to \infty$, $y(t) \to \infty$ (unbounded)

System is not stable.

SOL 3.6 Option (D) is correct.



$$f(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos \omega t + b_n \sin n\omega t)$$

- The given function $f(t)$ is an even function, therefore $b_n = 0$
- $f(t)$ is a non zero average value function, so it will have a non-zero value of a_0

$$a_0 = \frac{1}{(T/2)}\int_0^{T/2} f(t)\,dt \text{ (average value of } f(t))$$

- a_n is zero for all even values of n and non zero for odd n

$$a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega t)\,d(\omega t)$$

So, Fourier expansion of $f(t)$ will have a_0 and a_n, $n = 1, 3, 5 \dots \infty$

SOL 3.7 Option (A) is correct.

$$x(t) = e^{-t}$$

Laplace transformation

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$$X(s) = \frac{1}{s+1}$$

$$y(t) = e^{-2t}$$

$$Y(s) = \frac{1}{s+2}$$

Convolution in time domain is equivalent to multiplication in frequency domain.

$$z(t) = x(t) * y(t)$$

$$Z(s) = X(s)\,Y(s) = \left(\frac{1}{s+1}\right)\left(\frac{1}{s+2}\right)$$

By partial fraction and taking inverse Laplace transformation, we get

$$Z(s) = \frac{1}{s+1} - \frac{1}{s+2}$$

$$z(t) = e^{-t} - e^{-2t}$$

SOL 3.8 Option (D) is correct.

$$f(t) \xleftrightarrow{\mathcal{L}} F_1(s)$$

$$f(t-\tau) \xleftrightarrow{\mathcal{L}} e^{-s\tau} F_1(s) = F_2(s)$$

$$G(s) = \frac{F_2(s)\,F_1^*(s)}{|F_1(s)|^2} = \frac{e^{-s\tau} F_1(s)\,F_1^*(s)}{|F_1(s)|^2}$$

$$= \frac{e^{-sE}|F_1(s)|^2}{|F_1(s)|^2} \qquad \{\because F_1(s)\,F_1^*(s) = |F_1(s)|^2$$

$$= e^{-s\tau}$$

Taking inverse Laplace transform

$$g(t) = \mathcal{L}^{-1}[e^{-s\tau}] = \delta(t-\tau)$$

SOL 3.9 Option (C) is correct.

$$h(t) = e^{-t} + e^{-2t}$$

Laplace transform of $h(t)$ i.e. the transfer function

$$H(s) = \frac{1}{s+1} + \frac{1}{s+2}$$

For unit step input

$$r(t) = \mu(t)$$

or $R(s) = \frac{1}{s}$

Output, $Y(s) = R(s)\,H(s) = \frac{1}{s}\left[\frac{1}{s+1} + \frac{1}{s+2}\right]$

By partial fraction

$$Y(s) = \frac{3}{2s} - \frac{1}{s+1} - \left(\frac{1}{s+2}\right)\frac{1}{2}$$

Taking inverse Laplace

$$y(t) = \frac{3}{2}u(t) - e^{-t}u(t) - \frac{e^{-2t}u(t)}{2}$$
$$= u(t)[1.5 - e^{-t} - 0.5e^{-2t}]$$

SOL 3.10 Option (C) is correct.

System is given as

$$H(s) = \frac{2}{(s+1)}$$

Step input $R(s) = \frac{1}{s}$

Output $Y(s) = H(s)R(s) = \frac{2}{(s+1)}\left(\frac{1}{s}\right) = \frac{2}{s} - \frac{2}{(s+1)}$

Taking inverse Laplace transform

$$y(t) = (2 - 2e^{-t})u(t)$$

Final value of $y(t)$,

$$y_{ss}(t) = \lim_{t \to \infty} y(t) = 2$$

Let time taken for step response to reach 98% of its final value is t_s.

So,

$$2 - 2e^{-t_s} = 2 \times 0.98$$
$$0.02 = e^{-t_s}$$
$$t_s = \ln 50 = 3.91 \text{ sec.}$$

SOL 3.11 Option (D) is correct.

Period of $x(t)$,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.8\pi} = 2.5 \text{ sec}$$

SOL 3.12 Option (B) is correct.

Input output relationship

$$y(t) = \int_{-\infty}^{5t} x(\tau)\,d\tau, \qquad t > 0$$

Causality :

$y(t)$ depends on $x(5t), t > 0$ system is non-causal.

For example $t = 2$

$y(2)$ depends on $x(10)$ (future value of input)

Linearity :

Output is integration of input which is a linear function, so system is linear.

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SOL 3.13 Option (A) is correct.

Fourier series of given function

$$x(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$\because x(t) = - x(t) \text{ odd function}$$

So, $A_0 = 0$

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_0^T x(t) \sin n\omega_0 t\, dt$$

$$= \frac{2}{T}\left[\int_0^{T/2} (1) \sin n\omega_0 t\, dt + \int_{T/2}^{T} (-1) \sin n\omega_0 t\, dt\right]$$

$$= \frac{2}{T}\left[\left(\frac{\cos n\omega_0 t}{-n\omega_0}\right)_0^{T/2} - \left(\frac{\cos n\omega_0 t}{-n\omega_0}\right)_{T/2}^{T}\right]$$

$$= \frac{2}{n\omega_0 T}[(1 - \cos n\pi) + (\cos 2n\pi - \cos n\pi)]$$

$$= \frac{2}{n\pi}[1 - (-1)^n]$$

$$b_n = \begin{cases} \frac{4}{n\pi}, & n \text{ odd} \\ 0 \quad, & n \text{ even} \end{cases}$$

So only odd harmonic will be present in $x(t)$

For second harmonic component $(n = 2)$ amplitude is zero.

