Darbhanga College of Engineering

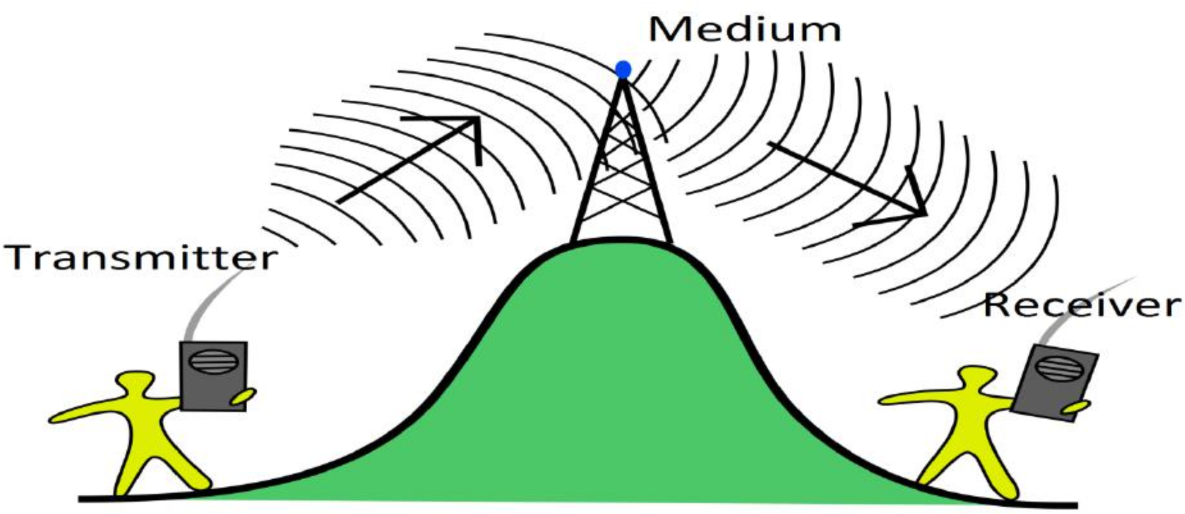
Darbhanga

Course File

Of

Analog and Digital Communication System

(PCC-EEE19)



Prepared by
Dr. Ravi Ranjan
Assistant Prof.
EEE Department, DCE Darbhanga

CONTENTS

Vision of the Institute

To produce young, dynamic, motivated and globally competent Engineering graduates with an aptitude for leadership and research, to face the challenges of modernization and globalization, who will be instrumental in societal development.

Mission of the Institute

1. To impart quality technical education, according to the need of the society.
2. To help the graduates to implement their acquired Engineering knowledge for society & community development.
3. To strengthen nation building through producing dedicated, disciplined, intellectual & motivated engineering graduates.
4. To expose our graduates to industries, campus connect programs & research institutions to enhance their career opportunities.
5. To encourage critical thinking and creativity through various academic programs.

Vision of EEE Department

To bring forth engineers with an emphasis on higher studies and a fervour to serve national and multinational organisations and, the society.

Mission of EEE Department

M1: - To provide domain knowledge with advanced pedagogical tools and applications.

M2: - To acquaint graduates to the latest technology and research through collaboration with industry and research institutes.

M3: - To instil skills related to professional growth and development.

M4: - To inculcate ethical valued in graduates through various social-cultural activities.

PEO of EEE

PEO 01 – The graduate will be able to apply the Electrical and Electrical Engineering concepts to excel in higher education and research and development.

PEO 02 – The graduate will be able to demonstrate the knowledge and skills to solve real life engineering problems and design electrical systems that are technically sound, economical and socially acceptable.

PEO 03 – The graduates will be able to showcase professional skills encapsulating team spirit, societal and ethical values.

Program Educational Objectives:-

PEO 1. Graduates will excel in professional careers and/or higher education by acquiring knowledge in Mathematics, Science, Engineering principles and Computational skills.

PEO 2. Graduates will analyze real life problems, design Electrical systems appropriate to the requirement that are technically sound, economically feasible and socially acceptable.

PEO 3. Graduates will exhibit professionalism, ethical attitude, communication skills, team work in their profession, adapt to current trends by engaging in lifelong learning and participate in Research & Development.

Program Outcomes of B.Tech in Electrical and Electronics Engineering

1.Engineering knowledge: Apply the knowledge of mathematics, science, engineeringfundamentals, and an engineering specialization to the solution of complex engineering problems.

2.Problem analysis: Identify, formulate, review research literature, and analyze complexengineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.

3.Design/development of solutions: Design solutions for complex engineering problems anddesign system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.

4.Conduct investigations of complex problems: Use research-based knowledge and researchmethods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.

5.Modern tool usage: Create, select, and apply appropriate techniques, resources, and modernengineering and IT tools including prediction and modelling to complex engineering activities with an understanding of the limitations.

6.The engineer and society: Apply reasoning informed by the contextual knowledge to assesssocietal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7.Environment and sustainability: Understand the impact of the professional engineering solutionsin societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8.Ethics: Apply ethical principles and commit to professional ethics and responsibilities and norms ofthe engineering practice.

9.Individual and team work: Function effectively as an individual, and as a member or leader indiverse teams, and in multidisciplinary settings.

10.Communication: Communicate effectively on complex engineering activities with the engineeringcommunity and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11.Project management and finance: Demonstrate knowledge and understanding of theengineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12.Life-long learning: Recognize the need and have the preparation and ability to engage in independent and life-long learning in the broadestcontext of technological change.

PSO 1. An ability to identify, formulate and solve problems in the areas of Electrical and Electronics Engineering.

PSO 2. An ability to use the techniques, skills and modern engineering tools necessary for innovation.

Scope and Objectives of the Course

Communication is the basic process of exchanging information. “Analog and Digital Communication System”, is the subject which deals with the techniques employed in communication for analog and digital data. The subject basically deals with the different aspects of a signal and spectra. It also deals with the modulation of signals and systems and different mathematical aspects related to signals. It gives a more analytical look into the basic entities such as those of signals, modulation, noise etc. which form the base for higher studies in telecommunication.

Course Objectives:

After the completion of this course the students will be able to:

CO1: Apply different mathematical concepts like Fourier series and Fourier transform and different circuit design concept to understand different modulation and demodulation techniques.

CO2: Compare the performance of different modulation techniques.

CO3: Understand working and operation of Digital communication principle.

CO4: Able to compute and analyse error correction codes.

CO5: Able to understand modern communication system.

Course Outcomes:

On completion of this course, the students will be able to

1. Understand different modulation and demodulation techniques analog and digital communication.
2. Apply signal and system analysis tools in the time and frequency domains, including Impulse response, convolution, frequency response, Fourier series, Fourier transform, and Hilbert transform.
3. Develop the ability to compare and contrast the strengths and weaknesses of various communication systems.
4. Able to understand error control coding techniques.
5. Prepare and deliver an oral presentation about a topic of current interest in the field of communications.

Mapping of CO's with PO's

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12	PSO 1	PSO 2
CO1	3	2	2	1	1	-	2	-	-	-	1	2	3	1
CO2	2	3	3	2	3	-	-	-	-	-	1	1	2	3
CO3	2	2	3	1	3	-	-	-	-	-	1	1	1	3
CO4	2	2	1	3	3	1	1	1	-	-	2	2	2	3
CO5	1	2	1	3	-	-	3	1	2	1	-	-	2	3

Syllabus

Subject Code: PCC-EEE19

Subject Name: Analog & Digital Communication System

Module 1: Basic blocks of Communication System. Analog Modulation - Principles of Amplitude Modulation, DSBSC, SSB-SC and VSB-SC. AM transmitters and receivers.

Module 2: Angle Modulation - Frequency and Phase Modulation. Transmission Bandwidth of FM signals, Methods of generation and detection. FM Transmitters and Receivers.

Module 3: Sampling theorem - Pulse Modulation Techniques - PAM, PWM and PPM concepts - PCM system – Data transmission using analog carriers (ASK, FSK, BPSK, QPSK).

Module 4: Error control coding techniques – Linear block codes- Encoder and decoder. Cyclic codes – Encoder, Syndrome Calculator. Convolution codes.

Module 5: Modern Communication Systems – Microwave communication systems - Optical communication system - Satellite communication system - Mobile communication system.

Text / References:

1. Simon Haykins, ‘Communication Systems’, John Wiley, 3rd Edition, 1995.

2. D.Roddy & J.Coolen, ‘Electronic Communications’, Prentice Hall of India, 4th Edition, 1999.

3. Kennedy G, ‘Electronic Communication System’, McGraw Hill, 1987.

DARBHANGA COLLEGE OF ENGINEERING, DARBHANGA

Electrical and Electronics Engineering
Semester – 5th, Session (2018-22)

Tuesday : 11 AM – 01 PM

Saturday : 11 AM – 01 PM

DARBHANGA COLLEGE OF ENGINEERING, DARBHANGA

5th Sem. Branch:- Electrical & Electronics Engineering Batch- (2018-22)

Subject :- ADC

S.No.	Name of Student	Roll No.	Registration No.
1	Nargis Nasreen	18-CS-14	18105111002
2	Abhishek Kumar	18-CS-21	18105111003
3	Soni Kumari	18-CS-48	18105111007
4	Ansh Shrivastava	18-CS-75	18105111008
5	Alamgir Ansari	18-CS-51	18105111009
6	Rimjhim Kumar	18-CS-74	18105111010
7	Rashi	18-CS-03	18105111011
8	Amit Kumar Thakur	18-CS-78	18105111013
9	Abhishek Raj	18-CS-59	18105111014
10	Muskan Gupta	18-CS-20	18105111015
11	Shivansh Sagar	18-CS-01	18105111016
12	Suman Kumari	18-CS-15	18105111017
13	Jemini Kumar	18-CS-04	18105111019
14	Harshit Raj	18-CS-17	18105111020
15	Shubham Kumar	18-CS-18	18105111021
16	Satyam Raj Shanu	18-CS-26	18105111022
17	Vishal Kumar	18-CS-25	18105111023
18	Manu Bharti	18-CS-50	18105111024
19	Nidhi	18-CS-42	18105111025
20	Sudhakar Kumar	18-CS-47	18105111026
21	Priyanka Kumari	18-CS-49	18105111027
22	Sneha Raj	18-CS-29	18105111028
23	Chandrika Bharti	18-CS-46	18105111029
24	Shalu Kumari	18-CS-55	18105111030
25	Anjali	18-CS-58	18105111031
26	Vikash Kumar Ray	18-CS-56	18105111032
27	Pooja Priya	18-CS-57	18105111033
28	Pragati	18-CS-52	18105111034
29	Shambhavi	18-CS-69	18105111035
30	Supriya Kumari	18-CS-68	18105111037
31	Santu Kumar	18-CS-67	18105111038
32	Ravishankar Kumar	18-CS-61	18105111039
33	Akshay Verma	18-CS-65	18105111040
34	Naman Raj	18-CS-70	18105111041
35	Neha Bharti	18-CS-77	18105111042

36	Md. Adil Khan	18-CS-80	18105111046
37	Md. Sahil Hussain	18-CS-09	18105111048
38	Abhishek Kumar	18-CS-45	18105111049
39	Aman Raj	18-CS-71	18105111050
40	Pranav Anand	18-CS-53	18105111052
41	Akancha	19LE-CS02	19105111901
42	Md Zakaullah	19LE-CS01	19105111902

COURSE HANDOUT

Institute/College Name:	Darbhanga College of Engineering
Program Name:	B.Tech (EEE, 5^{th} semester)
Course Code:	**PCC-EEE19**
Course Name:	Analog and Digital Communication System
Lecture/Tutorial(per week):	4/1
Course Credits:	3
Course Co-coordinator Name:	Dr. Ravi Ranjan

1. Scope and Objective of Course

Communication is the basic process of exchanging information. “Analog and Digital Communication System”, is the subject which deals with the techniques employed in communication for analog and digital data. The subject basically deals with the different aspects of a signal and spectra. It also deals with the modulation of signals and systems and different mathematical aspects related to signals. It gives a more analytical look into the basic entities such as those of signals, modulation, noise etc. which form the base for higher studies in telecommunication.

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3. Develop the ability to compare and contrast the strengths and weaknesses of various communication systems.
4. Able to understand error control coding techniques.

5. Prepare and deliver an oral presentation about a topic of current interest in the field of communications.

2. Textbooks

TB1: Simon Haykin, "Communication Systems", 4th edition, John Wiley & Sons, 2006, ISBN 812650904X, 9788126509041.

TB3: Allen V. Oppenheim & Allen S. Willsky, "Signals and Systems", 2nd edition, Prentice Hall, 1996, ISBN 0138147574

3. Reference Books

RB1: A. Bruce Carlson, Paul B. Crilly and Janet C. Rutledge, "Communication System" 4th edition, TMH, 2002, ISBN 0070111278

RB2: George Kennedy and Bernard Davis, "Electronics Communication Systems" 4th edition, TMH, 1999, ISBN 9780074636824

RB3: J. Proakis & M. Salehi, "Communication system engineering", 2nd edition, Prentice Hall, 2002, ISBN 0130617938, 9780130617934

Other readings and relevant websites

S. No.	Link of journals, Magazines, websites and Research papers
1.	http://nptel.ac.in/courses/117102059/
2.	http://ieeexplore.ieee.org/xpl/RecentIssue.jsp?punumber=35
3.	https://www.youtube.com/watch?v=F3slBe2r8vA&list=PLqGm0yRYwTgX2FkPVcY6io003-tZd8Ru
4.	http://onlinelibrary.wiley.com/journal/10.1002/(ISSN)1099-1131
5.	https://ocw.mit.edu/courses/aeronautics-and-astronautics/16-36-communication-systems-engineering-spring-2009/

Syllabus

Topics	No. of Lectures	Weightages (%)

Module 1: Basic blocks of Communication System. Analog Modulation - Principles of Amplitude Modulation, DSBSC, SSB-SC and VSB-SC. AM transmitters and receivers.	**8**	**25**
Module 2: Angle Modulation - Frequency and Phase Modulation. Transmission Bandwidth of FM signals, Methods of generation and detection. FM Transmitters and Receivers.	**8**	**20**
Module 3: Sampling theorem - Pulse Modulation Techniques - PAM, PWM and PPM concepts - PCM system – Data transmission using analog carriers (ASK, FSK, BPSK, QPSK).	**8**	**25**
Module 4: Error control coding techniques – Linear block codes- Encoder and decoder. Cyclic codes – Encoder, Syndrome Calculator. Convolution codes.	**8**	**15**
Module 5: Modern Communication Systems – Microwave communication systems - Optical communication system - Satellite communication system - Mobile communication system	**6**	**15**

Evaluation and Examination Blue Prints:

Internal assessment is done through quiz tests, presentations, assignments and projects work. Two sets of question paper are asked from each faculty and out of these two, without the knowledge of faculty, one question paper is chosen for the concerned examination. Examination rules and regulations are uploaded on the student's portals. Evaluation is a very transparent process and the answer sheets of sessional tests, internal assessment assignments are returned back to the students.

The components of the evaluation along with their weightage followed by the university are given below:

Component-1	Mid sem-1	20%
Component-2	Assignments, Quiz's, Test, Seminars	10%
Component-3	End Term Examination	70%
Totals		100%

Designation	**Name**
Course Coordinator	Dr. Ravi Ranjan
H.O.D	Mr. Prabhat Kumar
Principal	Dr. Achintya

Institute/College Name:	Darbhanga College of Engineering
Program Name:	B.Tech (EEE, 6th semester)
Course Code:	041603
Course Name:	Introduction to communication system
Lecture/Tutorial(per week):	4/1
Course Credits:	3
Course Co-coordinator Name:	Dr. Ravi Ranjan

Lecture Plan

Topics	No. of Lectures	Lecture Date
Module 1:		
Basic blocks of Communication System.	**2**	
Analog Modulation - Principles of Amplitude Modulation,	**3**	
DSBSC	**4**	
SSB-SC	**5**	
VSB-SC	**6**	
AM transmitters	**7**	
AM receivers	**8**	
Module 2:		
Module 2: Angle Modulation	**9**	
Frequency and Phase Modulation	**11**	
Transmission Bandwidth of FM signals	**13**	
Methods of generation and detection	**14**	
FM Transmitters and Receivers.	**16**	
Module 3:		
Module 3: Sampling theorem	**18**	
Pulse Modulation Techniques - PAM, PWM and PPM concepts	**19**	
PCM system	**21**	
Data transmission using analog carriers (ASK, FSK BPSK)	**23**	

ASSIGNMENT - I

Sub: Analog & Digital Communication System

Subject code: PCC-EEE 19

1. Define 'amplitude modulation'. Derive the relationship between the total transmitted power and carrier power in an AM system when several frequencies simultaneously modulate a carrier.

2. Describe the DSBSC wave generation process using balanced modulator.

3. A carrier of $10 \cos 8\pi \times 10^5 t$ is amplitude modulated by a message signal of $6 \cos \pi \times 10^4 t$.
   (i) find the parameters like, Bandwidth, total power (P_t), efficiency (η)
   (ii) plot spectral component & spectrum

4. A carrier of $10 \cos (4\pi \times 10^6 t)$ is DSB modulated by a message signal of $6 \cos (6\pi \times 10^4 t) + 8 \cos (\pi \times 10^5 t)$. Then plot the spectral component & spectrum

5. With the help of suitable diagram(s), derive the expression for SSB-AM signals

6. With the help of suitable block diagram, discuss electronic Communication System.

ASSIGNMENT - 2

Subject: Analog & Digital Communication System

Subject Code: PCC- EEE19

① Explain the relation between the frequency & phase modulation. Use the necessary diagrams.

② Compare between wideband FM and narrowband FM. Use Carson's rule to compare the bandwidth that would be required to transmit a baseband signal with frequency range from 300 Hz to 3 kHz using (i) NBFM with maximum deviation of 5 kHz and (ii) WBFM with maximum deviation of 75 kHz.

③ Explain the FM generation by Armstrong's indirect method.

④ A 100 MHz carrier is frequency modulated by 10 kHz wave. For a frequency deviation of 50 kHz, calculate the modulation index of the FM signal.

⑤ Determine the Bandwidth of a FM wave when the maximum deviation allowed is 75 kHz and the modulating signal has a frequency of 10 KHz.

⑥ Maximum frequency deviation and the maximum bandwidth allowed for commercial FM broadcast is:

(a) 80 kHz, 160 KHz

(b) 75 kHz, 200 kHz

(c) 60 kHz, 170 kHz

(d) 75 kHz, 250 kHz

⑦ What is the value of carrier frequency in the following equation for the FM signal.

$$v(t) = 5\cos(6600t + 12\sin 2500t)$$

(a) 1150 Hz

(b) 6600 Hz

(c) 2500 Hz

(d) 1050 Hz

Question Bank

Analog and Digital Communication

1.	Find the Fourier series for the square wave function $f(x)=-1$ for $-\pi<x<0$, $f(x)=1$, for $0<x<\pi$, and $f(0)=0$. Discuss the convergence (pointwise, uniform) of this Fourier series and find the limit function of it.
2.	Using the linearity of Fourier series and the previous problem, find the Fourier series of $f(x)=0$ for $-\pi<x<0$ and $f(x)=1$ for $0<x<\pi$
3.	Find the Fourier series for the function x(t) which has fundamental frequency ω_0 $x(t)=1+\sin\omega_0 t+2\cos\omega_0 t+\sin(2\omega_0 t+\pi/4)$
4.	Let $x_1(t)$ be a continuous-time periodic signal with fundamental frequency ω_1 and Fourier coefficients a_k. Given that $x_2(t)=x_1(1-t)+x_1(t-1)$, how is the fundamental frequency ω_2 of $x_2(t)$ related *to* ω_1? Also, find a relationship between the Fourier series coefficients b_k of $x_2(t)$ and the coefficients a_k.
5.	Draw the basic elements of communication system. Write function of communication channel in it.
6.	Define: 1. Modulation, 2. Modulation index of AM and 3. Deviation ratio of FM
7.	For an AM, DSBFC modulator with a carrier frequency fc=100KHz and a maximum modulating signal frequency fm=5KHz, determine: a. Frequency limits for upper and lower side band b. Bandwidth c. Draw the output frequency spectrum c. State two advantages and two disadvantages of FM over AM
8.	Find the carrier and modulating frequencies, the modulation index, and the maximum deviation of FM wave represented by the voltage equation v=10sin(5.5x108t + 4sin1250t). What power will this FM wave dissipate in a 15Ω resistor?
9.	For AM Fc = 100KHz, Fm = 5KHz determine: a. Upper and lower side band frequencies b. Bandwidth
10.	Draw the block diagram of Phase Lock Loop as FM detector and state the function of Voltage control oscillator.
11.	a) Explain AM with necessary expressions, waveforms and spectrums, b) The output power of an AM transmitter is 1KW when sinusoidally modulated to a depth of 100%. Calculate the power in each side band when the modulation depth is reduced to 50%.

12.	a) Discuss the main objectives of a communication system design? What are the primary resources of any communication system. b) The RC load for a diode envelope detector consists of a 1000 pF capacitor in parallel with a 10-K resistor. Calculate the maximum modulation depth that can be handled for sinusoidal modulation at a frequency of 10 KHz if diagonal peak clipping is to be avoided.
13.	a) Sketch the one cycle of AM wave and calculate the modulation index of it in terms of Vmax and Vmin voltages. b) A modulating signal consists of a symmetrical triangular wave having zero dc component and peak to peak voltage of 12V. It is used to amplitude modulate a carrier of peak voltage 10V. Calculate the modulation index and the ratio of the side lengths L1/L2 of the corresponding trapezoidal pattern.
14.	a) Explain the collector modulation method for generating AM wave with a neat circuit diagram and waveforms. b) An AM amplifier provides an output of 106W at 100% modulation. The internal loss is 20 W (i).What is unmodulated carrier power? (ii). What is the side band power?
15.	a) Explain operation of square law detector with circuit diagram and waveforms. b) An AM transmitter has un-modulated carrier power of 10 KW. It can be modulated by sinusoidal modulating voltage to a maximum depth of 40%, without overloading. If the maximum modulation index is reduced to 30%. What is the extent up to which the unmodulated carrier power can be increased to avoid over loading.
16.	a) Explain about the quadrature null effect of coherent detect . b) In DSB-SC, suppression of carrier so as to save transmitter power results in receiver complexity- Justify this statement
17.	a) Describe the time domain band-pass representation of SSB with necessary sketches. b) Find the percentage of power saved in SSB when compared with AM system.
18.	Find the various frequency components and their amplitude in the Voltage given below E=50(1+0.7cos5000t-0.3cos1000t) sin5x10^{6}t. Draw the single sided spectrum. Also evaluate the modulated and sideband powers.
19.	Describe the single tone modulation of SSB. Assume both modulating and carrier signals are sinusoids. Write SSB equation and plot all the waveforms and spectrums.
20.	Calculate the filter requirement to convert DSB signal to SSB Signal, given that the two side bands are separated by 200HZ. The suppressed carrier is 29MHZ.
21.	a) Explain about FM generation using transistor reactance modulator. b) Explain balanced ratio detector for detecting FM signal.
22.	a) Compute the bandwidth requirement for the transmission of FM signal having a frequency deviation 75 KHz and an audio bandwidth of 10KHz. b) An FM radio link has a frequency deviation of 30 kHz. The modulating frequency

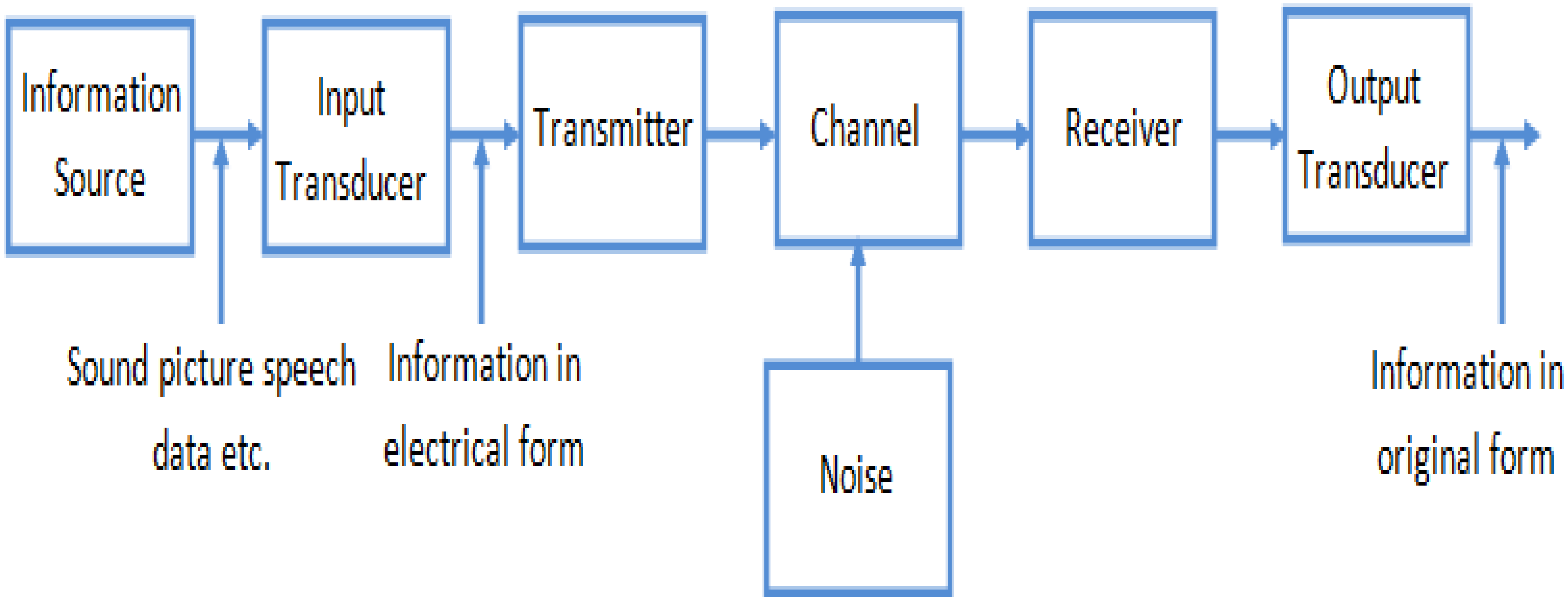
	is 3kHz. Calculate the bandwidth needed for the link. What will be the bandwidth if the deviation is reduced to 15 kHz?
23.	Determine the amplitude spectrum of the filter output for FM wave with modulation index $\beta = 1$ is transmitted through an ideal band pass filter with mid band frequency fc and bandwidth is 5fm, where fc is the carrier frequency and fm is the frequency of the sinusoidal modulating wave.
24.	a) List and discuss the factors influencing the choice of the intermediate frequency for a radio receiver. b) What is simple automatic gain control? What are its functions?
25.	In a broadcast super heterodyne receiver having no RF amplifier, the loaded Q of the antenna coupling circuit is 100. If the IF frequency is 455 kHz, determine the image frequency and its rejection ratio for tuning at 1.1. kHz a station.
26.	Explain the demodulation procedure for PWM signal demodulation.

Analog & Digital Communication System

Module-1

LECTURE-1

Basic blocks of Communication System



Information source :-

- The objective of any communication system is to convey information from one point to the other. The information comes from the information source, which originates it
- Information is a very generic word signifying at the abstract level anything intended for communication, which may include some thoughts, news, feeling, visual scene, and so on.
- The information source converts this information into physical quantity.
- The physical manifestation of the information is termed as message signal

Input Transducer :-

- Any device that converts input energy/power into another can be termed as transducer.
- An electrical transducer defined as an apparatus that converts some physical changeable (Pressure, force, temperature etc..) into similar variations within the electrical signal at its output .

Transmitter :-

- The objective of the transmitter block is to collect the incoming message signal and modify it in a suitable fashion (if needed), such that, it can be transmitted via the chosen channel to the receiving point.
- The functionality of the transmitter block is mainly decided by the type or nature of the channel chosen for communication.

Channel :-

- Channel is the physical medium which connects the transmitter with that of the receiver.
- The physical medium includes copper wire, coaxial cable, fibre optic cable, wave guide and free space or atmosphere.
- The choice of a particular channel depends on the feasibility and also the purpose of the communication system.

Noise:

- Noise is the undesirable electrical energy that enters the communication system and interferes with the desired signal. It is unpredictable in nature.

- It can be man made and natural
- It is produced at the transmitter channel and also at the receiver entirely.

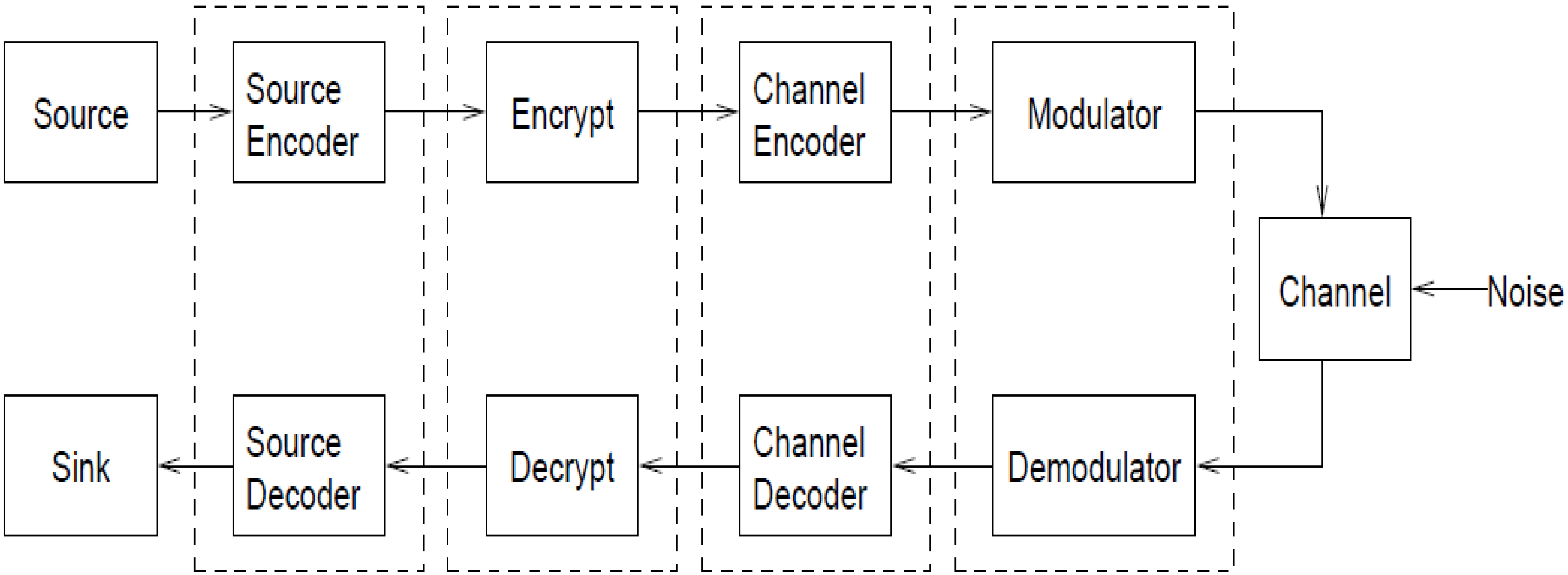
Receiver:-

- The receiver block receives the incoming modified version of the message signal from the channel and processes it to recreate the original (non-electrical) form of the message signal.
- There are a great variety of receivers in communication systems, depending on the processing required to recreate the original message signal and also final presentation of the message to the destination.

Destination:-

- The destination is the final block in the communication system which receives the message signal and processes it to comprehend the information present in it.
- Usually, humans will be the destination block.

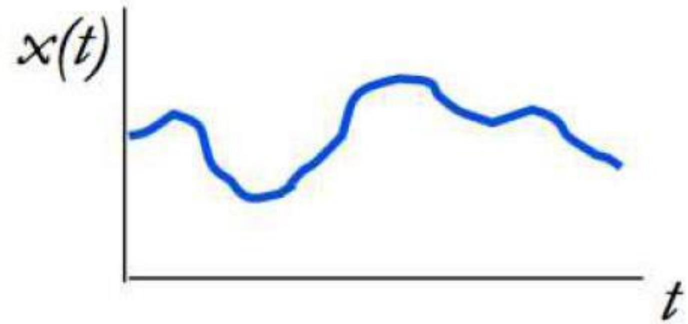
Block of Communication System (Advanced)



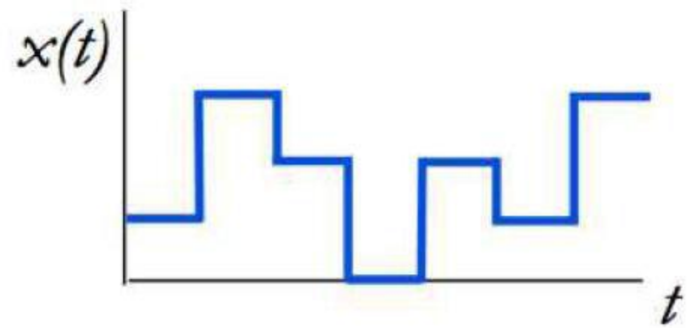
- Source encoder compresses message to remove redundancy
- Encryption protects against eavesdroppers and false messages
- Channel encoder adds redundancy for error protection

Analog Vs Digital Data

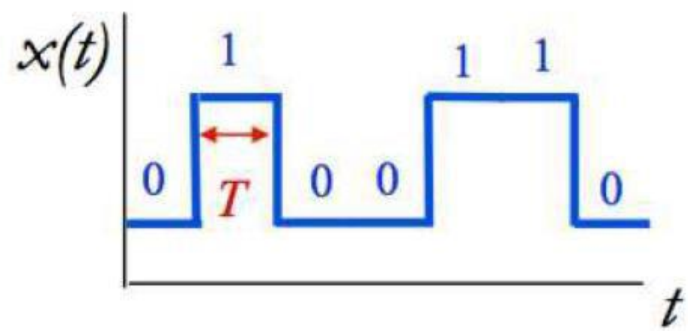
- Analog signals
  Values varies continously



- Digital signals
  Value limited to a finite set
  Digital systems are more robust



- Binary signals
  Have 2 possible values
  Used to represent bit values
  Bit time T needed to send 1 bit
  Data rate $R = 1/T$ bits per second



Modulation

"Modulation is the process of superimposing a low frequency signal on a high frequency carrier signal."

OR

"The process of modulation can be defined as varying the RF carrier wave in accordance with the intelligence or information in a low frequency signal."

OR

"Modulation is defined as the precess by which some characteristics, usually amplitude, frequency or phase, of a carrier is varied in accordance with instantaneous value of some other voltage, called the modulating voltage."

$m(t)=\cos(\omega t)$

Message signal: $m(t)=A\cos(\omega t+\Theta)$

Carrier Signal: $C(t)=A_C \cos(\omega_c t+\Theta)$

Where ω_c is much greater then ω

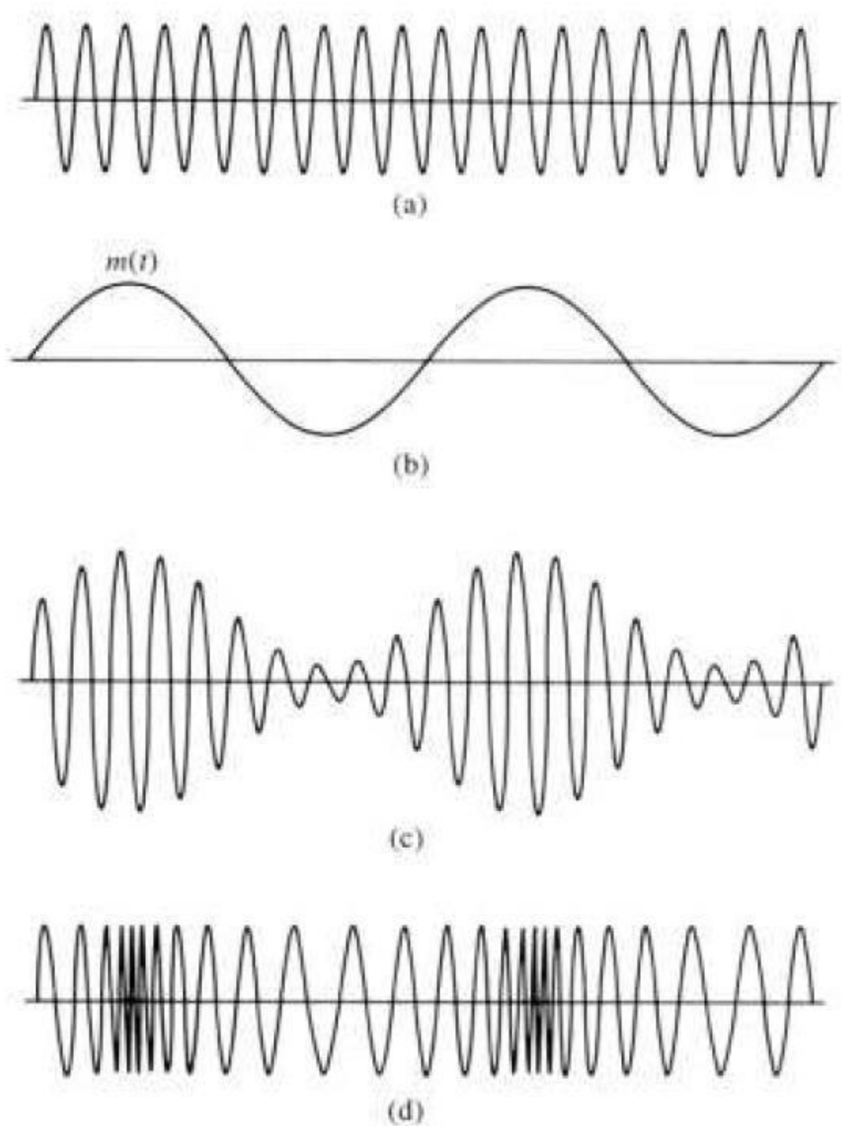
AM and FM Modulation

(a) Carrier

(b) Signal

(c) Amplitude modulated

(d) Frequency modulated



Need of Modulation

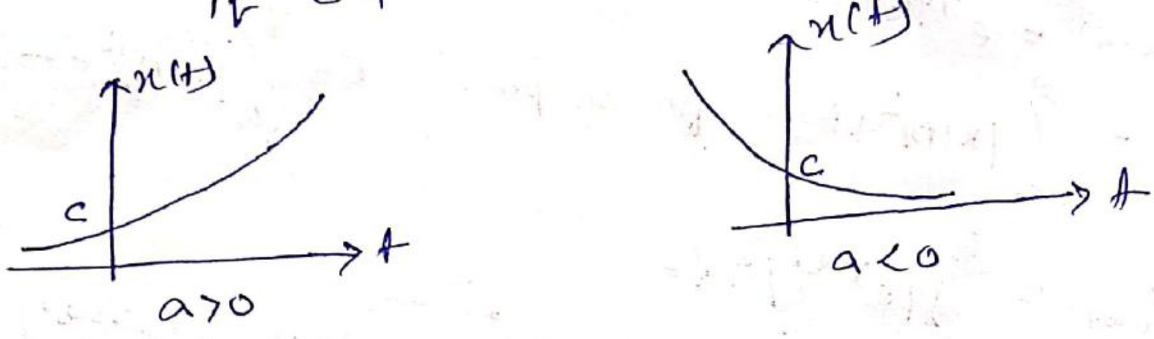
1. If two musical programs were played at the same time within distance, it would be difficult for anyone to listen to one source and not hear the second source. Since all musical sounds have approximately the same frequency range, form about 50 Hz to 10KHz. If a desired program is shifted up to a band of frequencies between 100KHz and 110KHz, and the second program shifted up to the band between 120KHz and 130KHz, Then both programs gave still 10KHz bandwidth and the listener can (by band selection) retrieve the program of his own choice. The receiver would down shift only the selected band of frequencies to a suitable range of 50Hz to 10KHz.
2. A second more technical reason to shift the message signal to a higher frequency is related to antenna size. It is to be noted that the antenna size is inversely proportional to the frequency to be radiated. This is 75 meters at 1 MHz but at 15KHz it has increased to 5000 meters (or just over 16,000 feet) a vertical antenna of this size is impossible.
3. The third reason for modulating a high frequency carrier is that RF (radio frequency) energy will travel a great distance than the same amount of energy transmitted as sound power.

- Antenna size gets reduced.
- No signal mixing occurs.
- Communication range increases.
- Multiplexing of signals occur.
- Adjustments in the bandwidth is allowed.
- Reception quality improves.

FOURIER SERIES

Let $x(t) = ce^{at}$ → aperiodic signal

if c & a are real



if a is purely imaginary and $c = 1$

$a \to j\omega_0$

and $x(t) = e^{j\omega_0 t}$ → periodic signal

Proof If we suppose that the $x(t) = e^{j\omega_0 t}$ is periodic signal

then, $e^{j\omega_0 t} = e^{j\omega_0 (t+T)}$

or, $e^{j\omega_0 t} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$

it means, for periodicity we must have, $e^{j\omega_0 T} = 1$

↳ if $\omega_0 = 0$, then $x(t) = 1$, which is periodic for any value of T

↳ if $\omega_0 \neq 0$, then the fundamental period (the smallest positive value of T) T_0 of $x(t)$ for which $e^{j\omega_0 T} = 1$ is,

$$T_0 = \frac{2\pi}{|\omega_0|}$$

$$\boxed{\begin{aligned} e^{j\omega_0 T} &= e^{j\omega_0 \cdot \frac{2\pi}{|\omega_0|}} = e^{j2\pi} \\ &= \cos 2\pi + j\sin 2\pi \\ &= 1 + j0 = 1 \\ \text{if } |\omega_0| &= -\omega_0 \text{ then also} \\ \cos(-2\pi) &+ j\sin(-2\pi) = 1 - j0 = 1 \end{aligned}}$$

↳ Thus the signals $e^{j\omega_0 t}$ and $e^{-j\omega_0 t}$ have the same fundamental period, and it is proved that,

$x(t) = e^{j\omega_0 t}$ is periodic signal having fundamental period of $T_0 = \frac{2\pi}{|\omega_0|}$

→ The periodic signal have infinite total energy but finite average power

Ex $x(t) = e^{j\omega_0 t}$

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ for one period, $E = \int_0^{T_0} |x(t)|^2 dt$

$$E_{period} = \int_0^{T_0} |e^{j\omega_0 t}|^2 dt$$

$$= \int_0^{T_0} 1 \cdot dt = T_0$$

$$P_{period} = \frac{1}{T_0} E_{period} = 1$$

Since, $|x(t)| = |e^{j\omega_0 t}|$

$= |\cos\omega_0 t + j\sin\omega_0 t|$

$= \sqrt{\cos^2\omega_0 t + \sin^2\omega_0 t} = 1$

→ $x(t) = e^{j\omega t}$ to be periodic

$e^{j\omega T_0} = 1$ → condition for periodic

which implies that ωT_0 is a multiple of 2π, ie

$\omega T_0 = 2\pi k$, $k = 0, \pm 1, \pm 2, \dots$ —— (A)

So, $\omega_0 = \frac{2\pi}{T_0}$

To satisfy (A) ω must be integer multiple of ω_0.

That is harmonically related set of complex exponentials is a set of periodic exponentials with fundamental frequencies that are all multiples of single frequency ω_0.

$$\phi_k(t) = e^{jk\omega_0 t}, \quad k = 0, \pm 1, \pm 2, \dots$$

for $k = 0$, $\phi_k(t)$ is a constant.

for $k \neq 0$, $\phi_k(t)$ is a periodic with fundamental frequency $|k|\omega_0$ and fundamental period, $\frac{2\pi}{|k|\omega_0} = \frac{T_0}{|k|}$

↳ The pair of equation that defines the Fourier Series of a periodic continuous time signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk(2\pi/T)t}$$

where, $$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{T}\int_T x(t)\, e^{-jk\left(\frac{2\pi}{T}\right)t}\, dt$$

The coefficient a_0 is the dc or constant component of $x(t)$ with $k=0$

so, $$a_0 = \frac{1}{T}\int_T x(t)\, dt$$

which is simply the average value of $x(t)$ over one period.