SOL 3.14 Option (D) is correct.

By parsval's theorem

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega = 2\pi \times 2 = 4\pi$$

SOL 3.15 Option (C) is correct.

Given sequences $x[n] = \{\underset{\uparrow}{1}, -1\}, \quad 0 \le n \le 1$

$$y[n] = \{\underset{\uparrow}{1}, 0, 0, 0, -1\}, \quad 0 \le n \le 4$$

If impulse response is $h[n]$ then

$$y[n] = h[n] * x[n]$$

Length of convolution $(y[n])$ is 0 to 4, $x[n]$ is of length 0 to 1 so length of $h[n]$ will be 0 to 3.

Let $h[n] = \{\underset{\uparrow}{a}, b, c, d\}$

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Convolution

	a	b	c	c
1	a	b	c	d
-1	$-a$	$-b$	$-c$	$-d$

$$y[n] = \{\underset{\uparrow}{a}, -a+b, -b+c, -c+d, -d\}$$

By comparing

$$\begin{aligned} a &= 1 \\ -a+b &= 0 \quad \Rightarrow b = a = 1 \\ -b+c &= 0 \quad \Rightarrow c = b = 1 \\ -c+d &= 0 \quad \Rightarrow d = c = 1 \end{aligned}$$

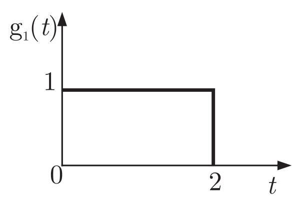
So, $$h[n] = \{\underset{\uparrow}{1}, 1, 1, 1\}$$

SOL 3.16 Option (D) is correct.

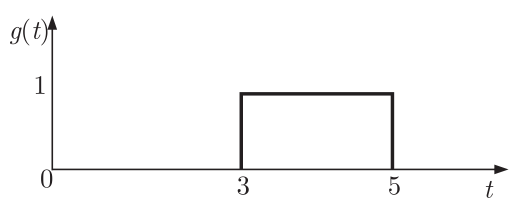
We can observe that if we scale $f(t)$ by a factor of $\frac{1}{2}$ and then shift, we will get $g(t)$.

First scale $f(t)$ by a factor of $\frac{1}{2}$

$$g_1(t) = f(t/2)$$



Shift $g_1(t)$ by 3, $\quad g(t) = g_1(t-3) = f\left(\frac{t-3}{2}\right)$



$$g(t) = f\left(\frac{t}{2} - \frac{3}{2}\right)$$

SOL 3.17 Option (C) is correct.

$g(t)$ can be expressed as

$$g(t) = u(t-3) - u(t-5)$$

By shifting property we can write Laplace transform of $g(t)$

$$G(s) = \frac{1}{s}e^{-3s} - \frac{1}{s}e^{-5s} = \frac{e^{-3s}}{s}(1-e^{-2s})$$

SOL 3.18 Option (D) is correct.

Let $x(t) \xleftrightarrow{\mathcal{L}} X(s)$

$y(t) \xleftrightarrow{\mathcal{L}} Y(s)$

$h(t) \xleftrightarrow{\mathcal{L}} H(s)$

So output of the system is given as

$$Y(s) = X(s)H(s)$$

Now for input $x(t-\tau) \xleftrightarrow{\mathcal{L}} e^{-s\tau}X(s)$ (shifting property)

$h(t-\tau) \xleftrightarrow{\mathcal{L}} e^{-s\tau}H(s)$

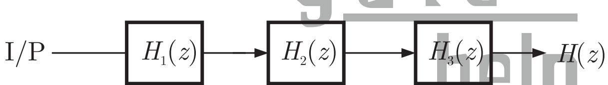
So now output is $Y'(s) = e^{-s\tau}X(s) \cdot e^{-\tau s}H(s)$

$$= e^{-2s\tau}X(s)H(s) = e^{-2s\tau}Y(s)$$

$$y'(t) = y(t-2\tau)$$

SOL 3.19 Option (B) is correct.

Let three LTI systems having response $H_1(z), H_2(z)$ and $H_3(z)$ are Cascaded as showing below



Assume $H_1(z) = z^2 + z^1 + 1$ (non-causal)

$H_2(z) = z^3 + z^2 + 1$ (non-causal)

Overall response of the system

$$H(z) = H_1(z)H_2(z)H_3(z)$$

$$H(z) = (z^2 + z^1 + 1)(z^3 + z^2 + 1)H_3(z)$$

To make $H(z)$ causal we have to take $H_3(z)$ also causal.

Let $H_3(z) = z^{-6} + z^{-4} + 1$

$$= (z^2 + z^1 + 1)(z^3 + z^2 + 1)(z^{-6} + z^{-4} + 1)$$

$$H(z) \to \text{causal}$$

Similarly to make $H(z)$ unstable atleast one of the system should be unstable.

SOL 3.20 Option (C) is correct.