↳ <u>Fourier Series Representation in Trigonometric form</u>

The representation of fourier series in trigonometry form

$$x(t) = a_0 + \sum_{k=1}^{\infty} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t)$$

where, $\omega_0 = \frac{2\pi}{T_0}$

$$a_k = \frac{2}{T_0}\int_{T_0} x(t) \cos k\omega_0 t\, dt$$

$$b_k = \frac{2}{T_0}\int_{T_0} x(t) \sin k\omega_0 t\, dt$$

a_0 is the dc component of the signal

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$$

Ex. Consider a periodic signal $x(t)$, with fundamental frequency 2π, that is expressed as,

$$x(t) = \sum_{k=-3}^{+3} a_k e^{jk2\pi t} \qquad \omega_0 = 2\pi$$

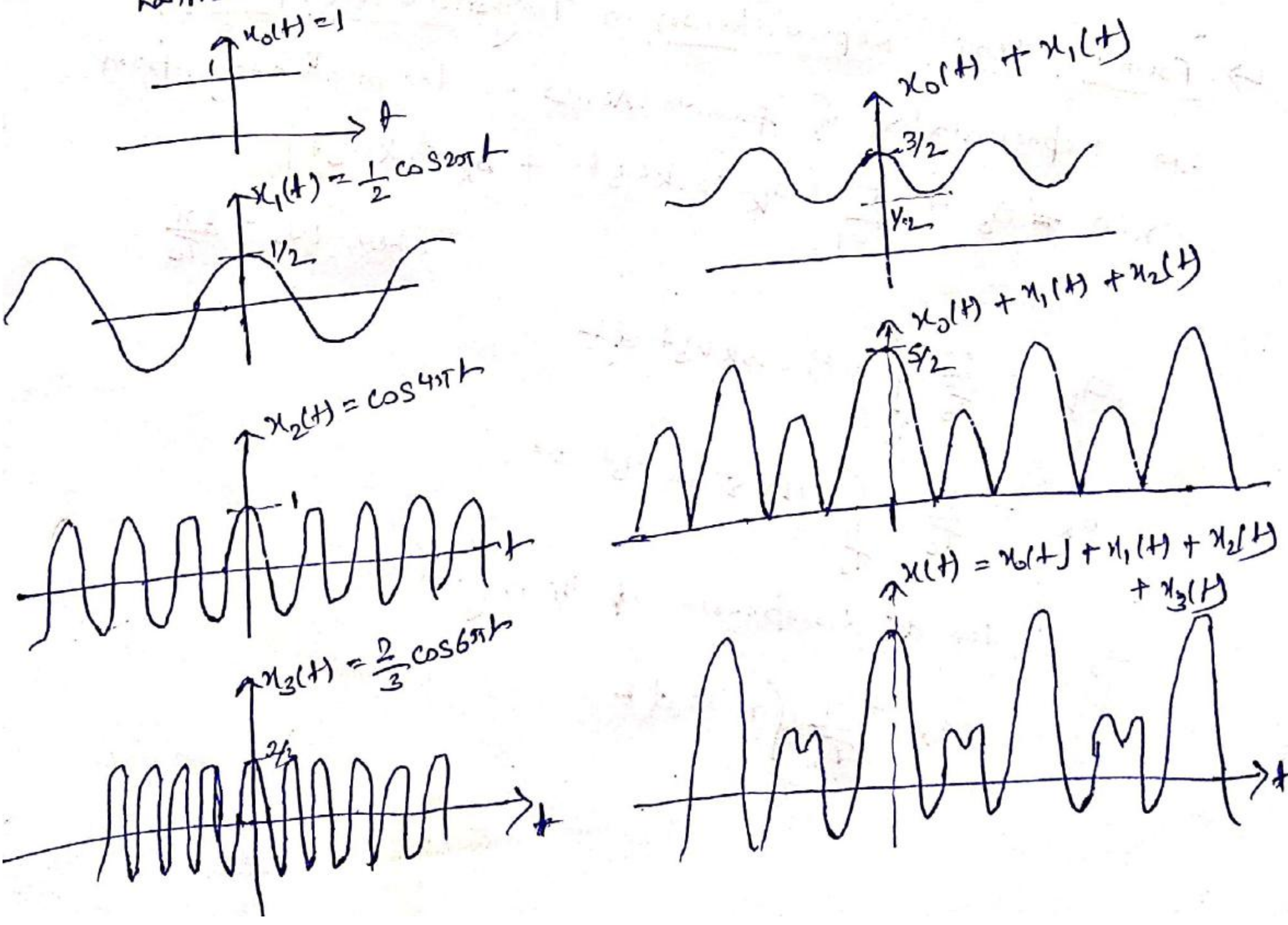
where, $a_0 = 1$, $a_1 = a_{-1} = \frac{1}{4}$, $a_2 = a_{-2} = \frac{1}{2}$

and, $a_3 = a_{-3} = \frac{1}{3}$

$$x(t) = 1 + \frac{1}{4}\left(e^{j2\pi t} + e^{-j2\pi t}\right) + \frac{1}{2}\left(e^{j4\pi t} + e^{-j4\pi t}\right) + \frac{1}{3}\left(e^{j6\pi t} + e^{-j6\pi t}\right)$$

$$= 1 + \frac{1}{2}\cos 2\pi t + \cos 4\pi t + \frac{2}{3}\cos 6\pi t$$

Now graphically we see the signal $x(t)$ is built from its harmonic components.



Ex $x(t) = \sin \omega_0 t$

$$\sin \omega_0 t = \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t}$$

So, $a_1 = \frac{1}{2j}$

$a_{-1} = -\frac{1}{2j}$

$a_k = 0 \quad k \neq \pm 1$

Ex $x(t) = 1 + \sin \omega_0 t + 2\cos \omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$

which has fundamental frequency ω_0

$$x(t) = 1 + \frac{1}{2j}\left[e^{j\omega_0 t} - e^{-j\omega_0 t}\right] + \left[e^{j\omega_0 t} + e^{-j\omega_0 t}\right] + \frac{1}{2}\left[e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}\right]$$

Collecting terms we obtain

$$x(t) = 1 + \left(1 + \frac{1}{2j}\right) e^{j\omega_0 t} + \left(1 - \frac{1}{2j}\right) e^{-j\omega_0 t} + \left(\frac{1}{2} e^{j(\pi/4)}\right) e^{j2\omega_0 t} + \left(\frac{1}{2} e^{-j(\pi/4)}\right) e^{-j2\omega_0 t}$$

Thus the fourier series coefficients

$a_0 = 1$

$a_1 = 1 + \frac{1}{2j} = 1 - \frac{1}{2}j$

$a_{-1} = 1 - \frac{1}{2j} = 1 + \frac{1}{2}j$

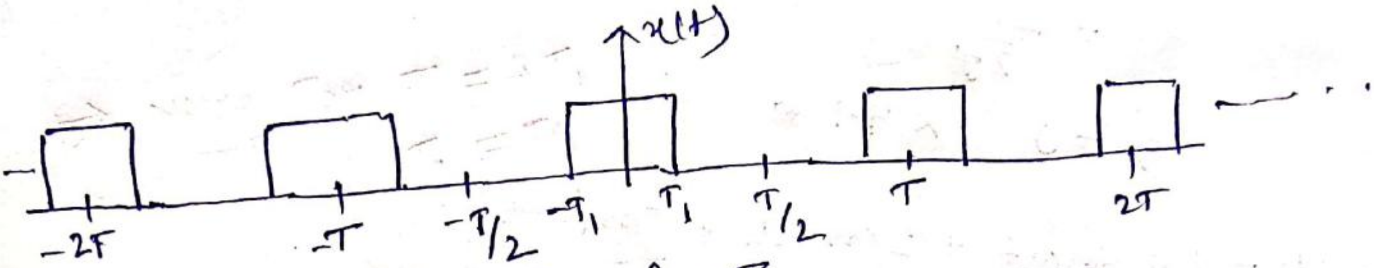
$a_2 = \frac{1}{2} e^{j(\pi/4)} = \frac{\sqrt{2}}{4}(1+j)$

$a_{-2} = \frac{1}{2} e^{-j(\pi/4)} = \frac{\sqrt{2}}{4}(1-j)$

$a_k = 0, \quad |k| > 2$

Ex The periodic square wave, sketched in figure and defined over one period as

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$



fundamental period $= T$

fundamental frequency $= \omega_0 = 2\pi/T$

Sol^n

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & T_1 < |t| < T/2 \end{cases}$$

Since, coefficient $a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt$

So, $a_0 = \frac{1}{T}\int_{-T_1}^{T_1} 1 \cdot 1\, dt = \frac{1}{T} \times 2T_1 = \frac{2T_1}{T}$

$$a_k = \frac{1}{T}\int_{-T_1}^{T_1} e^{-jk\omega_0 t}\, dt = -\frac{1}{jk\omega_0 T}\left[e^{-jk\omega_0 t}\right]_{-T_1}^{T_1}$$

$$a_k = -\frac{1}{jk\omega_0 T}\left\{e^{-jk\omega_0 T_1} - e^{+jk\omega_0 T_1}\right\}$$

$$= \frac{2}{k\omega_0 T}\left\{\frac{e^{jk\omega_0 T_1} - e^{-jk\omega_0 T_1}}{2j}\right\}$$

$$a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T} \quad \text{since, } \omega_0 = \frac{2\pi}{T}$$

$$= \frac{\sin(k\omega_0 T_1)}{k\pi}, \quad k \neq 0$$

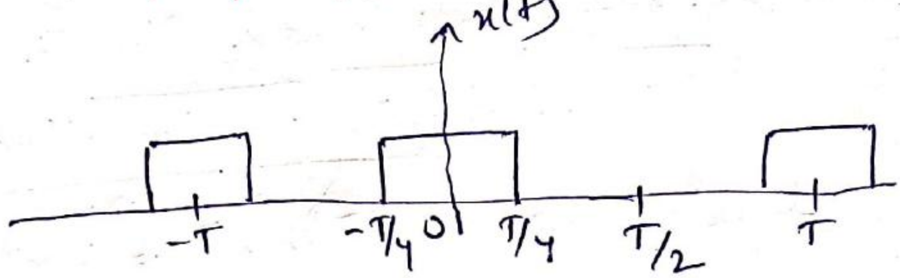
So, Fourier series of $x(t)$

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{\sin(k\omega_0 T_1)}{k\pi} e^{jk\omega_0 t} \qquad k \neq 0$$

$$\frac{2T}{T} e^{jk\omega_0 t} \qquad k = 0$$

→ Now, For $T = 4T_1$

so, $T_1 = T/4$



$$a_k = \frac{\sin(k\omega_0 T_1)}{k\pi} \qquad k \neq 0$$

so, $\omega_0 T_1 = \frac{2\pi}{T} \cdot \frac{T}{4} = \pi/2$

and $a_k = \frac{\sin(k\pi/2)}{k\pi}$

$$a_0 = \frac{2T_1}{T} = 2 \cdot \frac{T}{T\,4} = 1/2$$

So, the value of $a_k = \frac{\sin(k\pi/2)}{k\pi} = 0$ for k = even number

$a_1 = a_{-1} = 1/\pi$

$a_3 = a_{-3} = -1/3\pi$

$a_5 = a_{-5} = 1/5\pi$

FOURIER TRANSFORM

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

For aperiodic signals, the complex exponentials occur at a continuum of frequencies.

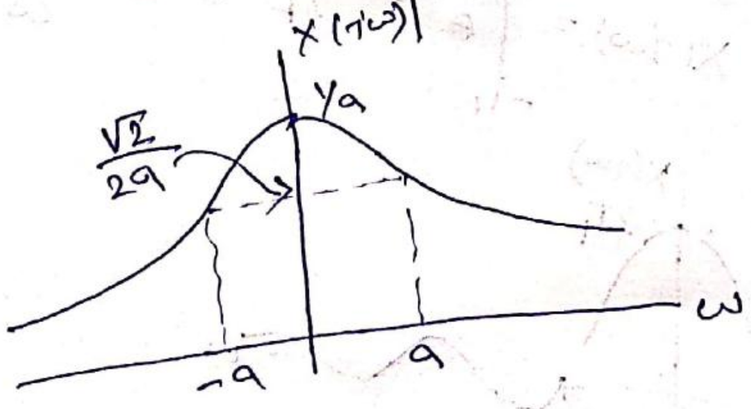
Ex $x(t) = e^{-at}u(t)$; $a>0$

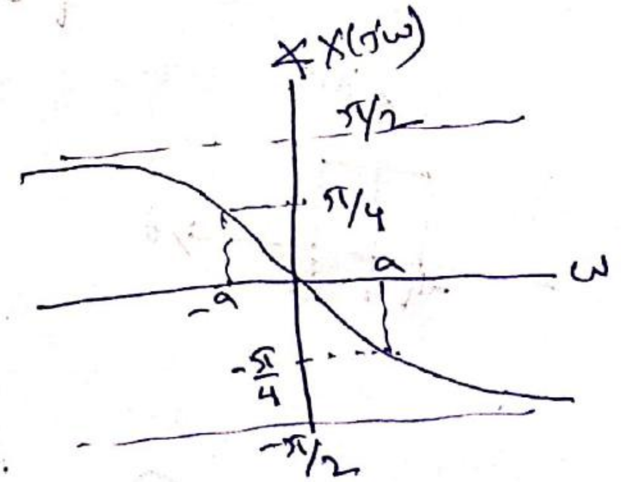
So $X(j\omega) = \int_0^{\infty} e^{-at}\cdot e^{-j\omega t}\,dt$

$$= -\frac{1}{a+j\omega}\cdot e^{-(a+j\omega)t}\Big|_0^{\infty}$$

that is, $X(j\omega) = \frac{1}{a+j\omega}$ $\quad a>0$

$|X(j\omega)| = \frac{1}{\sqrt{a^2+\omega^2}}$ $\qquad \sphericalangle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$





Ex $x(t) = e^{-a|t|} \quad a>0$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\,dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\,dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}.$$

Ex Fourier transform of unit impulse

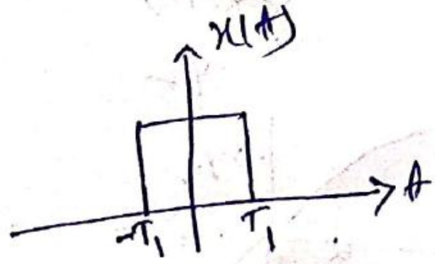
$$x(t) = \delta(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\,dt = 1$$

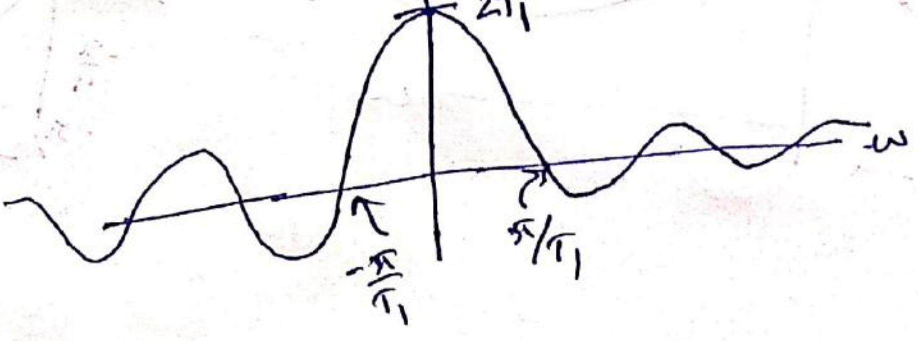
i.e. the unit impulse has a fourier transform consisting of equal contributions at all frequencies

Ex consider the rectangular pulse

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = \int_{-T_1}^{T_1} e^{-j\omega t}\,dt = 2\,\frac{\sin \omega T_1}{\omega}$$



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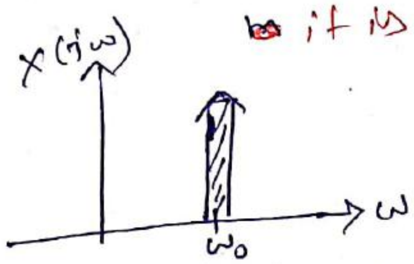
→ The Fourier Transform for periodic Signals.

We can construct the fourier transform of a periodic signal directly from its Fourier series representation. The resulting transform consists of a train of impulses in the frequency domain, with the areas of the impulses proportional to the Fourier series coefficients.

→ Let us consider a signal $x(t)$ with Fourier transform $X(j\omega)$

where, $X(j\omega) = 2\pi\delta(\omega - \omega_0)$ ——— (1)

it is single impulse of area 2π at $\omega = \omega_0$



To determine the signal $x(t)$ for which this is the Fourier transform, we can apply the inverse transform relation,

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega - \omega_0)\, e^{j\omega t}\, d\omega$$

→ at $\omega = \omega_0$ only the value exist, for rest it is zero

$$= e^{j\omega_0 t} \text{ ——— (11)}$$

So, in general, if $X(j\omega)$ is of the form of a linear combination of impulses equally spaced in frequency, that is,

$$X(j\omega) = \sum_{k=-\infty}^{\infty} 2\pi a_k\, \delta(\omega - k\omega_0) \text{ ——— (111)}$$

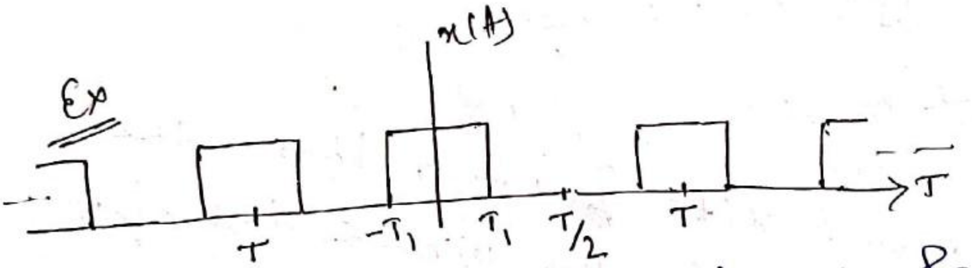
So, if we compare (1) (11) & (111)

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \quad \text{—— (iv)}$$

So, if we see the eqn (iv), it is exactly to the Fourier series representation of a periodic signal.

Thus the Fourier transform of a periodic signal with Fourier series coefficients $\{a_k\}$ can be interpreted as a train of impulses occuring at the harmonically related frequencies and for which the area of the impulses at the kth harmonic frequency $k\omega_0$ is 2π times the kth Fourier series coefficient a_k.

Ex



The Fourier series coefficient for the signal are

$$a_k = \frac{\sin k\omega_0 T_1}{\pi k}$$

So the Fourier transform of the signal is

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2\cancel{\pi} \cdot \frac{\sin k\omega_0 T_1}{\cancel{\pi} k} \delta(\omega - k\omega_0)$$

$$= \sum_{k=-\infty}^{+\infty} \frac{2\sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$$

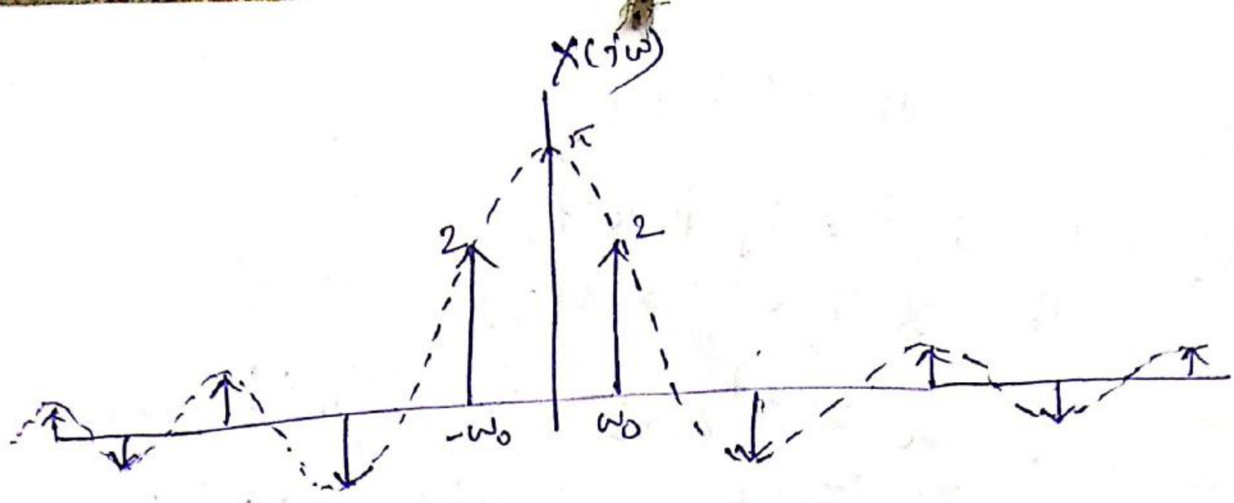
For, $T = 4T_1$, $T_1 = T/4$

$$X(j\omega) = \sum_{k=-\infty}^{+\infty} 2 \frac{\sin k\pi/2}{k} \delta(\omega - k\omega_0)$$

$\cancel{2 \sin k 2\pi . T / k 4}$

$$\frac{2 \sin k \cancel{2\pi} . \cancel{T}/ 4_2}{k}$$

$$\frac{2 \sin k\pi/2}{k}$$



Ex $x(t) = \sin \omega_0 t$

the fourier series coefficients for this signal

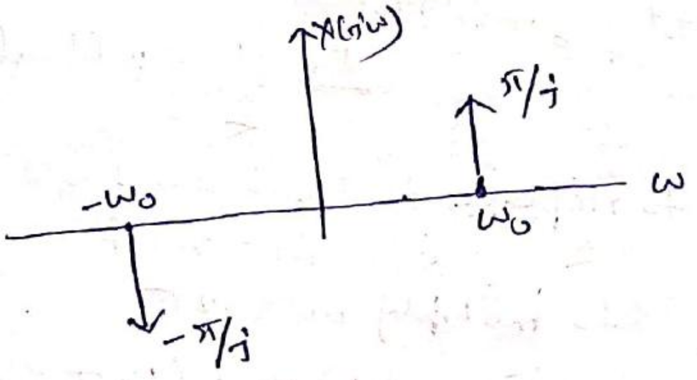
$$a_1 = \frac{1}{2j} \quad \text{and} \quad a_{-1} = -\frac{1}{2j}$$

$$a_k = 0, \quad k \neq 1, -1$$

So Fourier transform

$$X(j\omega) = \cancel{2}\pi \cdot \frac{1}{\cancel{2}j} \delta(\omega - \omega_0) + \cancel{2}\pi \cdot \frac{-1}{\cancel{2}j} \delta(\omega + \omega_0)$$

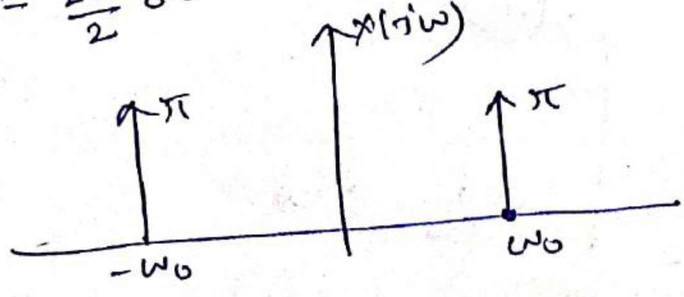
$$= \frac{\pi}{j} \delta(\omega - \omega_0) + \left(\frac{-\pi}{j}\right) \delta(\omega + \omega_0)$$



Ex $x(t) = \cos \omega_0 t$

$$a_1 = \frac{1}{2}, \quad a_{-1} = \frac{1}{2}, \quad a_k = 0 \quad k \neq \pm 1$$

$$X(j\omega) = \frac{2\pi}{2} \delta(\omega - \omega_0) + \frac{2\pi}{2} \delta(\omega + \omega_0) = \pi \delta(\omega - \omega_0) + \pi \delta(\omega + \omega_0)$$



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$\hookrightarrow$ Since, $\delta(t) \xleftrightarrow{F.T} 1$

Using frequency shifting property

if $x(t) \xleftrightarrow{F.T} X(f)$

then, $e^{j2\pi f_0 t} x(t) \xleftrightarrow{F.T} X(f-f_0)$

$e^{-j2\pi f_0 t} x(t) \xleftrightarrow{F.T} X(f+f_0)$

Using Duality property

if, $\delta(t) \xleftrightarrow{F.T} 1$

then, $1 \xleftrightarrow{F.T} \delta(-f) = \delta(f)$

So, $e^{j2\pi f_0 t} \cdot 1 \xleftrightarrow{F.T} \delta(f-f_0)$

$e^{-j2\pi f_0 t} \cdot 1 \xleftrightarrow{F.T} \delta(f+f_0)$

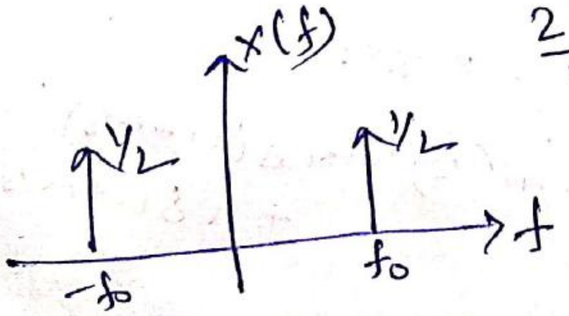
So, $\cos 2\pi f_0 t = \dfrac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \xleftrightarrow{F.T} \dfrac{1}{2}\{\delta(f-f_0)+\delta(f+f_0)\}$

if we want to replace it with ω then

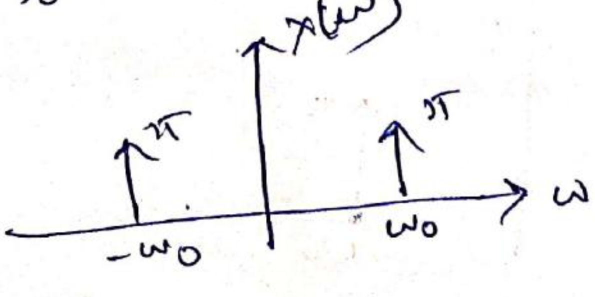
So, $\omega = 2\pi f$ So, we have to multiply with 2π.

$\dfrac{2\pi}{2}\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\}$

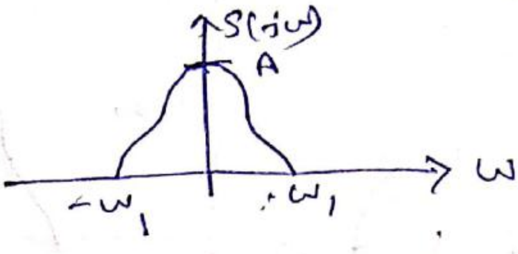
$= \pi\{\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\}$



or,



Ex Let $s(t)$ be a signal whose spectrum $S(j\omega)$ is shown in figure.



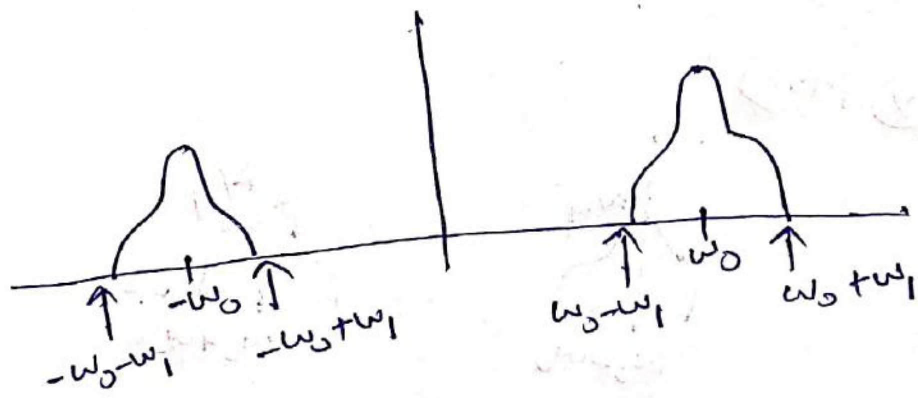
Another signal
$p(t) = \cos\omega_0 t$

then sketch the spectrum $R(j\omega)$

where $r(t) \xleftrightarrow{F.T} R(j\omega)$

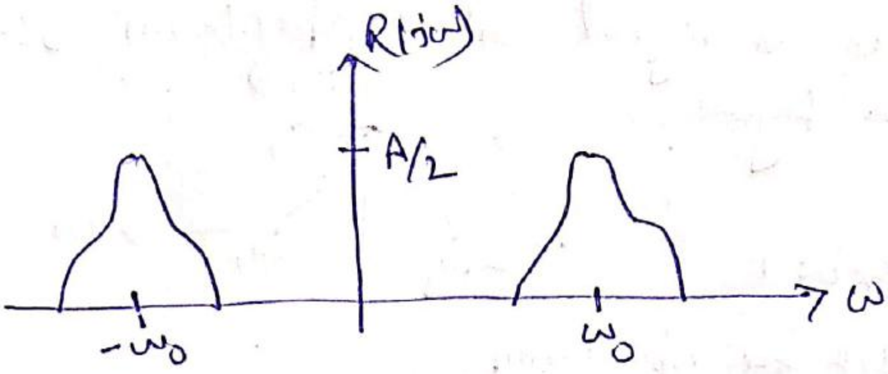
and, $r(t) = s(t)p(t)$

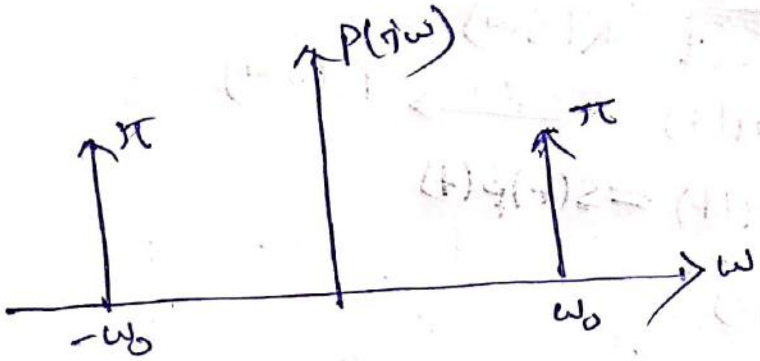
Since $r(t) = s(t)p(t)$
then $R(j\omega) = \frac{1}{2\pi}[S(j\omega) \cdot P(j\omega)]$

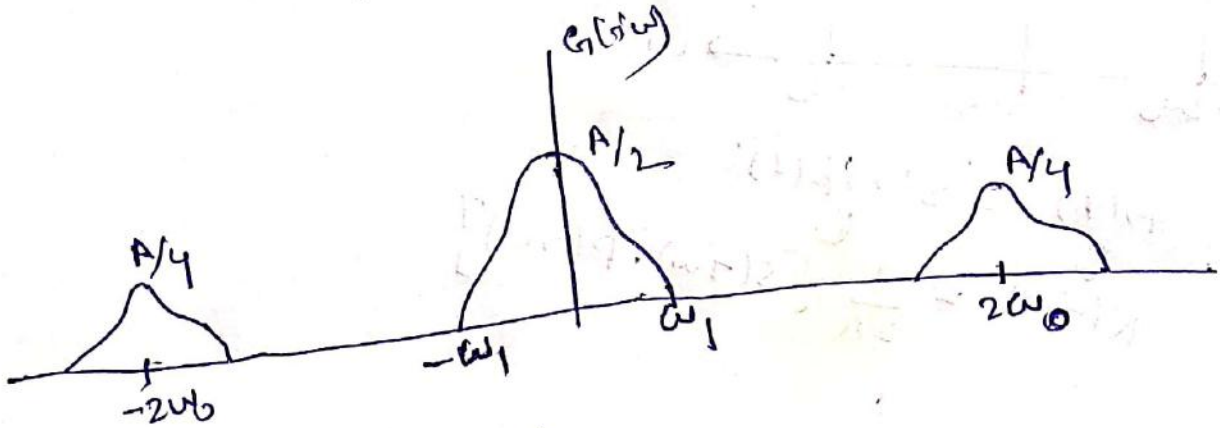


Ex Now, let us consider a signal
$g(t) = r(t)\,p(t)$

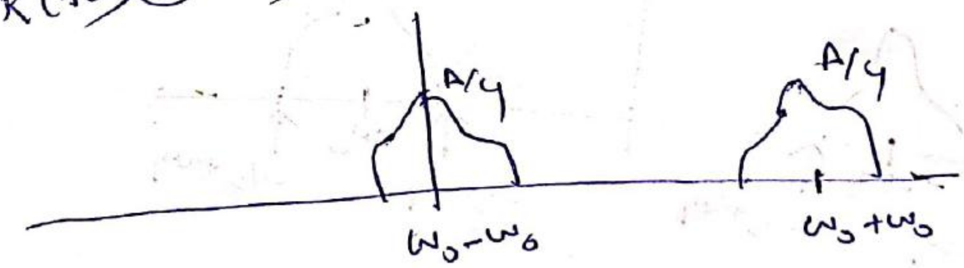
then, sketch the spectrum of $G(j\omega)$
where $G(j\omega)$ is the F.T of $g(t)$

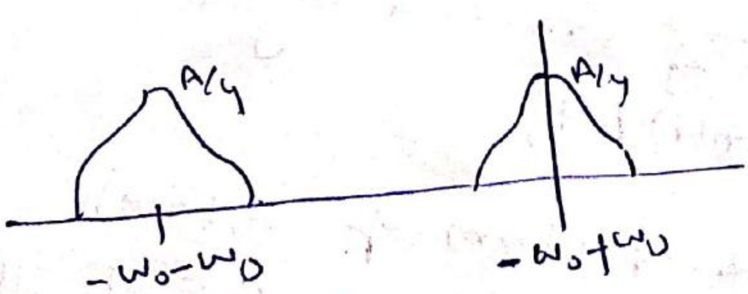






Actual, $R(j\omega) \otimes P(j\omega)$





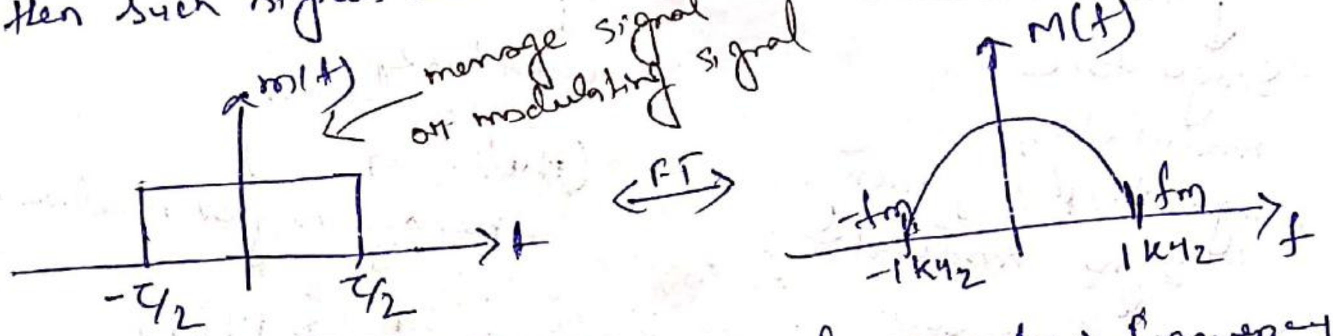
⇒ Concept of Modulation

If $x(t) \longleftrightarrow X(f)$

then $x(t)\cos 2\pi f_0 t \longleftrightarrow \dfrac{X(f-f_0)+X(f+f_0)}{2}$

$$x(t)\left[\frac{e^{j2\pi f_0 t}+e^{-j2\pi f_0 t}}{2}\right]$$

If a signal contains all the significant low frequency then such signals are called as "BASE BAND SIGNAL".

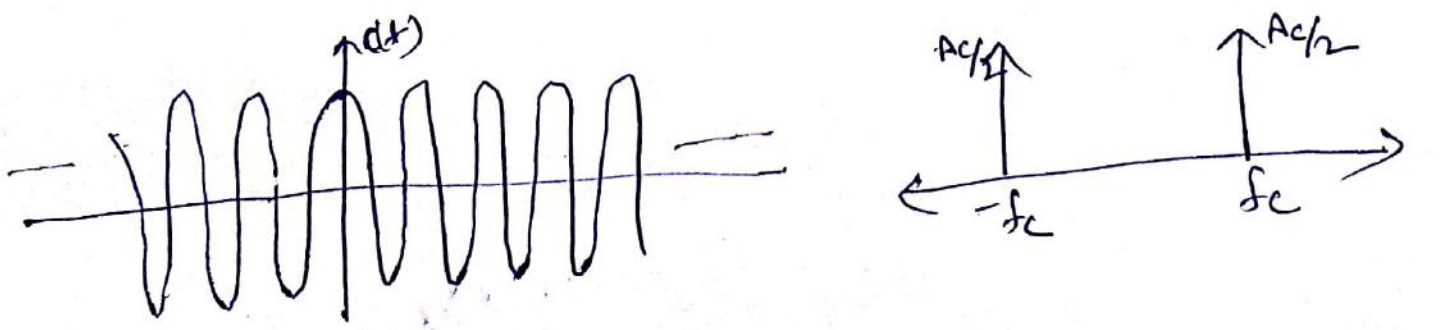


These base band signals have significant low frequency which means they require huge Antenna height, which is impossible to construct.

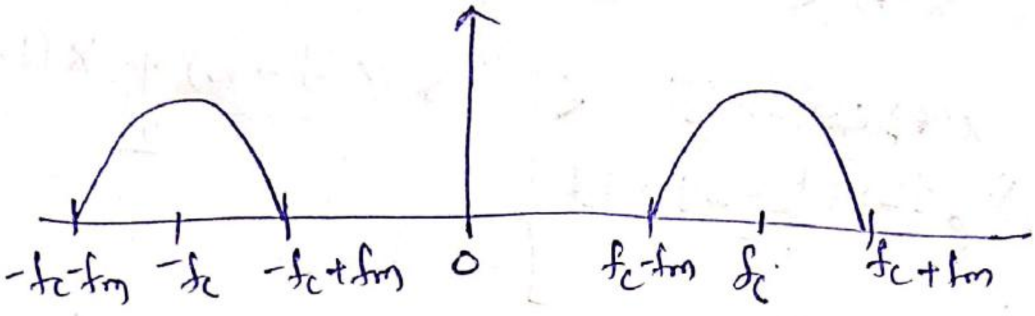
Hence the process of modulation is introduced such that the frequency is increased to Reduce Antenna height.

So that the carrier signal

$c(t) = A_c \cos 2\pi f_c t$, $f_c = 1\,MHz$
$= 1000\,kHz$



Sg $S(t) = c(t) \cdot m(t)$ → modulated signal.

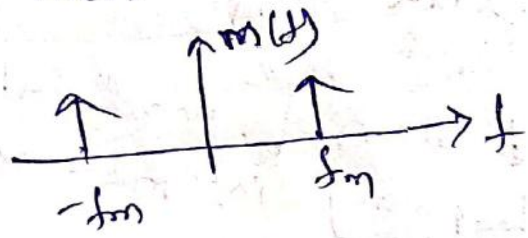


classification of modulation

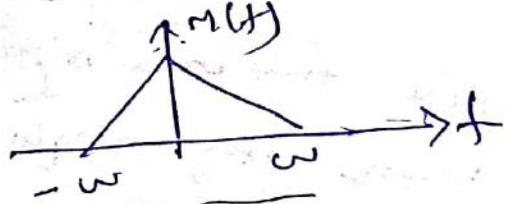
- Single tone modulation
  (single frequency message signal modulation)
- Multi tone modulation.
  (Multi frequency message signal modulation)

Here message signal

$m(t) = A_m \cos 2\pi f_m t$



$m(t) \xleftrightarrow{FT} M(f)$



Range of frequency

MODULATION

Modulation is the process in which one of the parameter (Amplitude, frequency or phase) of the carrier signal will be varied linearly in accordance with message signal amplitude variation.

⇒ Amplitude Modulation

It is the process in which amplitude of the carrier signal will be changed (varied) linearly in accordance with message signal Amplitude variation.

like, if $m(t)$ = message signal

$A_c \cos 2\pi f_c t$ = carrier signal $c(t)$

then the general expression for AM signal

$$S_{AM}(t) = A_c \{1 + K_a m(t)\} \cos 2\pi f_c t$$

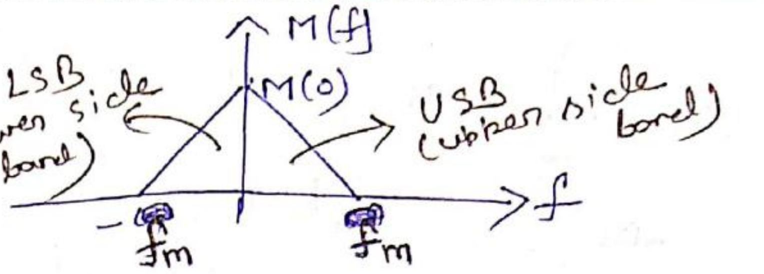
K_a = Amplitude sensitivity of AM modulator

$$S_{AM}(t) = \underbrace{A_c \cos 2\pi f_c t}_{\text{carrier signal}} + \underbrace{A_c K_a m(t) \cos 2\pi f_c t}_{\text{modulated signal}}$$

Adv → Due to additional carrier signal, the demodulation of the AM signal becomes easier and cheaper

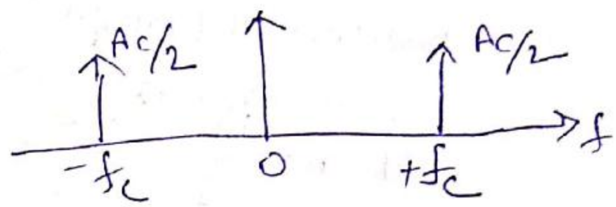
Disadvantage → Additional power is wasted in the ~~transmitting~~ transmitting of form of ~~transm~~ carrier signal.

$m(t) \longleftrightarrow M(f)$



$c(t) = A_c \cos 2\pi f_c t$

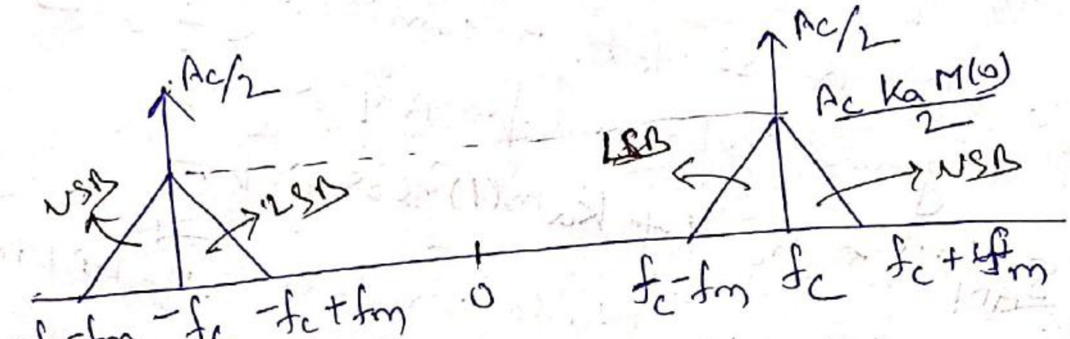
$\xleftrightarrow{F.T} \frac{A_c}{2}\{\delta(f+f_c) + \delta(f-f_c)\}$



$S_{AM}(t) = A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$

$\xleftrightarrow{F.T} \frac{A_c}{2}\{\delta(f+f_c) + \delta(f-f_c)\}$

$+ \frac{A_c K_a M(f)}{2}\{\delta(f+f_c) + \delta(f-f_c)\}$



AM Bandwidth $= (f_c + f_m) - (f_c - f_m)$

$= 2 f_m$

So, AM Bandwidth $= 2 \times$ message signal B.W

↳ **Single tone AM**

Let, $m(t) = A_m \cos 2\pi f_m t$

&, $S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$

using $m(t)$,

$S_{AM}(t) = A_c\{1 + K_a A_m \cos 2\pi f_m t\}\cos 2\pi f_c t$

$S_{AM}(t) = A_c\{1 + \mu \cos 2\pi f_m t\}\cos 2\pi f_c t$

Let, $S_{AM}(t) =$

So,

where, $\boxed{\mu = K_a \cdot A_m = \text{modulation index of AM}}$

$\boxed{\mu \times 100\ \% = \%\ \text{of modulation on depth of modulation}}$

→ The physical significance of depth of modulation is the content of message signal that is stored in the carrier signal is called as depth of modulation.

$$\left.\begin{array}{l}\mu < 1 \rightarrow \text{under modulation} \\ \mu = 1 \rightarrow \text{critical modulation}\end{array}\right\} \rightarrow \text{Generally used}$$

$\mu > 1 \rightarrow$ Over modulation

↳ demodulation of AM signal becomes difficult.

* To what extent the carrier signal is modulated by the message signal is specified by ~~that~~ MODULATION INDEX.