Given signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j2\pi kt/T}$$

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Let ω_0 is the fundamental frequency of signal $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \because \frac{2\pi}{T} = \omega_0$$

$$\begin{aligned} x(t) &= a_{-2} e^{-j2\omega_0 t} + a_{-1} e^{-j\omega_0 t} + a_0 + a_1 e^{j\omega_0 t} + a_2 e^{j2\omega_0 t} \\ &= (2-j)\, e^{-2j\omega_0 t} + (0.5+0.2j)\, e^{-j\omega_0 t} + 2j + \\ &\qquad + (0.5-0.2)\, e^{j\omega_0 t} + (2+j)\, e^{j2\omega_0 t} \\ &= 2[e^{-j2\omega_0 t} + e^{j2\omega_0 t}] + j[e^{j2\omega_0 t} - e^{-j2\omega_0 t}] + \\ &\qquad 0.5[e^{j\omega_0 t} + e^{-j\omega_0 t}] - 0.2j[e^{+j\omega_0 t} - e^{-j\omega_0 t}] + 2j \\ &= 2(2\cos 2\omega_0 t) + j(2j\sin 2\omega_0 t) + 0.5(2\cos\omega_0 t) - \\ &\qquad 0.2j(2j\sin\omega_0 t) + 2j \\ &= [4\cos 2\omega_0 t - 2\sin 2\omega_0 t + \cos\omega_0 t + 0.4\sin\omega_0 t] + 2j \end{aligned}$$

$$\mathrm{Im}[x(t)] = 2 \quad \text{(constant)}$$

SOL 3.21 Option (A) is correct.

Z-transform of $x[n]$ is

$$X(z) = 4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3$$

Transfer function of the system

$$H(z) = 3z^{-1} - 2$$

Output

$$Y(z) = H(z)X(z)$$

$$\begin{aligned} Y(z) &= (3z^{-1} - 2)(4z^{-3} + 3z^{-1} + 2 - 6z^2 + 2z^3) \\ &= 12z^{-4} + 9z^{-2} + 6z^{-1} - 18z + 6z^2 - 8z^{-3} - 6z^{-1} - 4 + 12z^2 - 4z^3 \\ &= 12z^{-4} - 8z^{-3} + 9z^{-2} - 4 - 18z + 18z^2 - 4z^3 \end{aligned}$$

Or sequence $y[n]$ is

$$y[n] = 12\delta[n-4] - 8\delta[n-3] + 9\delta[n-2] - 4\delta[n] - 18\delta[n+1] + 18\delta[n+2] - 4\delta[n+3]$$

$$y[n] \neq 0,\ n < 0$$

So $y[n]$ is non-causal with finite support.

SOL 3.22 Option (D) is correct.

Since the given system is LTI, So principal of Superposition holds due to linearity.

For causal system $h(t) = 0,\ t < 0$

Both statement are correct.

SOL 3.23 Option (C) is correct.

For an LTI system output is a constant multiplicative of input with same frequency.

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Here input $g(t) = e^{-\alpha t}\sin(\omega t)$

output $y(t) = \mathrm{K}e^{-\beta t}\sin(vt + \phi)$

Output will be in form of $\mathrm{K}e^{-\alpha t}\sin(\omega t + \phi)$

So $\alpha = \beta, v = \omega$

SOL 3.24 Option (D) is correct.

Input-output relation

$$y(t) = \int_{-\infty}^{-2t} x(\tau)\, d\tau$$

Causality :

Since $y(t)$ depends on $x(-2t)$, So it is non-causal.

Time-variance :

$$y(t) = \int_{-\infty}^{-2t} x(\tau - \tau_0)\, d\tau \neq y(t - \tau_0)$$

So this is time-variant.

Stability :

Output $y(t)$ is unbounded for an bounded input.

For example

Let $x(\tau) = e^{-\tau}$ (bounded)

$$y(t) = \int_{-\infty}^{-2t} e^{-\tau}\, d\tau = \left[\frac{e^{-\tau}}{-1}\right]_{-\infty}^{-2t} \longrightarrow \text{Unbounded}$$

SOL 3.25 Option (A) is correct.

Output $y(t)$ of the given system is

$$y(t) = x(t) * h(t)$$

Or $$Y(j\omega) = X(j\omega)\, H(j\omega)$$

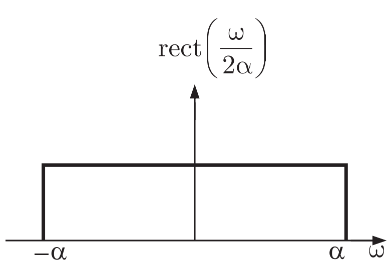
Given that, $x(t) = \operatorname{sinc}(\alpha t)$ and $h(t) = \operatorname{sinc}(\beta t)$

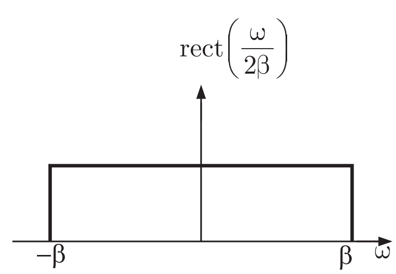
Fourier transform of $x(t)$ and $h(t)$ are

$$X(j\omega) = \mathcal{F}[x(t)] = \frac{\pi}{\alpha} rect\left(\frac{\omega}{2\alpha}\right), -\alpha < \omega < \alpha$$

$$H(j\omega) = \mathcal{F}[h(t)] = \frac{\pi}{\beta} rect\left(\frac{\omega}{2\beta}\right), -\beta < \omega < \beta$$

$$Y(j\omega) = \frac{\pi^2}{\alpha\beta} \mathrm{rect}\left(\frac{\omega}{2\alpha}\right) \mathrm{rect}\left(\frac{\omega}{2\beta}\right)$$





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So, $Y(j\omega) = \text{K rect}\left(\frac{\omega}{2\gamma}\right)$

Where $\gamma = \min(\alpha, \beta)$

And $y(t) = \text{K sinc}(\gamma t)$

SOL 3.26 Option (B) is correct.