Now, for single tone,

$$S_{AM}(t) = A_c \{1 + \mu \cos 2\pi f_m t\} \cos 2\pi f_c t$$

$$= A_c \cos 2\pi f_c t + A_c \mu \cos 2\pi f_m t \cdot \cos 2\pi f_c t$$

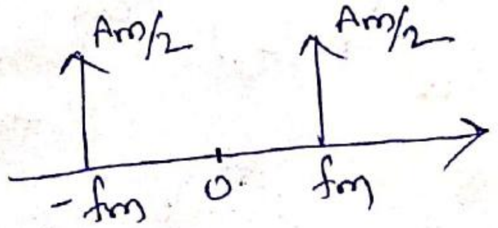
on expanding,

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$$

$A_c \cos 2\pi f_c t$ → carrier component

$\frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$ → Modulated signal

$m(t) = A_m \cos 2\pi f_m t$



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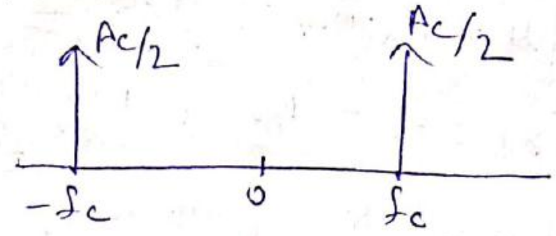
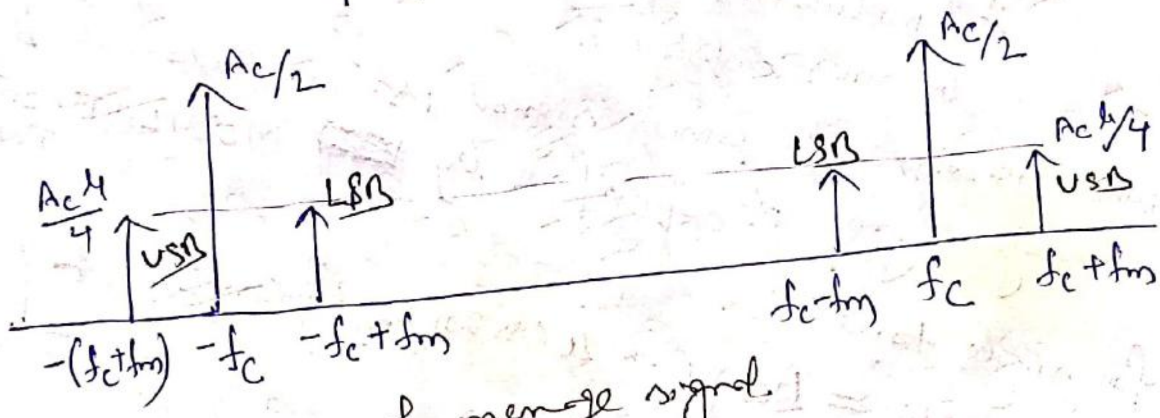
$c(t) = A_c \cos 2\pi f_c t$



$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t + \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m) t$$
$\uparrow$ USB $\quad \uparrow$ LSB

$\xleftrightarrow{F \cdot T}$

$$\frac{A_c}{2} \{ \delta(f - f_c) + \delta(f + f_c) \} + \frac{A_c \mu}{4} \{ \delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m)) \}$$
$$+ \frac{A_c \mu}{4} \{ \delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m)) \}$$



f_m = frequency of message signal.

$\Rightarrow$ <u>Power of the Signal</u>

The total power of AM signal is given as

$$P_t = \underbrace{P_c}_{\text{power of carrier}} + \underbrace{P_{USB}}_{\text{power of USB}} + \underbrace{P_{LSB}}_{\text{power of LSB}}$$

Now, For single tone,

since, $S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2} \cos 2\pi (f_c + f_m) t$
$$+ \frac{A_c \mu}{2} \cos 2\pi (f_c - f_m)$$

$$P_c = \frac{V_{RMS}^2}{R} = \left(\frac{A_c}{\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{A_c^2}{2R}$$

$$\left\{ P_{dc} = \frac{V_m^2}{R} \quad P_{ac} = \frac{V_{rms}^2}{R} \quad V_{rms} = \frac{V_m}{\sqrt{2}} \right.$$

$$P_{USB} = \left(\frac{A_c \mu}{2\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{A_c^2 \mu^2}{8R}$$

$$P_{LSB} = \left(\frac{A_c \mu}{2\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{A_c^2 \mu^2}{8R}$$

$$P_t = \frac{A_c^2}{2R} + \frac{A_c^2 \mu^2}{8R} + \frac{A_c^2 \mu^2}{8R}$$

$$P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\mu^2}{2}\right\}$$

$$P_t = P_c\left(1 + \frac{\mu^2}{2}\right)$$

power of carrier after modulation

Power of carrier before modulation

Now, $P_t = P_c + \frac{P_c \mu^2}{2} = P_c + P_{SB}$

where $P_{SB} = \frac{P_c \mu^2}{2} \Rightarrow P_{USB} + P_{LSB} = \frac{P_c \mu^2}{4}$

$\Rightarrow$ The side band power depends on μ (modulation index)

so, as μ increases, the P_{SB} also increases

at $\mu = 0$, then $P_t = P_c \rightarrow$ no modulation

$\mu = 1$, (100% modulation)

$$P_t = P_c + \frac{P_c}{4} = \frac{3}{2}P_c = 1.5 P_c = P_c + 0.5 P_c$$

and, $S_{AM}(t) = A_c \cos 2\pi f_c t + A_c \cos 2\pi f_m t \times \cos 2\pi f_c t$

→ As, μ increases, from 0 to 1, the total AM power increased by 50 %

→ NOW, $P_c = \frac{2}{3} P_t$

So, $P_c = 0.666 P_t$, So, $P_c = 66.66\%$ of P_t

Now, as, $P_t = P_c + P_{SB}$

$= \frac{2}{3} P_t + P_{SB}$

$\Rightarrow P_{SB} = \frac{1}{3} P_t$

So $P_{SB} = 33.33\%$ of P_t

So, Conclusion,

If, $\mu = 0 \Rightarrow P_c = 100\%$ of $P_t \Rightarrow P_{SB} = 0\%$ of P_t

If $\mu = 1 \Rightarrow P_c = 66.66\%$ of $P_t \Rightarrow P_{SB} = 33.33\%$ of P_t

⇒ Modulation Efficiency (η)

$$\eta = \frac{P_{SB}}{P_t} = \frac{\frac{P_c \mu^2}{2}}{P_c\left(1+\frac{\mu^2}{2}\right)} = \frac{\mu^2}{2+\mu^2}$$

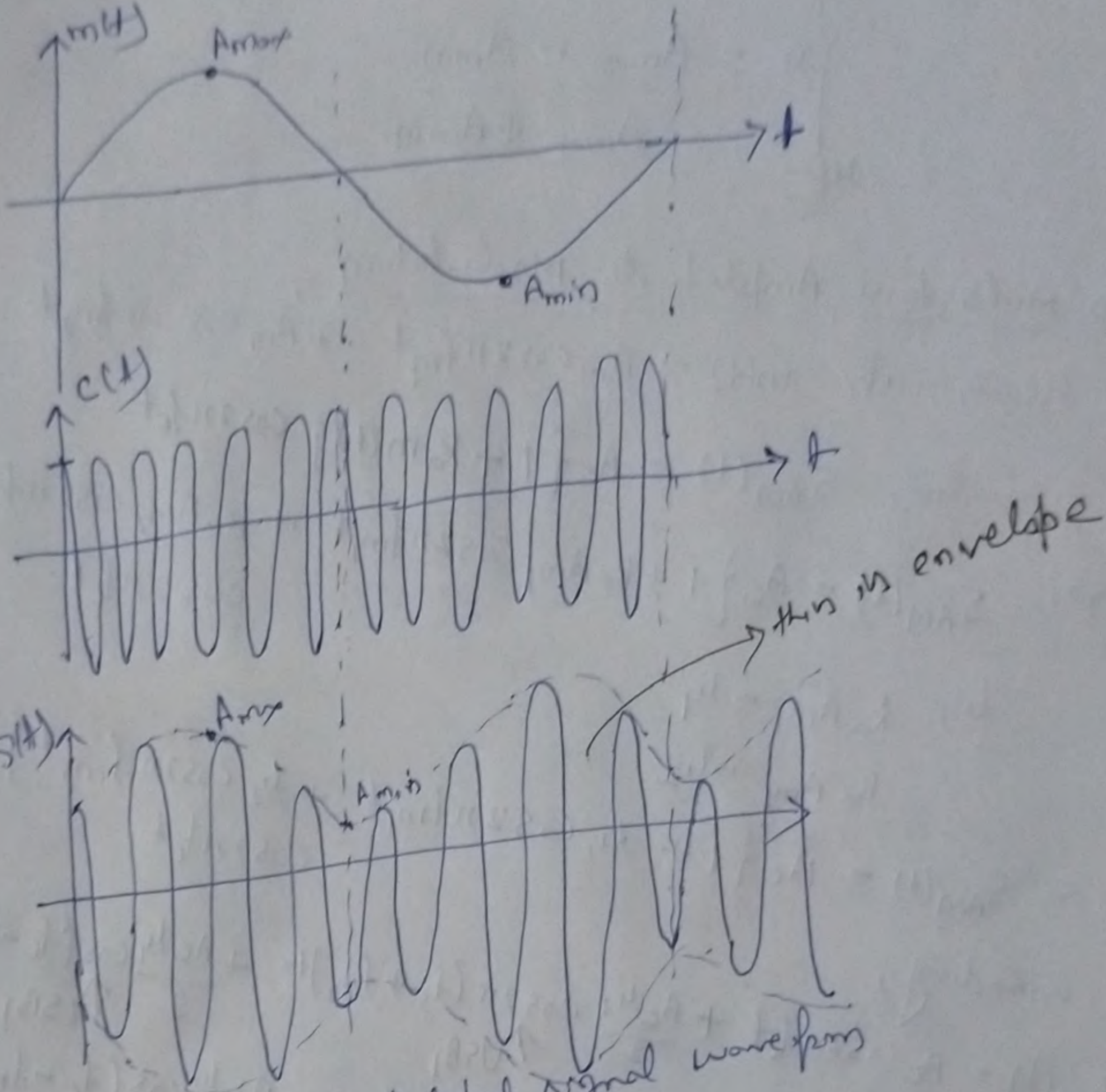
$$\boxed{\eta = \frac{\mu^2}{2+\mu^2}}$$

for, $\mu = 0$; $\eta = 0 \Rightarrow P_{SB} = 0\%$ of P_t, $P_c = 100\%$ of P_t

$\mu = 0.707$, $\eta = 0.2 \Rightarrow P_{SB} = 20\%$ P_t, $P_c = 80\%$ of P_t

$\mu = 1$, $\eta = 0.33$, $P_{SB} = 33.3\%$ of P_t, $P_c = 66.7\%$ of P_t

↳ Amplitude modulated waveform.



modulated signal wave form

[redacted]

Amplitude modulated wave form

$$s(t) = A_c(1+\mu\cos 2\pi f_m t)\cos 2\pi f_c t$$

since $\cos 2\pi f_c t|_{max} = 1$

$\cos 2\pi f_m t|_{max} = 1$

So $$\begin{cases} A_{max} = A_c(1+\mu) \\ A_{min} = A_c(1-\mu) \end{cases}$$

$$A_{max} + A_{min} = 2A_c$$

So, or $$A_c = \frac{A_{max} + A_{min}}{2}$$

Substituting the value of A_c in A_{max} & A_{min}

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

→ Multitone Amplitude Modulation

Assume, if $m(t) = A_m \cos 2\pi f_{m_1} t + A_m \cos 2\pi f_{m_2} t$

and, $S_{AM}(t) = A_C\{1 + K_a m(t)\} \cos 2\pi f_c t$

so, $S_{AM}(t) = A_C\{1 + K_a A_{m_1} \cos 2\pi f_{m_1} t + K_a A_{m_2} \cos 2\pi f_{m_2} t\} \cos 2\pi f_c t$

let $K_a A_{m_1} = \mu_1$

$K_a A_{m_2} = \mu_2$

So, $S_{AM}(t) = A_C\{1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t\} \cos 2\pi f_c t$

on expanding,

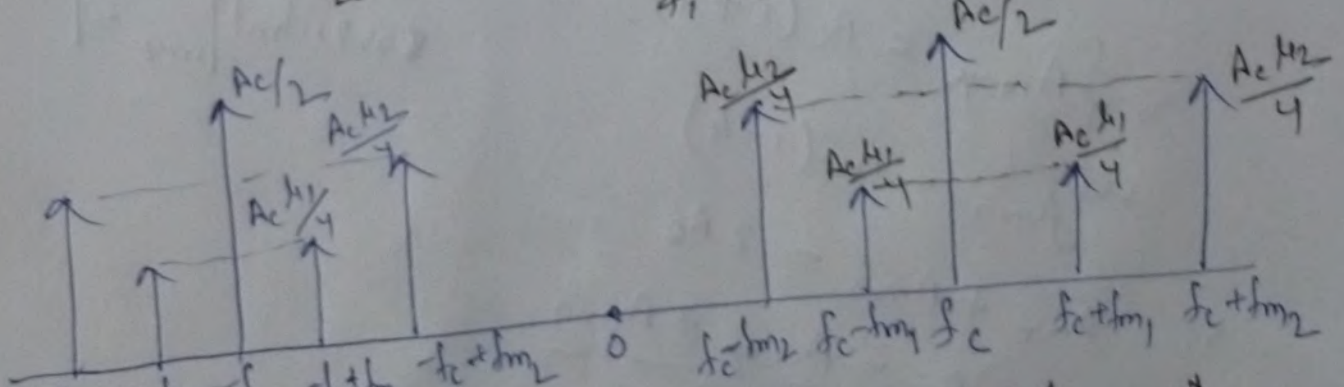
$$S_{AM}(t) = A_c \cos 2\pi f_c t + \underbrace{\frac{A_c \mu_1}{2} \cos 2\pi (f_c + f_{m_1}) t}_{USB_1} + \underbrace{\frac{A_c \mu_1}{2} \cos (f_c - f_{m_1}) t}_{LSB_1}$$

$$+ \underbrace{\frac{A_c \mu_2}{2} \cos 2\pi (f_c + f_m) t}_{USB_2} + \underbrace{\frac{A_c \mu_2}{2} \cos (f_c - f_m) t}_{LSB_2}$$

for plotting the spectrum

Let us assume, $f_{m_2} > f_{m_1}$

& $\mu_2 > \mu_1$



← $BW = 2 f_{m_2}$ →

$= 2 \times$ max^m frequency of message signal.

→ Total power of AM (multitone)

The total power (P_t) is given as:

$$P_t = P_c + P_{SB}$$

$$P_t = P_c + P_{USB(total)} + P_{LSB(total)}$$
$$= P_c + P_{USB_1} + P_{USB_2} + P_{LSB_1} + P_{LSB_2}$$

$$P_c = \frac{A_c^2}{2R} \quad ; \quad P_{USB_1} = \frac{\left(\frac{A_c \mu_1}{2}\right)^2}{2R} = \frac{A_c^2 \mu_1^2}{8R} = P_{LSB_1}$$

$$P_{USB_2} = \frac{\left(\frac{A_c \mu_2}{2}\right)^2}{2R} = \frac{A_c^2 \mu_2^2}{8R} = P_{LSB_2}$$

So $$P_t = \frac{A_c^2}{2R} + 2 \cdot \frac{A_c^2 \mu_1^2}{8R} + 2 \cdot \frac{A_c^2 \mu_2^2}{8R}$$

$$= \frac{A_c^2}{2R}\left\{1 + \frac{\mu_1^2 + \mu_2^2}{2}\right\}$$

So $$P_t = P_c\left\{1 + \frac{\mu_t^2}{2}\right\}$$

where $\mu_t = \sqrt{\mu_1^2 + \mu_2^2}$
↓
total modulation index

So, efficiency $$\eta = \frac{\mu_t^2}{2 + \mu_t^2}$$

Q. An AM signal is given by

$$s(t) = \{20 + 4\cos 8\pi \times 10^4 t + 8\cos \pi \times 10^5 t\}\cos\omega_c t$$

find, μ_t, P_t, BW & η?

Ans $$s(t) = 20\left\{1 + \frac{4}{20}\cos 8\pi \times 10^4 t + \frac{8}{20}\cos\pi \times 10^5 t\right\}\cos 2\pi f_c t$$

and $$s_{AM}(t) = A_c\{1 + \mu_1 \cos 2\pi f_{m_1} t + \mu_2 \cos 2\pi f_{m_2} t\}$$

So, $A_c = 20\,V$, $\mu_1 = \frac{4}{20}$, $\mu_2 = 8/20$

$f_{m_1} = 40\, kHz,\ f_{m_2} = 50\, kHz$

So, $BW = 2 f_{max} = 2 \times 50\, kHz = 100\, kHz$

$\mu_t = \sqrt{\mu_1^2 + \mu_2^2} = \sqrt{(0.2)^2 + (0.4)^2} = 0.44$

$P_c = \frac{A_c^2}{2R} = \frac{400}{2 \times 1} = 200\, W$

$P_t = P_c \left\{1 + \frac{\mu_t^2}{2}\right\} = 200 \left\{1 + \frac{(0.44)^2}{2}\right\}$

$= 220\, W$

So, $\eta = \frac{\mu_t^2}{2 + \mu_t^2} = \frac{(0.44)^2}{2 + (0.44)^2} = 0.09 = 9\%$

→ **Current & Voltage Relation in AM**

$P_t = P_c \left\{1 + \frac{\mu^2}{2}\right\}$

So, $I_t^2 R = I_c^2 R \left\{1 + \frac{\mu^2}{2}\right\}$ or, $\frac{V_t^2}{R} = \frac{V_c^2}{R}\left\{1 + \frac{\mu^2}{2}\right\}$

$\Rightarrow \boxed{I_t = I_c \sqrt{1 + \frac{\mu^2}{2}}}$ or, $\boxed{V_t = V_c \sqrt{1 + \frac{\mu^2}{2}}}$

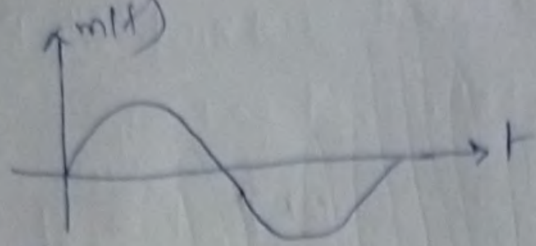
→ AM waveform under critical and over modulation

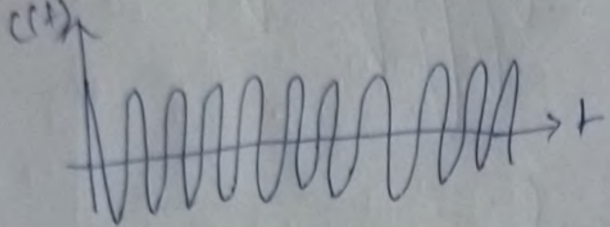
For under modulation

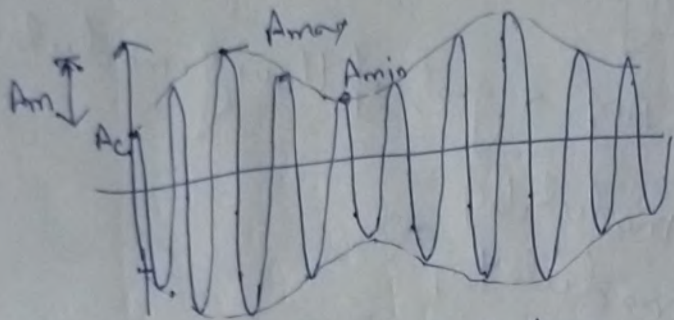
$\mu < 1$

$\mu = \frac{A_m}{A_c} < 1 \Rightarrow A_m < A_c$

$A_{min} = A_c\{1 - \mu\} = +ve$





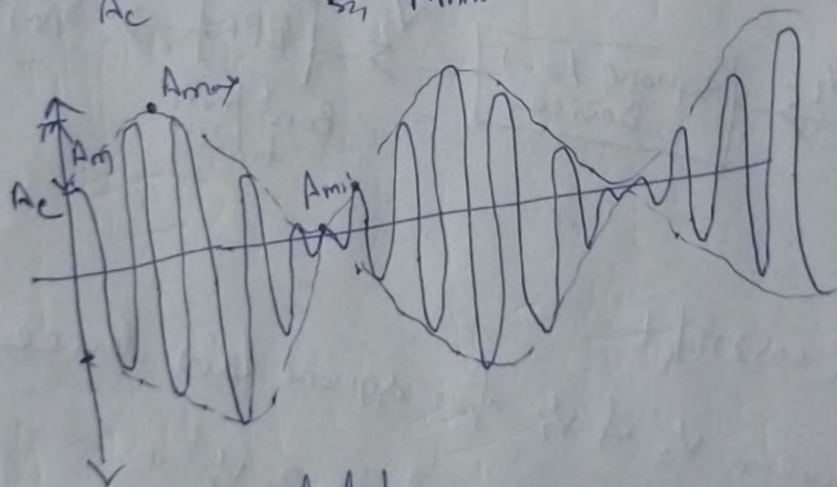


↳ for critical modulation

$\mu = 1$

$\frac{A_m}{A_c} = 1 \Rightarrow A_m = A_c$

so $A_{min} = A_c\{1-\mu\} = 0$

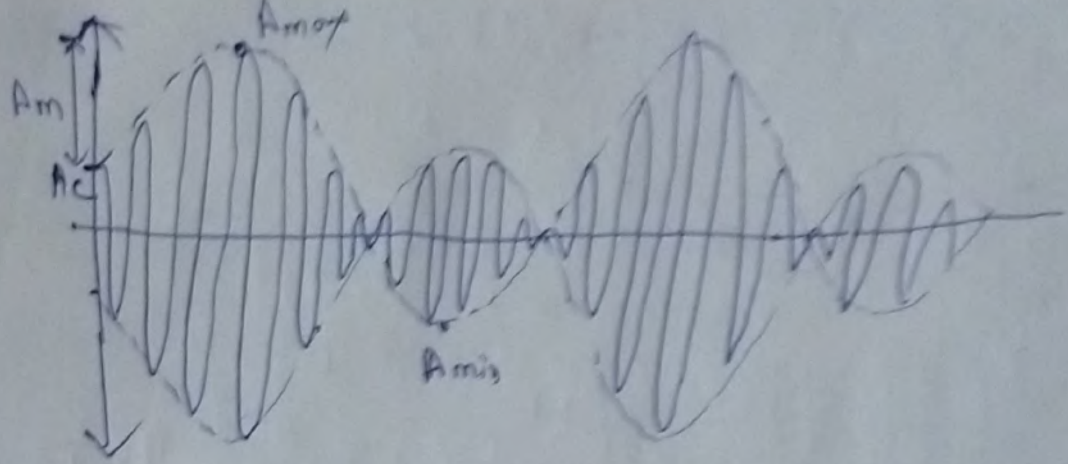


↳ for over modulation,

$\mu > 1$

$\frac{A_m}{A_c} > 1 \Rightarrow A_m > A_c$

so $A_{min} = A_c\{1-\mu\} = -ve$



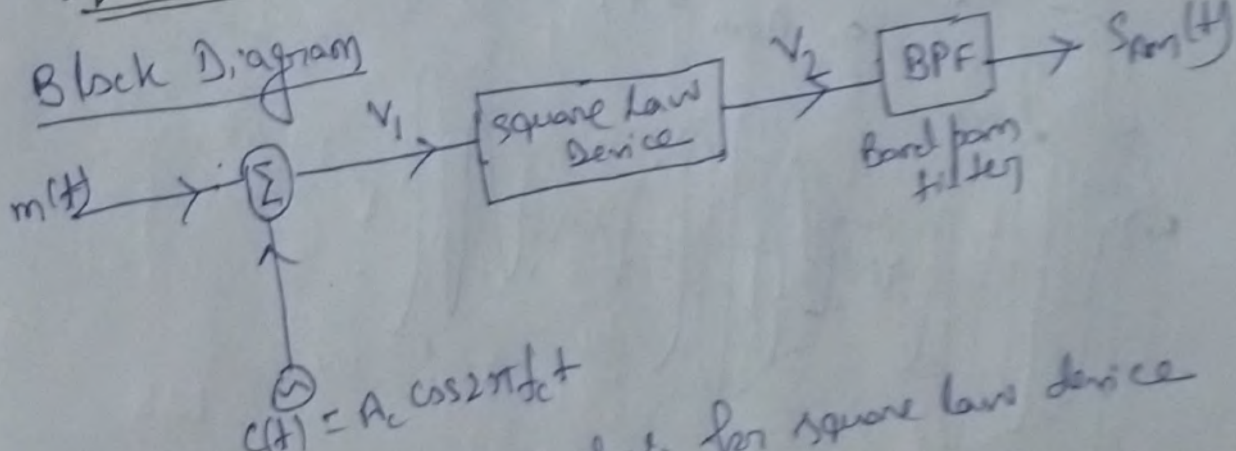
⇒ **Generation of AM Signal :-**

For the generation of AM signal, following modulators are used:

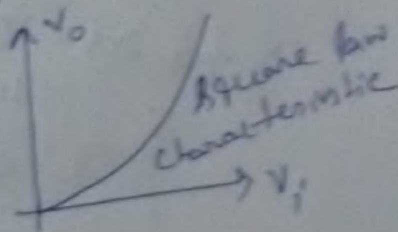
i) square law modulator
ii) Switching modulator

· **Square Law Modulator**

Block Diagram



The relation between v_0 & v_i for square law device is given as: $v_0 = a_0 v_i + a_1 v_i^2 + a_2 v_i^3 + \dots$

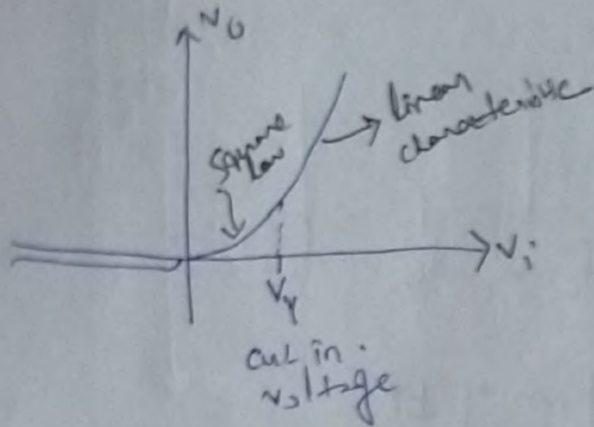


where, $a_0, a_1, a_2 \dots$ are square law constant.

so, from the block diagram,

$$v_1 = m(t) + c(t)$$
$$= m(t) + A_c \cos 2\pi f_c t$$

The square law characteristic we achieve through diode (63)



when, the applied voltage $V_i < V_\gamma$ (cut in voltage of diode) then diode exhibits square law characteristics.

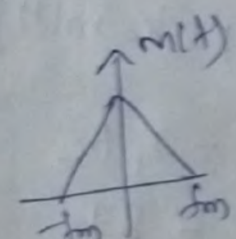
So, if $V_1 = m(t) + c(t)$
then, $m(t)$ & $c(t)$ should be such that the peak voltage of V_1 must be less than V_γ of diode.

→ The o/p of square law device,

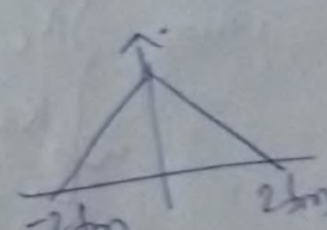
$$V_2 = a_1 V_1 + a_2 V_1^2 + \underbrace{a_3 V_1^3 + \cdots}_{\text{we neglect the other terms}}$$

$$V_2 = a_1\{m(t) + A_c \cos 2\pi f_c t\} + a_2\{m^2(t) + A_c^2 \cos^2 2\pi f_c t + 2A_c m(t) \cos 2\pi f_c t\} \quad \text{— (A)}$$

Now, let, $m(t) \longleftrightarrow M(f)$



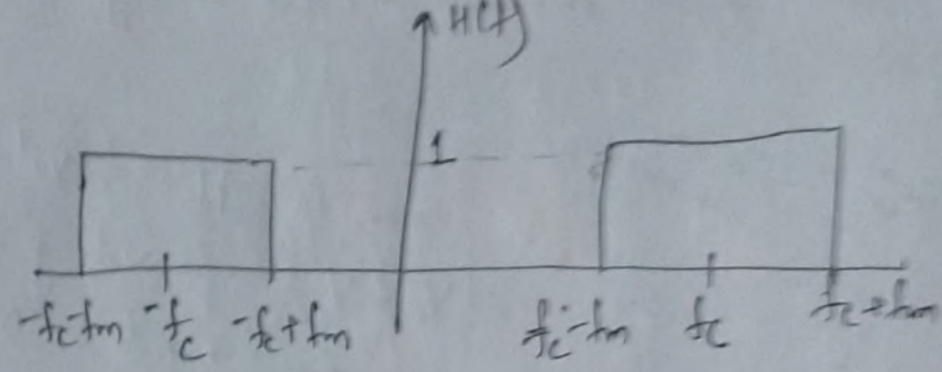
So, $m^2(t) \longleftrightarrow M(f) \circledast M(f)$



$$\cos^2 2\pi f_c t = \frac{1 + \cos 4\pi f_c t}{2}$$

$$= \underbrace{\frac{1}{2}}_{\text{dc term}} + \frac{\cos 4\pi f_c t}{2} \rightarrow \text{Its F.T occurs at } 2f_c$$

So, if we see carefully, eq^n (A) and, if our BPF filter are like,



↳ then the o/p of v_2 after passing BPF

$$(BPF)_{o/p} = a_1 A_c \cos 2\pi f_c t + a_2 \cdot 2A_c\, m(t) \cos 2\pi f_c t$$

the signal exist which is around the 'f_c'.

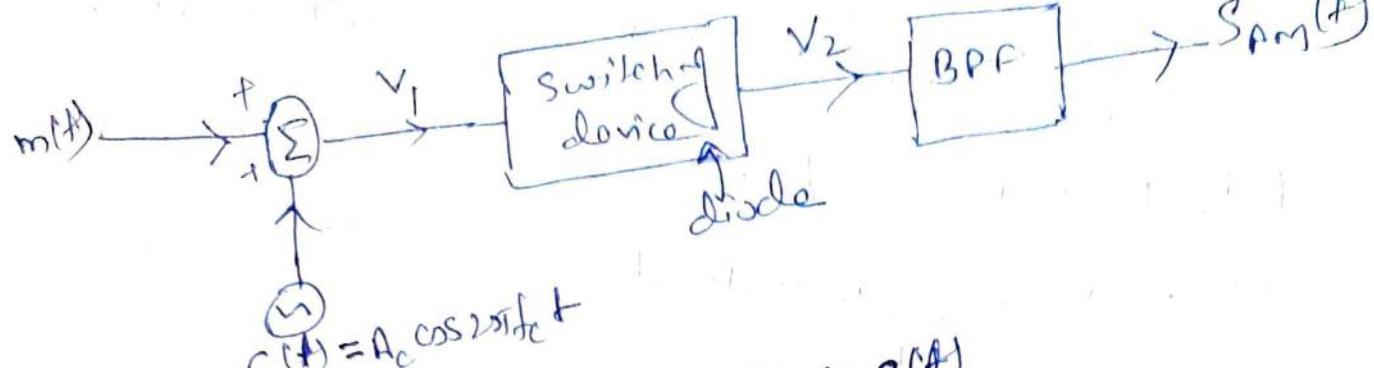
$$(BPF)_{o/p} = a_1 A_c \left[1 + \frac{2a_2}{a_1} m(t)\right] \cos 2\pi f_c t = S_{Am}(t)$$

comparing with standard AM signal

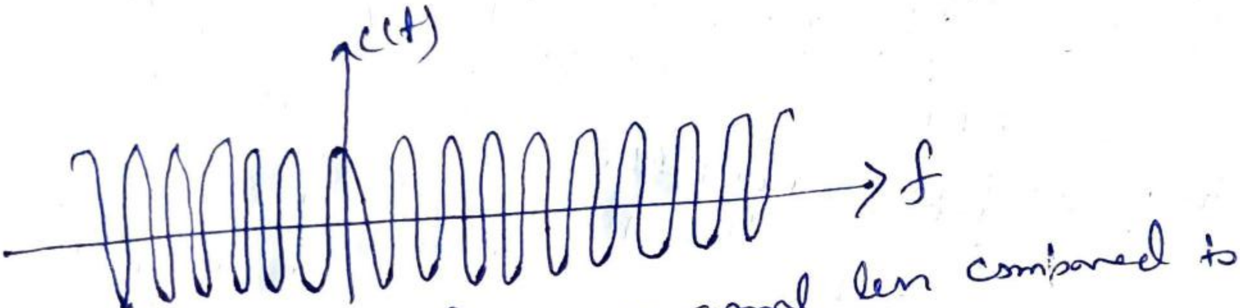
$$A_c \{1 + k_a m(t)\} \cos 2\pi f_c t$$

where,
$$A_c = a_1 A_c$$
$$k_a = \frac{2a_2}{a_1}$$

↳ Switching Modulator



i/p of diode $\Rightarrow V_1 = m(t) + c(t)$
$= m(t) + A_c \cos 2\pi f_c t$



In general the strength of message signal less compared to carrier signal so, the diode is mainly controlled by carrier signal.

so, when c(t) is +ve, diode is forward bias i.e, short ckt

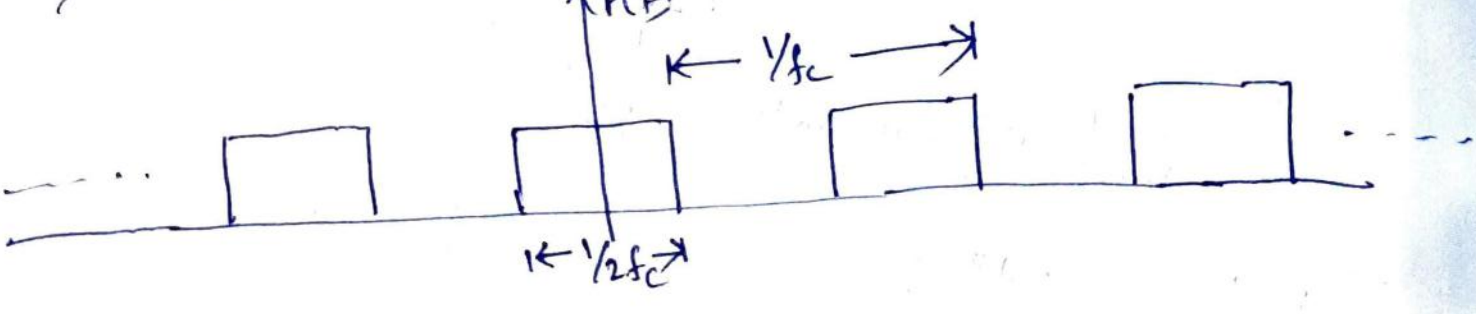
so, $V_2 = V_1$

when c(t) is -ve diode is Reverse bias i.e, open ckt

then, $V_2 = 0$

So, the o/p of diode switches between V_1 and 0 with the time interval of $1/f_c$
(period)

↳ So, we can write, $V_2 = V_1 \cdot p(t)$ —— (A)



So, if we find out the fourier series

$$f(t) = a_0 + \sum_{n=1}^{\infty} \{a_n \cos n\omega_0 t + b_n \sin n\omega_0 t\} \; ; \omega_0 = \frac{2\pi}{T}$$

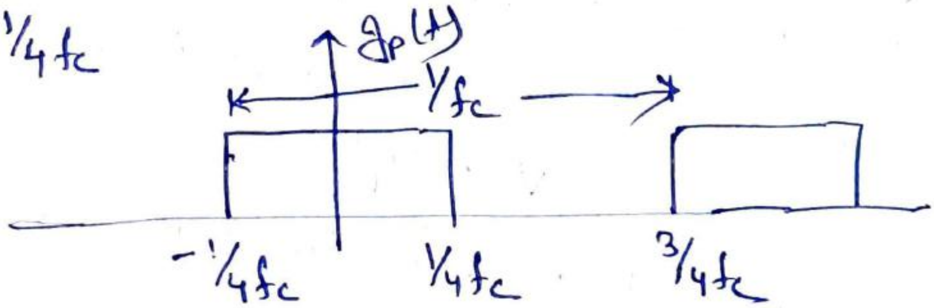
Since, $p(t)$ is even, so, $b_n = 0$

$$p(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t$$

Now, $$\boxed{a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt} = \frac{1/2f_c}{1/f_c} = \frac{1}{2}$$

and, $$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} p(t) \cos n\omega_0 T\, dt$$

$$= \frac{2}{1/f_c}\int_{-1/4f_c}^{3/4f_c} p(t) \cos n\, 2\pi f_c t\, dt$$



So, $$a_n = \frac{2}{1/f_c}\int_{-1/4f_c}^{3/4f_c} 1 \cdot \cos 2\pi n f_c t\, dt$$

$$= 2f_c \times \left.\frac{\sin 2\pi n f_c t}{2\pi n f_c}\right|_{-\frac{1}{4f_c}}^{1/4f_c}$$

$$= \frac{1}{n\pi}\left\{\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right\}$$

$$\boxed{a_n = \frac{2}{n\pi}\sin\frac{n\pi}{2}}$$

So, $p(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin\frac{n\pi}{2} \cdot \cos n 2\pi f_c t$

So, from equation (A)

$$V_2 = V_1 \cdot p(t)$$
$$= \left\{ m(t) + A_c \cos 2\pi f_c t \right\} \left\{ \frac{1}{2} + \frac{2}{\pi} \cos 2\pi f_c t - \frac{2}{3\pi} \cos 2\pi (3f_c) t + \frac{2}{5\pi} \cos 2\pi (5f_c t) + \cdots \right\}$$

So, again, if V_2 is then passed through a BPF so its o/p is given as

$$(BPF)_{o/p} = S_{AM}(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \cos 2\pi f_c t$$
$$= \frac{A_c}{2} \left\{ 1 + \frac{4}{\pi A_c} m(t) \right\} \cos 2\pi f_c t$$

Comparing with the standard equation

$$A_c' = \frac{A_c}{2}$$

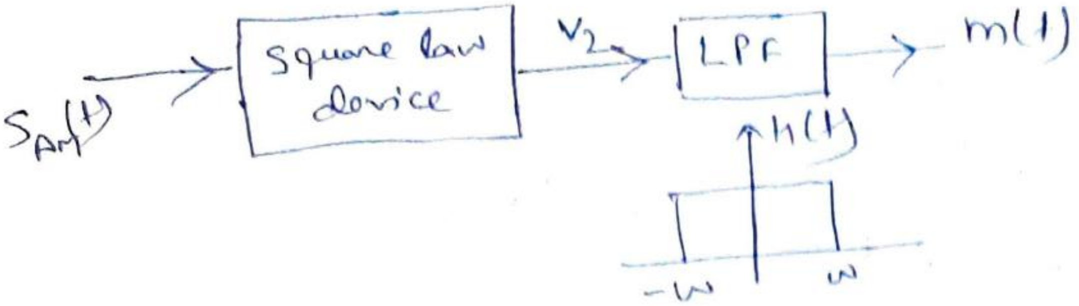
and, $K_a = \frac{4}{\pi A_c}$

⇒ Demodulation of AM Signal.

The demodulation that are used

(i) Square law demodulator
(ii) Envelope detector (diode detector) } $\mu \leq 1$
(iii) Synchronous detector } any value of μ

→ Square Law Demodulator.



As, $S_{AM}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$

$= A_c \cos 2\pi f_c t + K_a m(t) \cos 2\pi f_c t \cdot A_c$

The o/p of the square law device may be given as

$$V_2 = a_1 S_{AM}(t) + a_2\{S_{AM}(t)\}^2$$

$$= a_1\{A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t\}$$
$$+ a_2\{A_c^2 \cos^2 2\pi f_c t + K_a^2 m^2(t) \cos^2 2\pi f_c t \cdot A_c^2$$
$$+ 2K_a A_c^2 m(t) \cos^2 2\pi f_c t\}$$

blocked by blocking capacitor

So, $V_2 = \left[\cancel{a_1 A_c \cos 2\pi f_c t} + \cancel{a_1 A_c K_a m(t) \cos 2\pi f_c t} + \frac{a_2 A_c^2}{2}(1 + \cancel{\cos 4\pi f_c t})\right.$

$\left. + \frac{a_2 A_c^2 K_a^2 m^2(t)}{2}(1 + \cancel{\cos 4\pi f_c t}) + \frac{a_2 2 A_c^2 K_a m(t)}{2}(1 + \cancel{\cos 4\pi f_c t})\right]$

After passing through LPF

$$(LPF)_{o/p} = \underbrace{\frac{a_2 A_c^2 K_a^2 m^2(t)}{2}}_{\text{unwanted signal or noise}} + \underbrace{a_2 A_c^2 K_a m(t)}_{\text{expected signal}}$$

(19)

If, $\frac{S}{N}$ (signal to noise ratio) $>>> 1$; $m(t)$ can be reconstructed

$\frac{S}{N} < 1$ $m(t)$ can't be reconstructed

So, $$\frac{S}{N} = \frac{a_2 A_c^2 k_a m(t)}{\frac{a_2 A_c^2 k_a^2 m^2(t)}{2}} = \frac{2}{k_a m(t)}$$

If $m(t) = A_m \cos 2\pi f_m t$

so $$\frac{S}{N} = \frac{2}{k_a \cdot A_m \cos 2\pi f_m t}$$

So, $$\frac{S}{N} = \frac{2}{\mu \cos 2\pi f_m t}$$

As, $-1 \leq \cos 2\pi f_m t \leq 1$

So, for max^n $\frac{S}{N} = \frac{2}{\mu}$ where, $\cos 2\pi f_m t = 1$

So, $\frac{S}{N} \propto \frac{1}{\mu}$

for, getting $\frac{S}{N} = 100$, $\mu = 0.02$

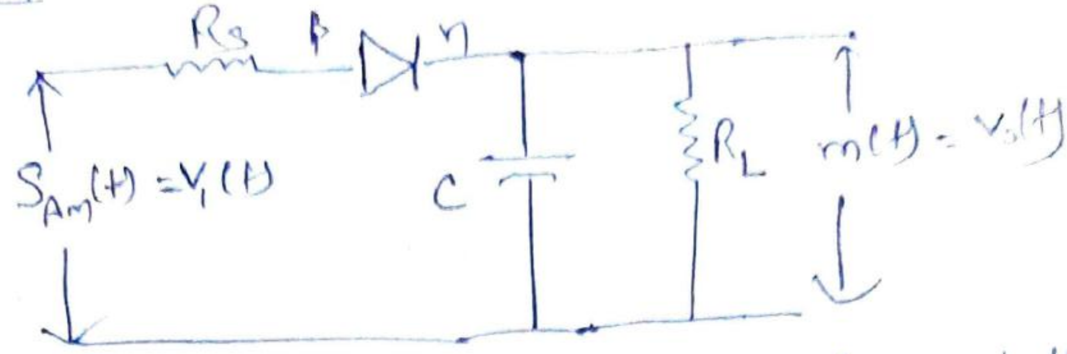
for, such small value of μ, η (efficiency) will be worstly affected i.e, low share of sideband power in AM signal.

So, this method is practically not preferred for AM demodulation.

→ Envelope Detector

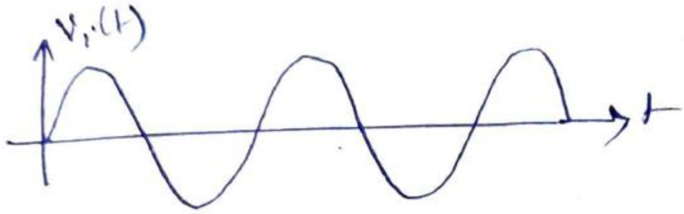
it is very simple & cheaper

Ckt Diagram

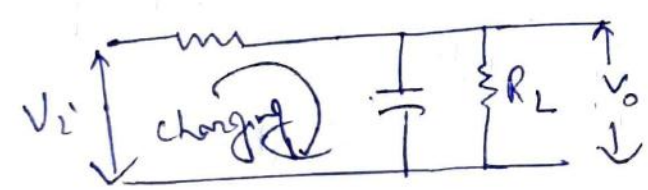


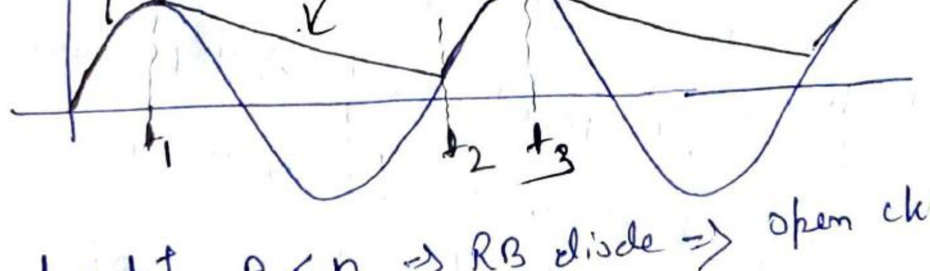
the +ve envelope of applied signal will produces at the o/p so it is called as Envelope Detector.

like,

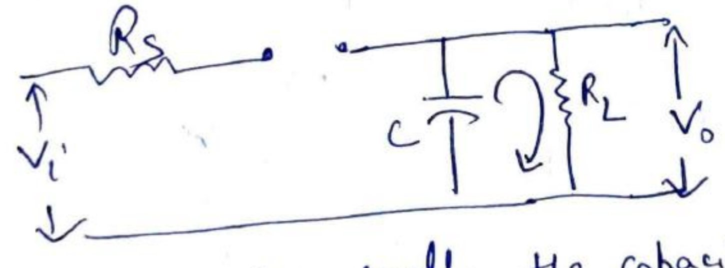


if, $t = 0^+$ $p > n$ diode is S.C





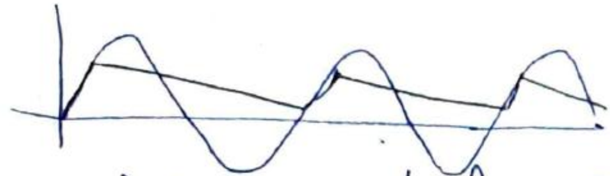
for $t = t_1^+$ $p < n \Rightarrow$ RB diode $\Rightarrow$ open ckt



if $\tau = R_L \cdot C$ is very small, the capacitor rapidly discharges to zero

if τ(time const.) $= R_L \cdot C$ is very high, the capacitor discharges slowly.