Let a_k is the Fourier series coefficient of signal $x(t)$

Given $y(t) = x(t - t_0) + x(t + t_0)$

Fourier series coefficient of $y(t)$

$$b_k = e^{-jk\omega t_0} a_k + e^{jk\omega t_0} a_k$$

$$b_k = 2a_k \cos k\omega t_0$$

$$b_k = 0 \text{ (for all odd } k)$$

$$k\omega t_0 = \frac{\pi}{2}, \text{ k} \rightarrow \text{odd}$$

$$k\frac{2\pi}{T} t_0 = \frac{\pi}{2}$$

For $k = 1$, $t_0 = \frac{T}{4}$

SOL 3.27 Option ( ) is correct.

SOL 3.28 Option (D) is correct.

Given that $X(z) = \frac{z}{(z-a)^2}, \ |z| > a$

Residue of $X(z) z^{n-1}$ at $z = a$ is

$$= \frac{d}{dz}(z-a)^2 X(z) z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz}(z-a)^2 \frac{z}{(z-a)^2} z^{n-1}\Big|_{z=a}$$

$$= \frac{d}{dz} z^n\Big|_{z=a} = nz^{n-1}\Big|_{z=a} = na^{n-1}$$

SOL 3.29 Option (C) is correct.

Given signal

$$x(t) = \text{rect}\left(t - \frac{1}{2}\right)$$

So, $x(t) = \begin{cases} 1, & -\frac{1}{2} \le t - \frac{1}{2} \le \frac{1}{2} \quad \text{or} \quad 0 \le t \le 1 \\ 0, & \text{elsewhere} \end{cases}$

Similarly

$$x(-t) = \text{rect}\left(-t - \frac{1}{2}\right)$$

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$$x(-t) = \begin{cases} 1, & -\frac{1}{2} \le -t - \frac{1}{2} \le \frac{1}{2} \quad \text{or} \quad -1 \le t \le 0 \\ 0, & \text{elsewhere} \end{cases}$$

$$\mathcal{F}[x(t) + x(-t)] = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt + \int_{-\infty}^{\infty} x(-t)\, e^{-j\omega t}\, dt$$

$$= \int_0^1 (1)\, e^{-j\omega t}\, dt + \int_{-1}^0 (1)\, e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{-j\omega t}}{-j\omega}\right]_0^1 + \left[\frac{e^{-j\omega t}}{-j\omega}\right]_{-1}^0 = \frac{1}{j\omega}(1 - e^{-j\omega}) + \frac{1}{j\omega}(e^{j\omega} - 1)$$

$$= \frac{e^{-j\omega/2}}{j\omega}(e^{j\omega/2} - e^{-j\omega/2}) + \frac{e^{j\omega/2}}{j\omega}(e^{j\omega/2} - e^{-j\omega/2})$$

$$= \frac{(e^{j\omega/2} - e^{-j\omega/2})(e^{-j\omega/2} + e^{j\omega/2})}{j\omega}$$

$$= \frac{2}{\omega}\sin\left(\frac{\omega}{2}\right) \cdot 2\cos\left(\frac{\omega}{2}\right) = 2\cos\frac{\omega}{2}\operatorname{sinc}\left(\frac{\omega}{2\pi}\right)$$

SOL 3.30 Option (B) is correct.

In option (A)

$$z_1[n] = x[n-3]$$
$$z_2[n] = z_1[4n] = x[4n-3]$$
$$y[n] = z_2[-n] = x[-4n-3] \neq x[3-4n]$$

In option (B)

$$z_1[n] = x[n+3]$$
$$z_2[n] = z_1[4n] = x[4n+3]$$
$$y[n] = z_2[-n] = x[-4n+3]$$

In option (C)

$$v_1[n] = x[4n]$$
$$v_2[n] = v_1[-n] = x[-4n]$$
$$y[n] = v_2[n+3] = x[-4(n+3)] \neq x[3-4n]$$

In option (D)

$$v_1[n] = x[4n]$$
$$v_2[n] = v_1[-n] = x[-4n]$$
$$y[n] = v_2[n-3] = x[-4(n-3)] \neq x[3-4n]$$

SOL 3.31 Option ( ) is correct.

The spectrum of sampled signal $s(j\omega)$ contains replicas of $U(j\omega)$ at frequencies $\pm nf_s$.

Where $n = 0, 1, 2.......$

$$f_s = \frac{1}{T_s} = \frac{1}{1\,\text{m sec}} = 1\,\text{kHz}$$

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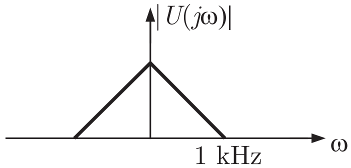
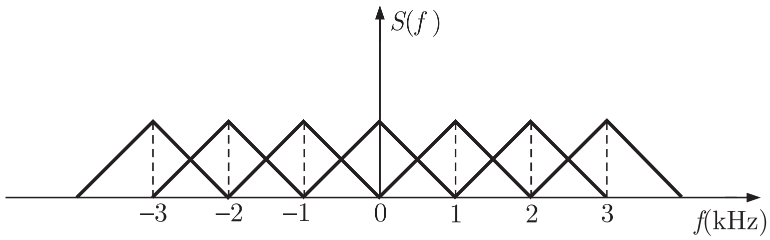
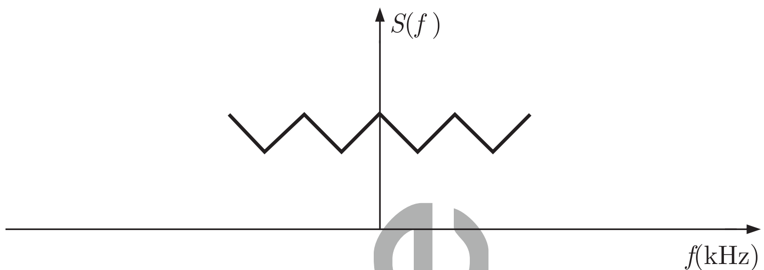






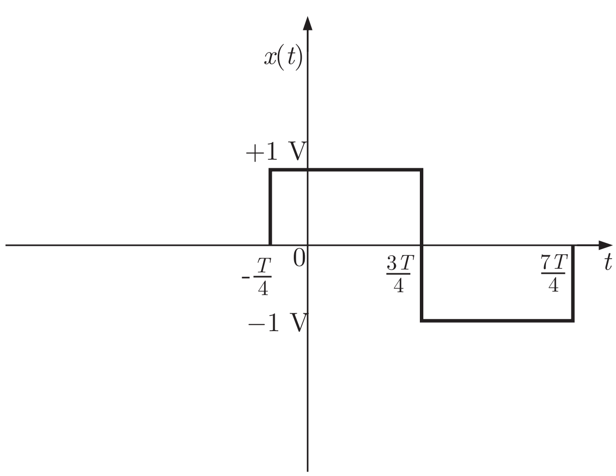
SOL 3.32 Option (D) is correct.

For an LTI system input and output have identical wave shape (i.e. frequency of input-output is same) within a multiplicative constant (i.e. Amplitude response is constant)

So F must be a sine or cosine wave with $\omega_1 = \omega_2$

SOL 3.33 Option (C) is correct.

Given signal has the following wave-form



Function x(t) is periodic with period $2T$ and given that

$$x(t) = -x(t+T) \text{ (Half-wave symmetric)}$$

So we can obtain the fourier series representation of given function.

SOL 3.34 Option (C) is correct.

Output is said to be distortion less if the input and output have identical wave shapes within a multiplicative constant. A delayed output that retains input waveform is also considered distortion less.

Thus for distortion less output, input-output relationship is given as

$$y(t) = Kg(t - t_d)$$

Taking Fourier transform.

$$Y(\omega) = KG(\omega)\, e^{-j\omega t_d} = G(\omega)\, H(\omega)$$

$H(\omega) \Rightarrow$ transfer function of the system

So, $$H(\omega) = Ke^{-j\omega t_d}$$

Amplitude response $|H(\omega)| = K$

Phase response, $\theta_n(\omega) = -\omega t_d$

For distortion less output, phase response should be proportional to frequency.

SOL 3.35 Option (A) is correct.

$$G(z)\big|_{z=e^{j\omega}} = \alpha e^{-j\omega} + \beta e^{-3j\omega}$$

for linear phase characteristic $\alpha = \beta$.

SOL 3.36 Option (A) is correct.

System response is given as

$$H(z) = \frac{G(z)}{1 - KG(z)}$$

$$g[n] = \delta[n-1] + \delta[n-2]$$

$$G(z) = z^{-1} + z^{-2}$$

So $$H(z) = \frac{(z^{-1} + z^{-2})}{1 - K(z^{-1} + z^{-2})} = \frac{z+1}{z^2 - Kz - K}$$

For system to be stable poles should lie inside unit circle.

$$|z| \le 1$$

$$z = \frac{K \pm \sqrt{K^2 + 4K}}{2} \le 1 \quad K \pm \sqrt{K^2 + 4K} \le 2$$

$$\sqrt{K^2 + 4K} \le 2 - K$$

$$K^2 + 4K \le 4 - 4K + K^2$$

$$8K \le 4$$

$$K \le 1/2$$

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SOL 3.37 Option (C) is correct.

Given Convolution is,

$$h(t) = u(t+1) * r(t-2)$$

Taking Laplace transform on both sides,

$$H(s) = \mathcal{L}[h(t)] = \mathcal{L}[u(t+1)] * \mathcal{L}[r(t-2)]$$

We know that, $\mathcal{L}[u(t)] = 1/s$

$$\mathcal{L}[u(t+1)] = e^{s}\left(\frac{1}{s^2}\right) \qquad \text{(Time-shifting property)}$$

and

$$\mathcal{L}[r(t)] = 1/s^2$$

$$\mathcal{L}\, r(t-2) = e^{-2s}\left(\frac{1}{s^2}\right) \qquad \text{(Time-shifting property)}$$

So

$$H(s) = \left[e^{s}\left(\frac{1}{s}\right)\right]\left[e^{-2s}\left(\frac{1}{s^2}\right)\right]$$

$$H(s) = e^{-s}\left(\frac{1}{s^3}\right)$$

Taking inverse Laplace transform

$$h(t) = \frac{1}{2}(t-1)^2 u(t-1)$$

SOL 3.38 Option (C) is correct.