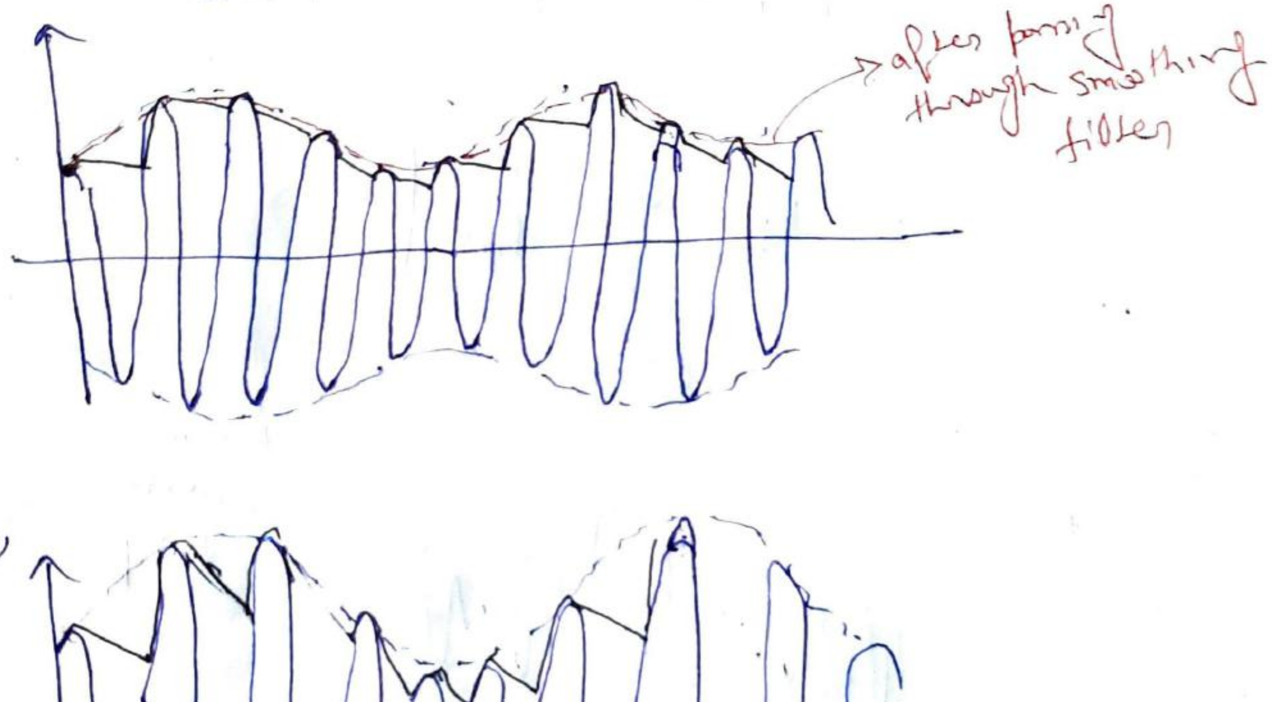
If the input, $R_s \cdot C$ is very high



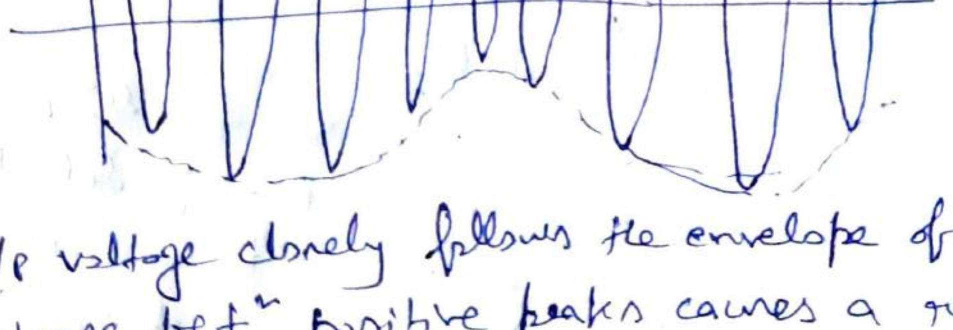
If $R_s \cdot C$ is very high, even before capacitor voltage reaches to peak voltage of input, diode becomes Reverse bias and capacitor will be discharged.

↳ For efficient demodulation, the input $R_s C$ should be very small and $R_L C$ should be high.

↳ If AM signal is applied to the diode detector



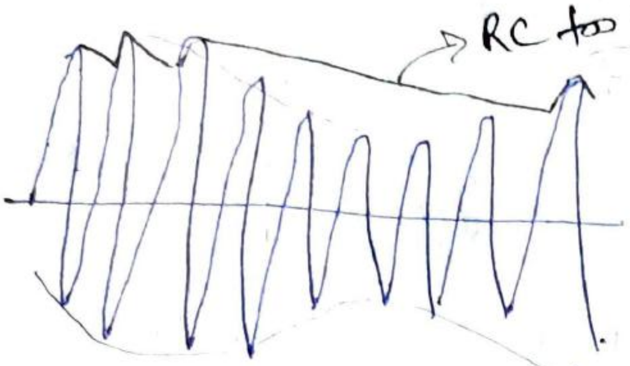
Analysis,



The o/p voltage closely follows the envelope of the input. Capacitor discharge betn positive peaks causes a ripple signal of frequency ω_c in the output. This ripple can be reduced by increasing the time constant RC so that the capacitor discharge very little betn the positive peaks

$$RC >> 1/\omega_c \text{ or, } 1/f_c$$

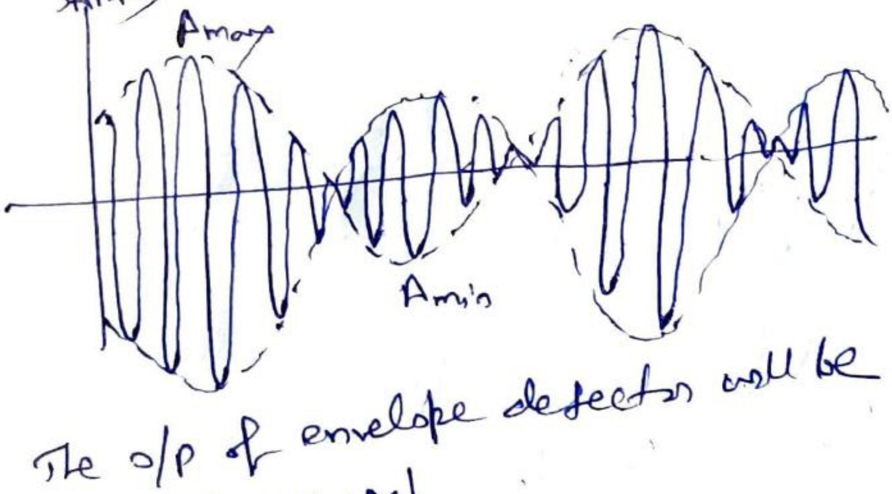
But making RC too large, however would make it impossible for the capacitor voltage to follow the envelope



Thus RC should be large compared to $1/f_c$ but should be small compared to $1/f_m$

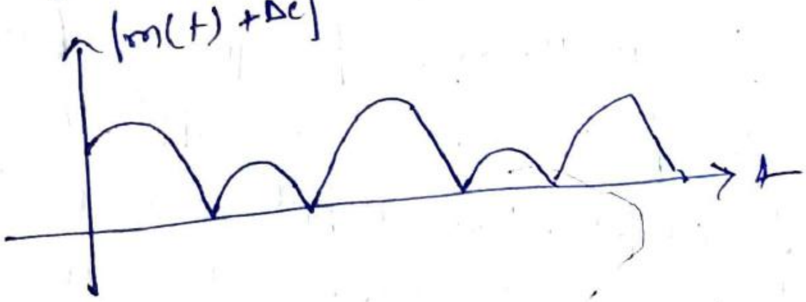
So, $$\boxed{\frac{1}{f_c} <<< R_L C << \frac{1}{f_m}}$$

↳ In case of overmodulation,



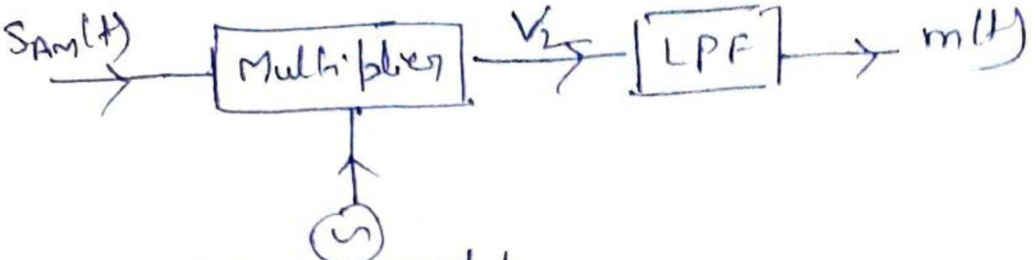
A_{min} = -ve
overmodulated
$\mu > 1$

The o/p of envelope detector will be



So, for, $\mu > 1$ the Envelope detector fails to construct original signal.

↳ Synchronous Detection



$c(t) = A_c \cos 2\pi f_c t$

$$S_{Am}(t) = A_c\{1 + K_a m(t)\} \cos 2\pi f_c t$$
$$= A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t$$

then multiplier o/p, $V_2 = S_{Am}(t) \times c(t)$

$$= \{A_c \cos 2\pi f_c t + A_c K_a m(t) \cos 2\pi f_c t\} A_c \cos 2\pi f_c t$$

$$= A_c^2 \cos^2 2\pi f_c t + A_c^2 K_a m(t) \cos^2 2\pi f_c t$$
$$= \frac{A_c^2}{2}(1 + \cos 4\pi f_c t) + \frac{A_c^2 K_a m(t)}{2}(1 + \cos 4\pi f_c t)$$

So, $\left\{ (LPF)_{o/p} = \frac{A_c^2}{2} K_a m(t) \right\}$

Double Side Band Suppressed Carrier (DSB-SC)

Assume, message signal = $m(t)$

carrier signal = $c(t) = A_c \cos 2\pi f_c t$

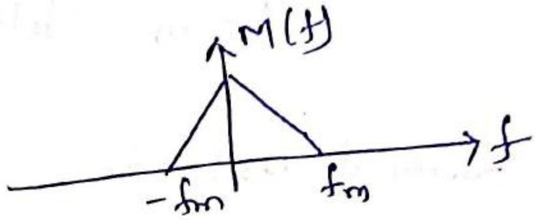
↳ General exp of DSB-SC signal:

$$S_{DSB}(t) = m(t)\, c(t)$$

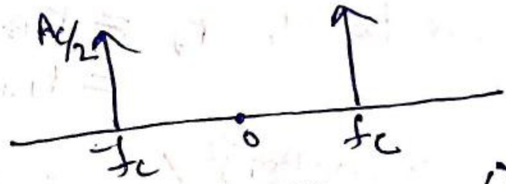
$$\boxed{S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t}$$

frequency spectrum :-

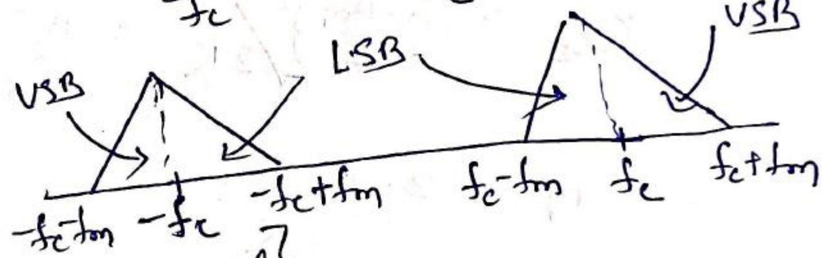
$m(t) \xleftrightarrow{F.T}$



$c(t) = A_c \cos 2\pi f_c t$



$S_{DSB}(t) \xleftrightarrow{F.T}$



$\boxed{\text{Bandwidth} = 2 \times \text{message signal}}$

↳ Single tone DSB

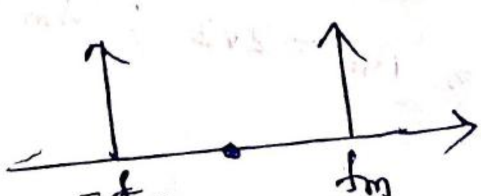
$m(t) = A_m \cos 2\pi f_m t$

so, $S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$

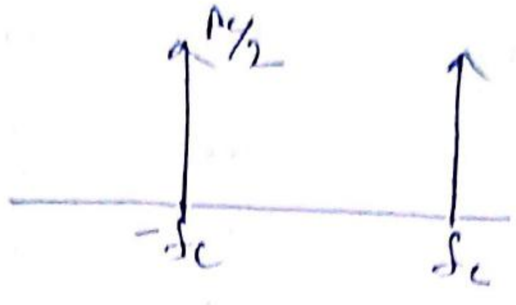
$= A_c A_m \cos 2\pi f_m t \cdot \cos 2\pi f_c t$

$$S_{DSB}(t) = \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c + f_m) t}_{USB} + \underbrace{\frac{A_c A_m}{2} \cos 2\pi (f_c - f_m) t}_{LSB}$$

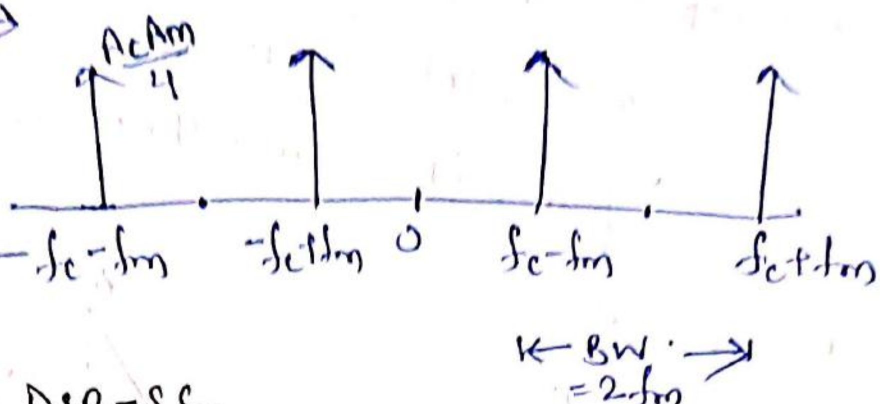
$m(t) \xrightarrow{F.T}$



$c(t) = A_c \cos 2\pi f_c t \longleftrightarrow$



$S_{DSB}(t) \longleftrightarrow$



↳ The total power of DSB-SC

The total power, $P_t = P_{SB}$

$= P_{USB} + P_{LSB}$

$$P_{USB} = \left(\frac{A_c A_m}{2}\right)^2 \Big/ 2R = \frac{A_c^2 A_m^2}{8R} = P_{LSB}$$

so, $P_t = 2 \cdot \dfrac{A_c^2 A_m^2}{8R}$

or, $\boxed{P_t = \dfrac{A_c^2 A_m^2}{4R}}$

↳ Modulation efficiency

as, $\eta = \dfrac{P_{SB}}{P_t} = \dfrac{P_t}{P_t} = 1 = 100\%$

Q. A carrier of $20\sqrt{2} \cos \pi \times 10^5 t$ is DSB modulated by a message signal of $2\sqrt{2} \cos \pi \times 10^3 t$. Find all the parameter and plot the spectrum.

since, $m(t) = 2\sqrt{2} \cos \pi \times 10^3 t$

so, $A_m = 2\sqrt{2}$; $f_m = \dfrac{10^3}{2}$ Hz $= 0.5$ kHz

$c(t) = 20\sqrt{2}\cos\pi \times 10^5 t$

$A_c = 20\sqrt{2}$; $f_c = \frac{10 \times 10^4}{2} = 50$ kHz

→ Now, $P_t = \frac{A_c^2 A_m^2}{4R} = \frac{800 \times 8}{4 \times 1}$.

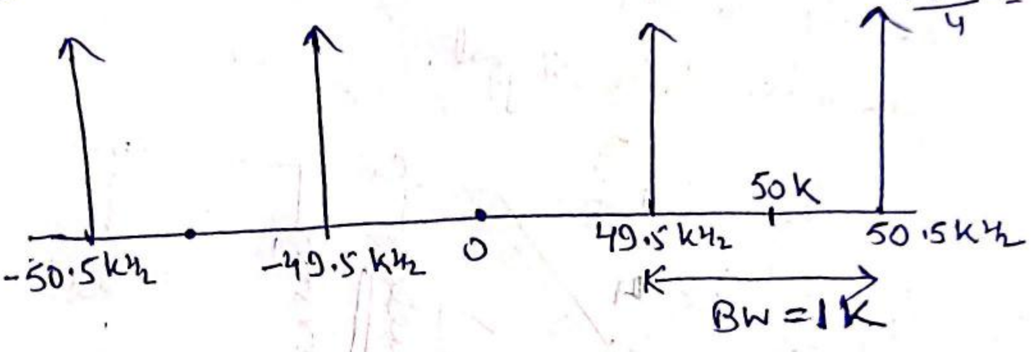
So, $P_t = 1600$ W   Normalized power for $R = 1\Omega$

→ Bandwidth $= 2 f_m = 2 \times 0.5$ K
$= 1$ KHz

→ $P_{USB} = P_{LSB} = \frac{P_t}{2} = 800$ W

→ $\eta = 100\%$

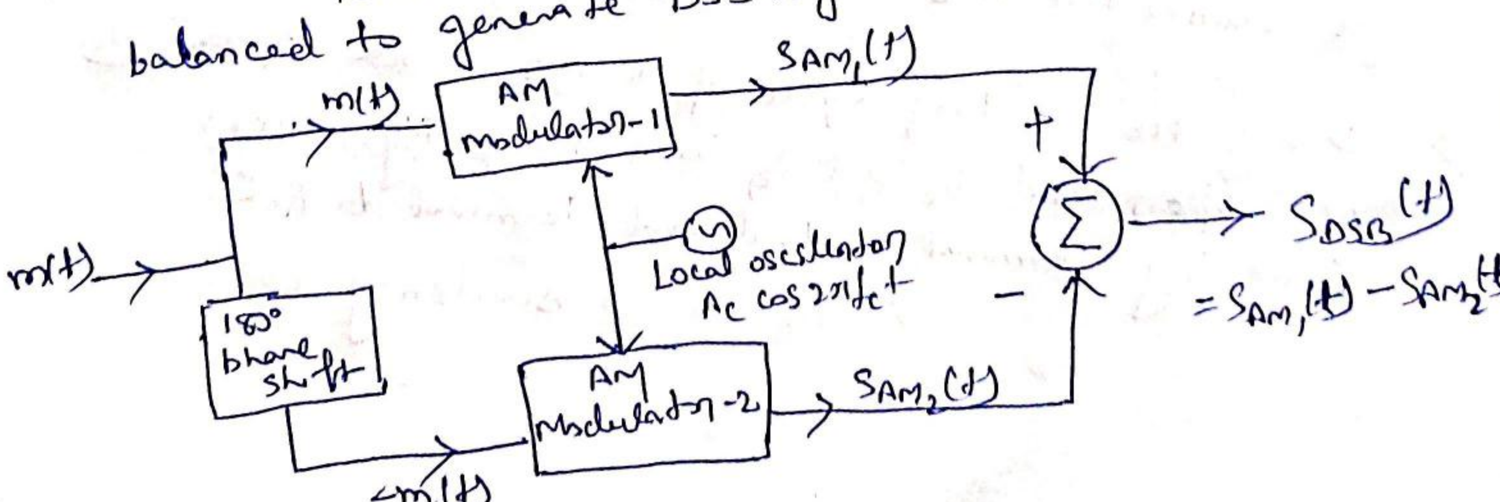
→ Spectrum.



→ Generation of DSB-SC Signal.

The generation method are:
(i) Balanced Modulator
(ii) Ring Modulator

→ Balanced Modulator: Here two AM modulator are connected in balanced to generate DSB signal.