Impulse response of given LTI system.

$$h[n] = x[n-1] * y[n]$$

Taking z-transform on both sides.

$$H(z) = z^{-1}X(z)\,Y(z) \qquad \because \; x[n-1] \xleftrightarrow{\mathcal{Z}} z^{-1}x(z)$$

We have $X(z) = 1 - 3z^{-1}$ and $Y(z) = 1 + 2z^{-2}$

So

$$H(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})$$

Output of the system for input $u[n] = \delta[n-1]$ is,

$$y(z) = H(z)\,U(z) \qquad U[n] \xleftrightarrow{\mathcal{Z}} U(z) = z^{-1}$$

So

$$Y(z) = z^{-1}(1-3z^{-1})(1+2z^{-2})\,z^{-1}$$

$$= z^{-2}(1-3z^{-1}+2z^{-2}-6z^{-3}) = z^{-2}-3z^{-3}+2z^{-4}-6z^{-5}$$

Taking inverse z-transform on both sides we have output.

$$y[n] = \delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - 6\delta[n-5]$$

SOL 3.39 Option (B) is correct.

A bounded signal always possesses some finite energy.

$$E = \int_{-t_0}^{t_0} |g(t)^2|\,dt < \infty$$

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SOL 3.40 Option (C) is correct.

Trigonometric Fourier series is given as

$$x(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

Since there are no sine terms, so $b_n = 0$

$$b_n = \frac{2}{T_0}\int_0^{T_0} x(t) \sin n\omega_0 t \, dt$$

$$= \frac{2}{T_0}\left[\int_0^{T_0/2} x(\tau) \sin n\omega_0\tau \; d\tau + \int_{T_0/2}^{T} x(t) \sin n\omega_0 t \, dt\right]$$

Where $\tau = T - t \;\; \Rightarrow d\tau = -\, dt$

$$= \frac{2}{T_0}\left[\int_{T_0}^{T_0/2} x(T-t) \sin n\omega_0 (T-t)\,(-\,dt) + \int_{T_0/2}^{T} x(t) \sin n\omega_0 t \, dt\right]$$

$$= \frac{2}{T_0}\left[\int_{T_0/2}^{T_0} x(T-t) \sin n\left(\frac{2\pi}{T}T - t\right) dt + + \int_{T_0/2}^{T} x(t) \sin n\omega_0 t \, dt\right]$$

$$= \frac{2}{T_0}\left[\int_{T_0/2}^{T_0} x(T-t) \sin(2n\pi - n\omega_0)\, dt + \int_{T_0/2}^{T_0} x(t) \sin n\omega_0 t \, dt\right]$$

$$= \frac{2}{T_0}\left[-\int_{T_0/2}^{T_0} x(T-t) \sin(n\omega_0 t)\, dt + + \int_{T_0/2}^{T_0} x(t) \sin n\omega_0 t \, dt\right]$$

$b_n = 0$ if $x(t) = x(T-t)$

From half wave symmetry we know that if

$$x(t) = -\,x\left(t \pm \frac{T}{2}\right)$$

Then Fourier series of $x(t)$ contains only odd harmonics.

SOL 3.41 Option (C) is correct.

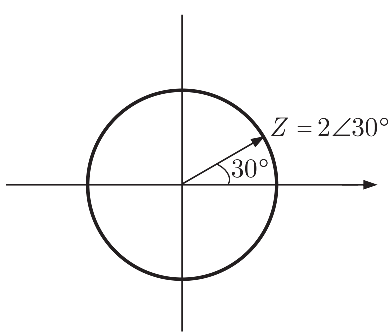
Z-transform of a discrete all pass system is given as

$$H(z) = \frac{z^{-1} - z_0^*}{1 - z_0 z^{-1}}$$

It has a pole at z_0 and a zero at $1/z_0^*$.

Given system has a pole at

$$z = 2\angle 30^\circ = 2\frac{(\sqrt{3} + j)}{2} = (\sqrt{3} + j)$$



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system is stable if $|z| < 1$ and for this it is anti-causal.

SOL 3.42 Option (A) is correct.

According to given data input and output Sequences are

$$x[n] = \{-1,\underset{\uparrow}{2}\}, -1 \le n \le 0$$

$$y[n] = \{-1,\underset{\uparrow}{3}, -1, -2\}, -1 \le n \le 2$$

If impulse response of system is $h[n]$ then output

$$y[n] = h[n] * x[n]$$

Since length of convolution $(y[n])$ is -1 to 2, $x[n]$ is of length -1 to 0 so length of $h[n]$ is 0 to 2.

Let $\quad h[n] = \{\underset{\uparrow}{a}, b, c\}$

Convolution

	a	b	c
-1	$-a$	$-b$	$-c$
2	$2a$	$2b$	$2c$

$$y[n] = \{-a, \underset{\uparrow}{2a-b}, 2b-c, 2c\}$$

$$y[n] = \{-1,\underset{\uparrow}{3}, -1, -2\}$$

So,

$a = 1$

$$2a - b = 3 \ \Rightarrow b = -1$$

$$2a - c = -1 \ \Rightarrow c = -1$$

Impulse response $h[n] = \{\underset{\uparrow}{1}, -1, -1\}$

SOL 3.43 Option ( ) is correct.

SOL 3.44 Option (D) is correct.

Output $\quad y(t) = e^{-|x(t)|}$

If $x(t)$ is unbounded, $|x(t)| \rightarrow \infty$

$$y(t) = e^{-|x(t)|} \rightarrow 0 \quad \text{(bounded)}$$

So $y(t)$ is bounded even when $x(t)$ is not bounded.

SOL 3.45 Option (B) is correct.

Given $\quad y(t) = \int_{-\infty}^{t} x(t')\,dt'$

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Laplace transform of $y(t)$

$$Y(s) = \frac{X(s)}{s}, \text{ has a singularity at } s = 0$$

For a causal bounded input, $y(t) = \int_{-\infty}^{t} x(t')\,dt'$ is always bounded.

SOL 3.46 Option (A) is correct.

RMS value is given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

Where

$$V(t) = \begin{cases} \left(\frac{2}{T}\right)t, & 0 \le t \le \frac{T}{2} \\ 0, & \frac{T}{2} < t \le T \end{cases}$$

So
$$\frac{1}{T}\int_0^T V^2(t)\,dt = \frac{1}{T}\left[\int_0^{T/2}\left(\frac{2t}{T}\right)^2 dt + \int_{T/2}^{T}(0)\,dt\right]$$

$$= \frac{1}{T}\cdot\frac{4}{T^2}\int_0^{T/2} t^2\,dt = \frac{4}{T^3}\left[\frac{t^3}{3}\right]_0^{T/2}$$

$$= \frac{4}{T^3}\times\frac{T^3}{24} = \frac{1}{6}$$

$$V_{rms} = \sqrt{\frac{1}{6}}\ \text{V}$$

SOL 3.47 Option (A) is correct.

By final value theorem

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} s\,F(s) = \lim_{s\to 0} s\frac{(5s^2+23s+6)}{s(s^2+2s+2)}$$

$$= \frac{6}{2} = 3$$

SOL 3.48 Option (D) is correct.

$$f(x) = \sin^2 x = \frac{1-\cos 2x}{2}$$

$$= 0.5 - 0.5\ \cos 2x$$

$$f(x) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 x + b_n \sin n\omega_0 x$$

$f(x) = \sin^2 x$ is an even function so $b_n = 0$

$$A_0 = 0.5$$

$$a_n = \begin{cases} -0.5, & n = 1 \\ 0, & \text{otherwise} \end{cases}$$

$$\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{T} = 2$$

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SOL 3.49 Option (B) is correct.

Z-transform $F(z) = \frac{1}{z+1} = 1 - \frac{z}{z+1} = 1 - \frac{1}{1+z^{-1}}$

so, $f(k) = \delta(k) - (-1)^k$

Thus $(-1)^k \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{1+z^{-1}}$

SOL 3.50 Option (A) is correct.

Root mean square value is given as

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T I^2(t)\,dt}$$

From the graph, $I(t) = \begin{cases} -\left(\frac{12}{T}\right)t, & 0 \le t < \frac{T}{2} \\ 6, & T/2 < t \le T \end{cases}$

So $\frac{1}{T}\int_0^T I^2\,dt = \frac{1}{T}\left[\int_0^{T/2}\left(\frac{-12t}{T}\right)^2 dt + \int_{T/2}^T (6)^2\,dt\right]$

$$= \frac{1}{T}\left(\frac{144}{T^2}\left[\frac{t^3}{3}\right]_0^{T/2} + 36[t]_{T/2}^T\right)$$

$$= \frac{1}{T}\left[\frac{144}{T^2}\left(\frac{T^3}{24}\right) + 36\left(\frac{T}{2}\right)\right] = \frac{1}{T}[6T + 18T] = 24$$

$$I_{rms} = \sqrt{24} = 2\sqrt{6}\text{ A}$$

SOL 3.51 Option (B) is correct.

Total current in wire

$$I = 10 + 20\sin\omega t$$

$$I_{rms} = \sqrt{(10)^2 + \frac{(20)^2}{2}} = 17.32\text{ A}$$

SOL 3.52 Option (C) is correct.

Fourier series representation is given as

$$f(t) = A_0 + \sum_{n=1}^{\infty} a_n\cos n\omega_0 t + b_n\sin n\omega_0 t$$

From the wave form we can write fundamental period T = 2 sec

$$f(t) = \begin{cases} \left(\frac{4}{T}\right)t, & -\frac{T}{2} \le t \le 0 \\ -\left(\frac{4}{T}\right)t, & 0 \le t \le \frac{T}{2} \end{cases}$$

$f(t) = f(-t)$, $f(t)$ is an even function

So, $b_n = 0$

$$A_0 = \frac{1}{T}\int_T f(t)\,dt$$

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$$= \frac{1}{T}\left[\int_{-T/2}^{0}\left(\frac{4}{T}\right)tdt + \int_{0}^{T/2}\left(-\frac{4}{T}\right)tdt\right]$$

$$= \frac{1}{T}\left(\frac{4}{T}\left[\frac{t^2}{2}\right]_{-T/2}^{0} - \frac{4}{T}\left[\frac{t^2}{2}\right]_{0}^{T/2}\right)$$

$$= \frac{1}{T}\left[\frac{4}{T}\left(\frac{T^2}{8}\right) - \frac{4}{T}\left(\frac{T^2}{8}\right)\right] = 0$$

$$a_n = \frac{2}{T}\int_T f(t)\cos n\omega_0 t\, dt$$

$$= \frac{2}{T}\left[\int_{-T/2}^{0}\left(\frac{4}{T}\right)t\cos n\omega_0 t + \int_{0}^{T/2}\left(-\frac{4}{T}\right)t\cos n\omega_0 tdt\right]$$

By solving the integration

$$a_n = \begin{cases}\frac{8}{n^2\pi^2}, & n \text{ is odd}\\ 0, & n \text{ is even}\end{cases}$$

So,

$$f(t) = \frac{8}{\pi^2}\left[\cos \pi t + \frac{1}{9}\cos(3\pi t) + \frac{1}{25}\cos(5\pi t) + ...\right]$$

SOL 3.53 Option (A) is correct.

Response for any input $u(t)$ is given as

$$y(t) = u(t) * h(t) \qquad h(t) \to \text{impulse response}$$

$$y(t) = \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$

Impulse response $h(t)$and step response $s(t)$ of a system is related as

$$h(t) = \frac{d}{dt}[s(t)]$$

So $$y(t) = \int_{-\infty}^{\infty} u(\tau)\frac{d}{dt}s[t-\tau]\,d\tau = \frac{d}{dt}\int_{-\infty}^{\infty} u(\tau)\,s(t-\tau)\,d\tau$$

SOL 3.54 Option (B) is correct.

Final value theorem states that

$$\lim_{t\to\infty} y(t) \;\; \lim_{s\to\infty} Y(s)$$

SOL 3.55 Option (D) is correct.

$$V_{rms} = \sqrt{\frac{1}{T_0}\int_{T_0} V^2(t)\, dt}$$

here $T_0 = \pi$

$$\frac{1}{T_0}\int_{T_0} V^2(t)\,dt = \frac{1}{\pi}\left[\int_0^{\pi/3}(100)^2\,dt + \int_{\pi/3}^{2\pi/3}(-100)^2\,dt + \int_{2\pi/3}^{\pi}(100)^2\,dt\right]$$

$$= \frac{1}{\pi}\left[10^4\left(\frac{\pi}{3}\right) + 10^4\left(\frac{\pi}{3}\right) + 10^4\left(\frac{\pi}{3}\right)\right] = 10^4 \text{ V}$$

$$V_{rms} = \sqrt{10^4} = 100 \text{ V}$$

SOL 3.56 Option (D) is correct.

Let $h(t)$ is the impulse response of system

$$y(t) = u(t) * h(t)$$

$$y(t) = \int_0^t u(\tau)\,h(t-\tau)\,d\tau$$

$$= \int_0^t (2+t-\tau)\,e^{-3(t-\tau)}u(\tau)\,d\tau$$

So $\quad h(t) = (t+2)\,e^{-3t}u(t), t > 0$

Transfer function

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{(s+3)^2} + \frac{2}{(s+3)}$$

$$= \frac{1+2s+6}{(s+3)^2} = \frac{(2s+7)}{(s+3)^2}$$

SOL 3.57 Option (B) is correct.

Fourier series representation is given as

$$v(t) = A_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

period of given wave form $T = 5$ ms

DC component of v is

$$A_0 = \frac{1}{T}\int_T v(t)\,dt$$

$$= \frac{1}{5}\left[\int_0^3 1\,dt + \int_3^5 -1\,dt\right]$$

$$= \frac{1}{5}[3-5+3] = \frac{1}{5}$$

SOL 3.58 Option (A) is correct.

Coefficient, $a_n = \dfrac{2}{T}\displaystyle\int_T v(t)\cos n\omega_0 t\,dt$

$$= \frac{2}{5}\left[\int_0^3 (1)\cos nwt\,dt + \int_3^5 (-1)\cos nwt\,dt\right]$$

$$= \frac{2}{5}\left(\left[\frac{\sin n\omega t}{n\omega}\right]_0^3 - \left[\frac{\sin n\omega t}{n\omega}\right]_3^5\right)$$

Put $\quad \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{5}$

$$a_n = \frac{1}{n\pi}[\sin 3n\omega - \sin 5n\omega + \sin 3n\omega]$$

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$$= \frac{1}{n\pi}\left[2\sin\left(3n\frac{2\pi}{5}\right) - \sin\left(5n\frac{2\pi}{5}\right)\right]$$

$$= \frac{1}{n\pi}\left[2\sin\left(\frac{6\pi n}{5}\right) - \sin(2n\pi)\right]$$

$$= \frac{2}{n\pi}\sin\left(\frac{6\pi n}{5}\right)$$

Coefficient, $b_n = \frac{2}{T}\int_T v(t)\sin n\omega_0 t\,dt$

$$= \frac{2}{5}\left[\int_0^3 (1)\sin nwt\,dt + \int_3^5 (-1)\sin nwt\,dt\right]$$

$$= \frac{2}{5}\left(\left[-\frac{\cos n\omega t}{n\omega}\right]_0^3 - \left[-\frac{\cos n\omega t}{n\omega}\right]_3^5\right)$$

put $\omega = \frac{2\pi}{T} = \frac{2\pi}{5}$

$$b_n = \frac{1}{n\pi}[-\cos 3n\omega + 1 + \cos 5n\omega - \cos 3n\omega]$$

$$= \frac{1}{n\pi}[-2\cos 3n\omega + 1 + \cos 5n\omega]$$

$$= \frac{1}{n\pi}\left[-2\cos\left(3n\frac{2\pi}{5}\right) + 1 + \cos\left(5n\frac{2\pi}{5}\right)\right]$$

$$= \frac{1}{n\pi}\left[-2\cos\left(\frac{6\pi n}{5}\right) + 1 + 1\right]$$

$$= \frac{2}{n\pi}\left[1 - \cos\left(\frac{6\pi n}{5}\right)\right]$$

Amplitude of fundamental component of v is

$$v_f = \sqrt{a_1^2 + b_1^2}$$

$$a_1 = \frac{2}{\pi}\sin\left(\frac{6\pi}{5}\right),\ b_1 = \frac{2}{\pi}\left(1 - \cos\frac{6\pi}{5}\right)$$

$$v_f = \frac{2}{\pi}\sqrt{\sin^2\frac{6\pi}{5} + \left(1 - \cos\frac{6\pi}{5}\right)^2}$$

$$= 1.20 \text{ Volt}$$

****

GATE Multiple Choice Questions
For Electrical Engineering

By RK Kanodia & Ashish Murolia

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- There are a variety of problems on each topic
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