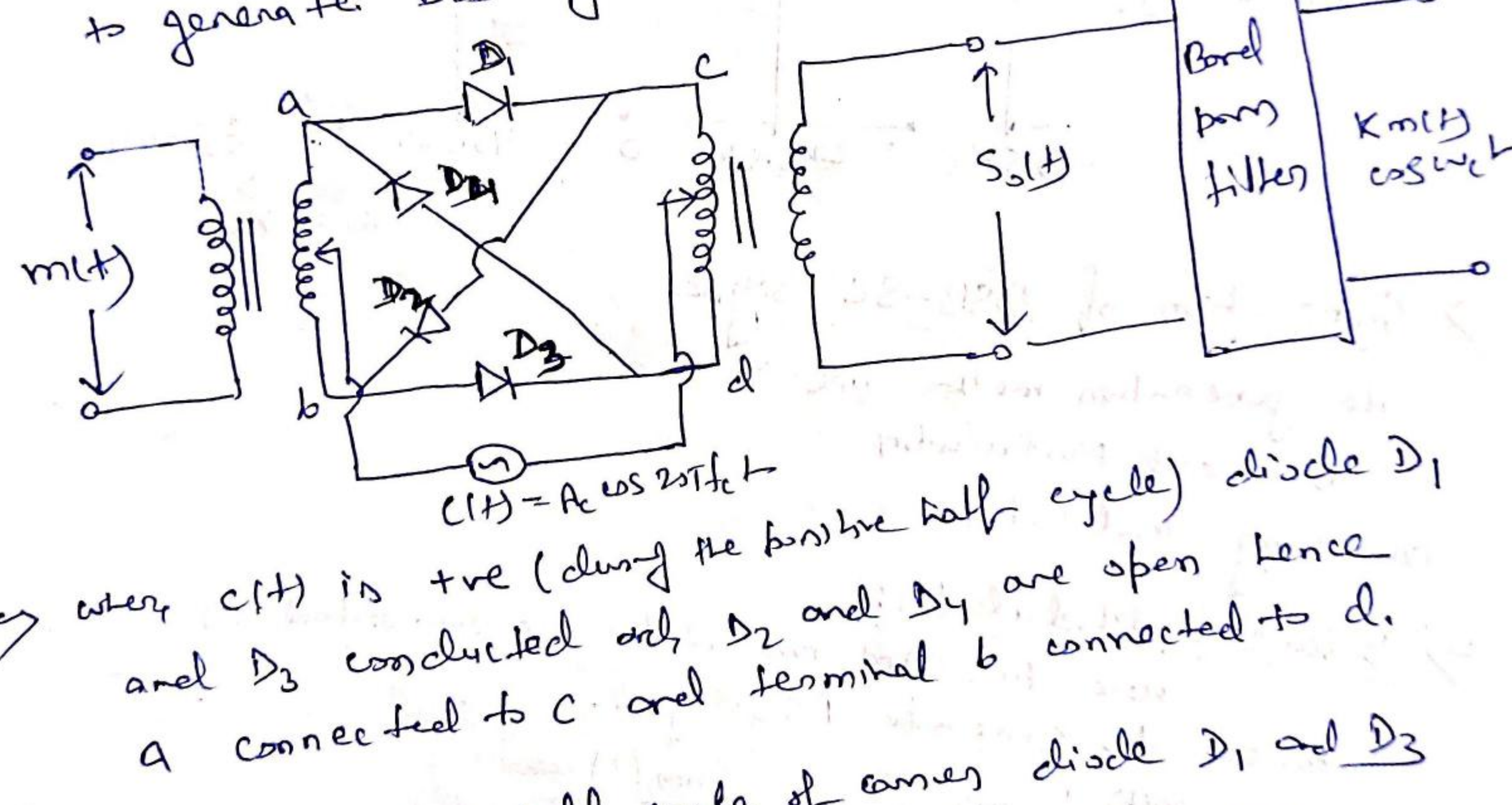
$$S_{AM_1}(t) = A_c\{1 + K_a m(t)\}\cos 2\pi f_c t$$

&, $$S_{AM_2}(t) = A_c\{1 - k_a m(t)\}\cos 2\pi f_c t$$

So, $$S_{DSB}(t) = S_{AM_1}(t) - S_{AM_2}(t)$$
$$= 2 A_c K_a m(t) \cos 2\pi f_c t$$

or, $$S_{DSB}(t) = A_c' m(t) \cos 2\pi f_c t$$
where, $A_c' = 2 A_c k_a$

→ Ring Modulator: Here 4 diodes are connected in the form of RING to generate DSB signal.

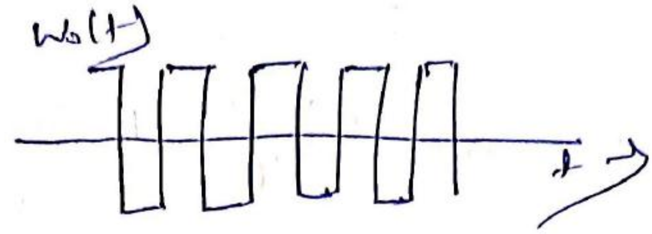


$c(t) = A_c \cos 2\pi f_c t$

→ when c(t) is +ve (during the positive half cycle) diode D_1 and D_3 conducted and D_2 and D_4 are open hence a connected to c and terminal b connected to d.

→ During the -ve half cycle of carrier diode D_1 and D_3 are open and D_2 and D_4 are conducting thus connecting terminal a to d and terminal b to c.
So, it is also a switching ckt.

→ the output is proportional to m(t) during the +ve half cycle and −m(t) during the −ve half cycle. in effect m(t) is multiplied by a square pulse train $w_0(t)$



Fourier series of $w_0(t)$

$$w_0(t) = \frac{4}{\pi}\left(\cos w_c t - \frac{1}{3}\cos 3w_c t + \frac{1}{5}\cos 5w_c t - \cdots\cdots\right)$$

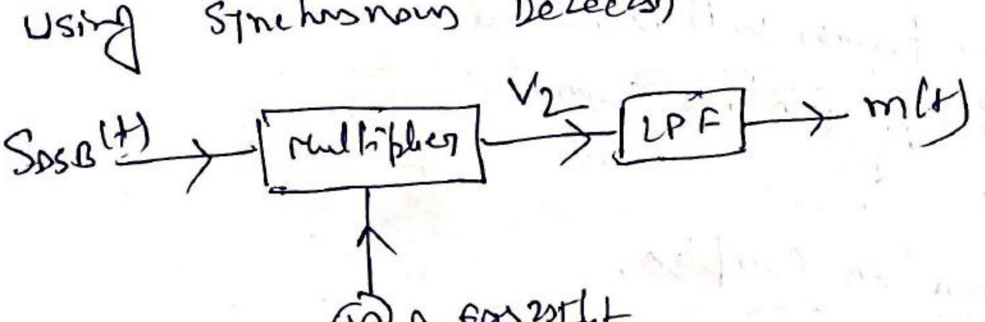
Hence $s_0(t) = m(t)\, w_0(t)$

$$= \frac{4}{\pi}\left[m(t)\cos w_c t - \frac{1}{3} m(t)\cos 3w_c t + \frac{1}{5} m(t)\cos 5w_c t \cdots\cdots\right]$$

after passing through band pass filter the output signal is $\frac{4}{\pi} m(t)\cos w_c t$.

→ Demodulation of DSB-SC Signal

using Synchronous Detector



Since, $S_{DSB}(t) = A_c\, m(t) \cos 2\pi f_c t$

case 1 · perfect synchronous,

$$v_2(t) = A_c\, m(t)\cos 2\pi f_c t \times A_c \cos 2\pi f_c t$$
$$= A_c^2\, m(t)\cos^2 2\pi f_c t$$

$$= \frac{A_c^2}{2} m(t) \{1 + \cos 2\pi f_c t\}$$

$$(L.P.F)_{o/p} = \frac{A_c^2 m(t)}{2}$$

Case 2 $(\text{Local oscillator})_{o/p} = A_c \cos(2\pi f_c t + \phi)$

no phase synchronous

$$V_2(t) = A_c m(t) \cos 2\pi f_c t \times A_c \cos(2\pi f_c t + \phi)$$

$$= A_c^2 m(t) \cos 2\pi f_c t \cdot \cos(2\pi f_c t + \phi)$$

$$= \frac{A_c^2 m(t)}{2} \cos(2\pi f_c t + \phi) + \frac{A_c^2 m(t)}{2} \cos\phi$$

So, $(LPF)_{o/p} = \frac{A_c^2 m(t)}{2} \cos\phi$

→ To maintain the ϕ constant additional ckt has to be utilized

→ when $\phi = 90°$, $o/p = 0$, hence it suffers the problem of Quadrature Null effect (QNE)

Advantage of DSB

Transmitter power will be saved

used for long distance comm^n

Drawbacks

Demodulation is complex

It needs high transmission BW

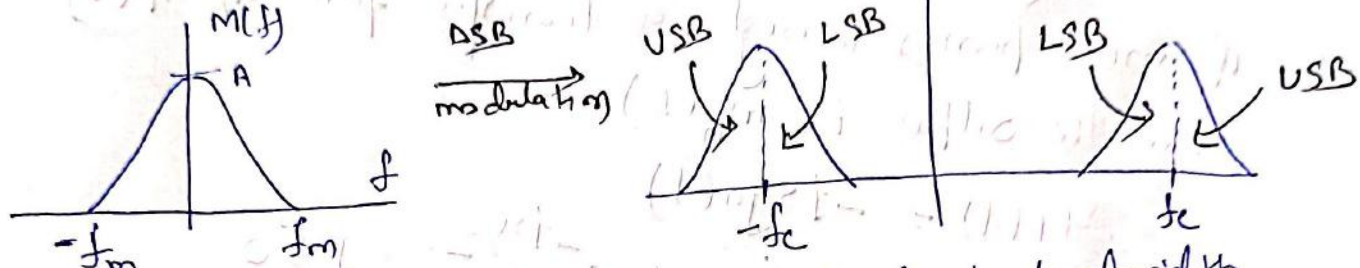
affected by QNE

SINGLE SIDE BAND - SUPPRESSED CARRIER

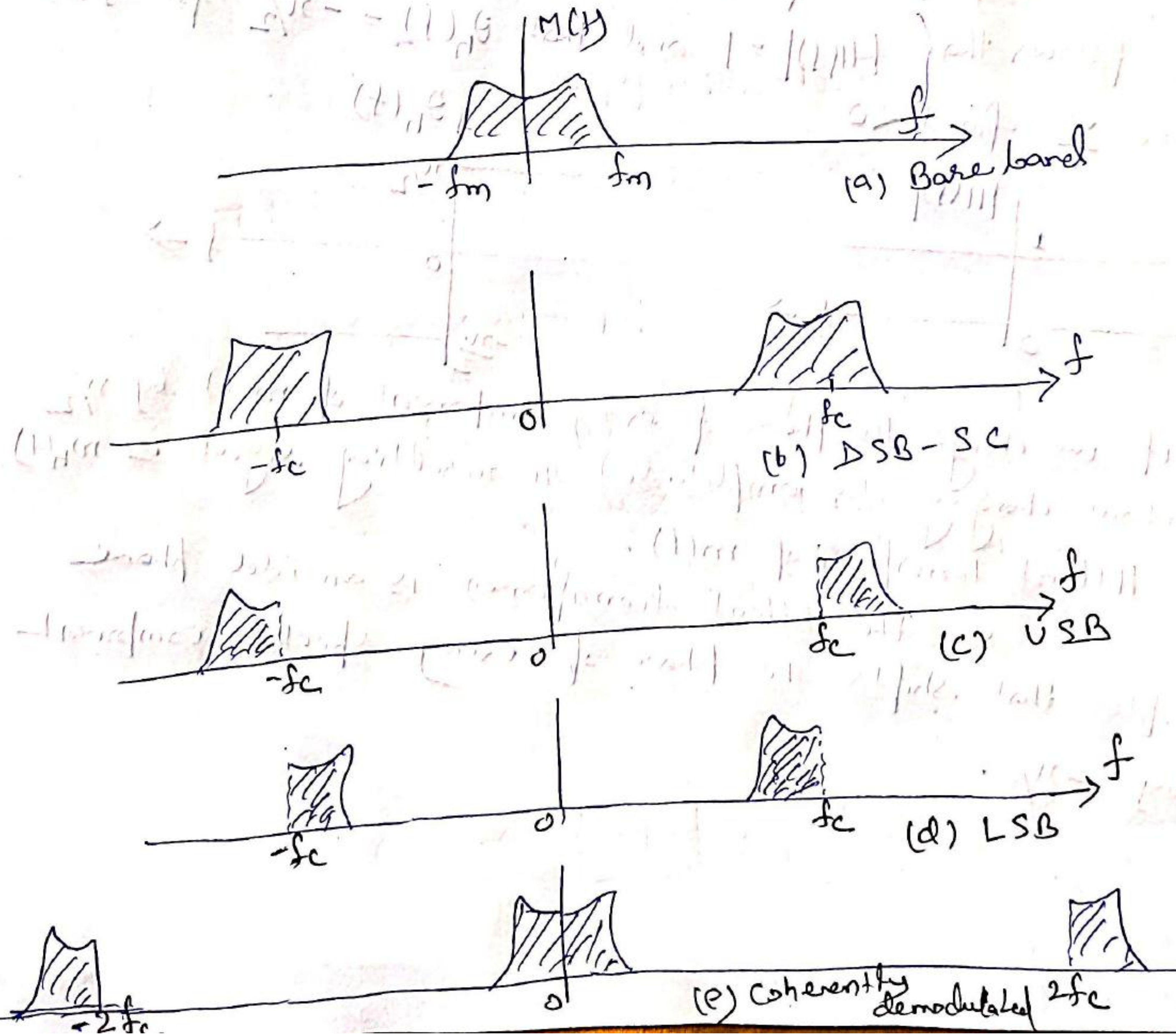
The advantage of SSB over AM and DSB is both the transmitter power and BW will be saved.

In SSB modulation, which removes either the LSB or the USB that uses only Bandwidth of B Hz for one message signal $m(t)$.

→ Single Side band (SSB)



∴ The redundant bandwidth consumption in DSB modulation.



↳ Hilbert transform (this will use in latter on)

Hilbert transform of $x(t)$

$$x_h(t) = H\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\, d\alpha = x(t) \circledast \frac{1}{\pi t}$$

Since, the F.T of $\frac{1}{\pi t} \longleftrightarrow -j\,sgn(f)$

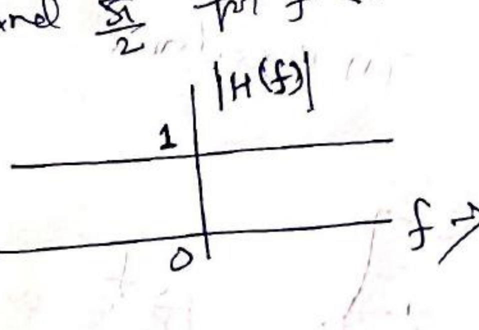
So, $X_h(f) = -j\,X(f)\,sgn(f)$

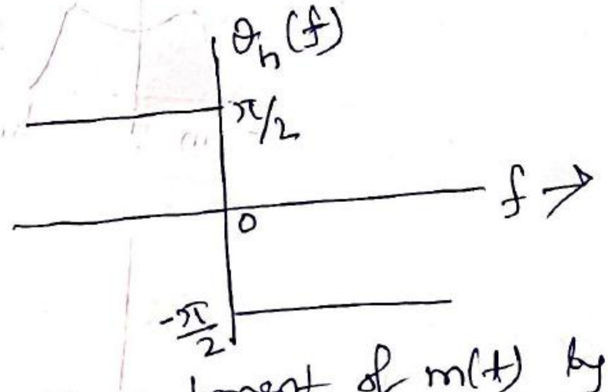
So, if $m(t)$ passes through a transfer function $H(f) = -j\,sgn(f)$ then the output is $m_h(t)$

$$H(f) = -j\,sgn(f)$$

$$= \begin{cases} -j = 1 \cdot e^{-j\pi/2} & f > 0 \\ j = 1 \cdot e^{j\pi/2} & f < 0 \end{cases}$$

It follows that $|H(f)| = 1$ and that $\theta_h(f) = -\pi/2$ for, $f > 0$ and $\frac{\pi}{2}$ for $f < 0$

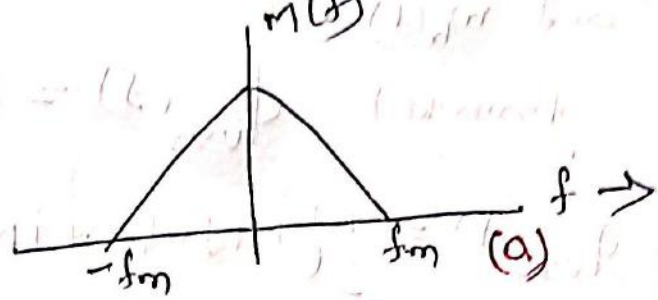




So, if we change the phase of every component of $m(t)$ by $\pi/2$ (without changing its amplitude), the resulting signal is $m_h(t)$, the Hilbert transform of $m(t)$.

So, the Hilbert transformer is an ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$.

Time domain representation of SSB signals.



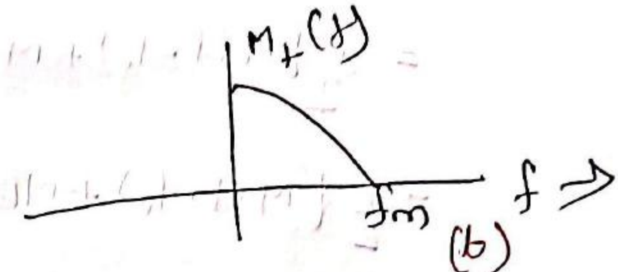
from figure (b)

$$M_+(f) = M(f)\cdot u(f)$$

$$= M(f)\cdot \frac{1}{2}\left[1 + sgn(f)\right]$$

$$= \frac{1}{2}\left[M(f) + M(f)\, sgn(f)\right]$$

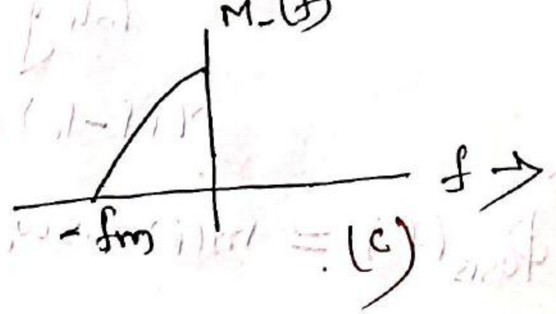
$$= \frac{1}{2}\left[M(f) + \frac{(-j)}{-j} M(f)\, sgn(f)\right]$$



Since, $M_h(f) = -j\, M(f)\, sgn(f)$

$$= \frac{1}{2}\left[M(f) - \frac{1}{j}\cdot\frac{j}{j}\, M_h(f)\right]$$

So, $$\boxed{M_+(f) = \frac{1}{2}\left[M(f) + j M_h(f)\right]}$$



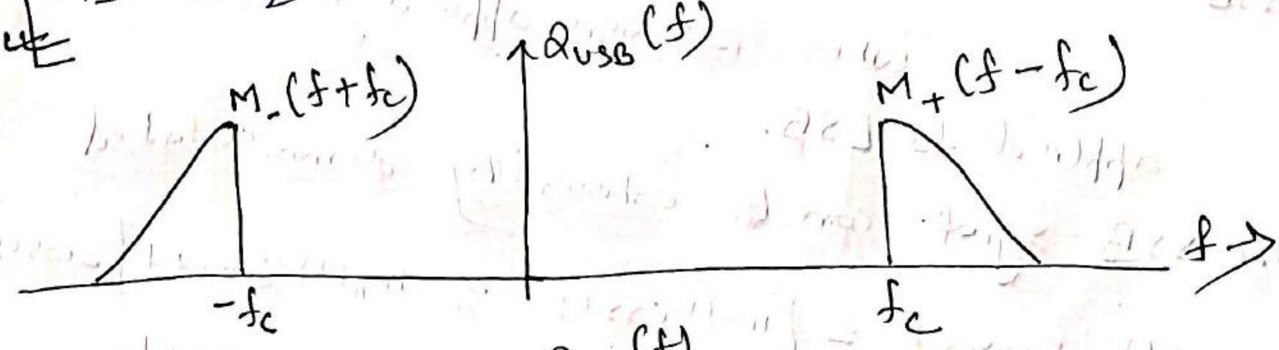
Similarly, from figure (c)

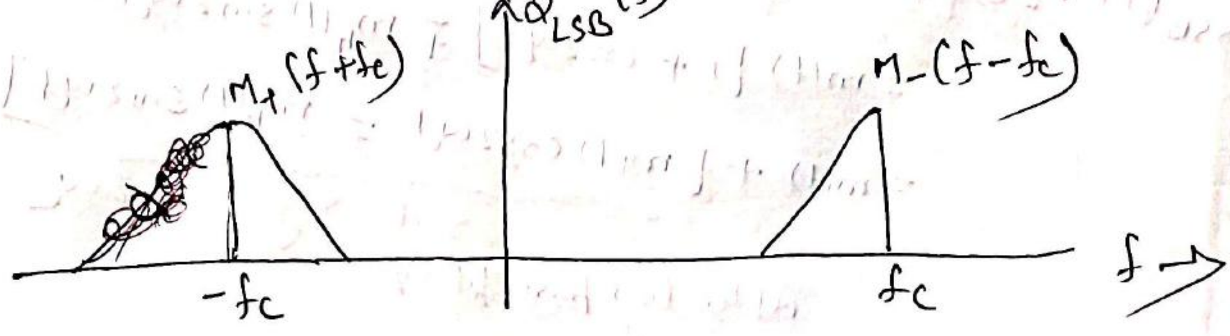
$$M_-(f) = M(f)\, u(-f)$$

$$= M(f)\, \frac{1}{2}\left[1 - sgn(f)\right]$$

$$= \frac{1}{2}\left[M(f) - M(f)\, sgn(f)\cdot\frac{j}{j}\right] = \frac{1}{2}\left[M(f) + \frac{1}{j} M_h(f)\right]$$

$$\boxed{M_-(f) = \frac{1}{2}\left[M(f) - j M_h(f)\right]}$$





Now we express the SSB signal in terms of $m(t)$ and $m_h(t)$

from (d), $\Phi_{USB}(f) = M_+(f-f_c) + M_-(f+f_c)$

So, $\Phi_{USB}(f) = \frac{1}{2}\left[M(f-f_c) + jM_h(f-f_c)\right] + \frac{1}{2}\left[M(f+f_c) - jM_h(f+f_c)\right]$

$= \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] + \frac{j}{2}\cdot\frac{j}{j}\left[M_h(f-f_c) - M_h(f+f_c)\right]$

$= \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right] - \frac{1}{2j}\left[M_h(f-f_c) - M_h(f+f_c)\right]$

taking inverse fourier transform

$M(f-f_c) \xleftrightarrow{\mathcal{F}} m(t)\, e^{j2\pi f_c t}$

So, $\boxed{\Phi_{USB}(t) = m(t)\cos\omega_c t - m_h(t)\sin\omega_c t}$

Similarly for LSB,

$\boxed{\Phi_{LSB}(t) = m(t)\cos\omega_c t + m_h(t)\sin\omega_c t}$

Hence in general SSB can be expressed as:-

$\boxed{\Phi_{SSB}(t) = m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t}$

where, (−) sign applied to USB and (+) sign applied to LSB.

→ The SSB signal can be coherently demodulated

$\Phi_{SSB}(t)\cdot 2\cos\omega_c t = \left[m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t\right]\cdot 2\cos\omega_c t$

$= m(t)\left[1 + \cos 2\omega_c t\right] \mp m_h(t)\sin 2\omega_c t$

$= m(t) + \underbrace{\left[m(t)\cos 2\omega_c t \mp m_h(t)\sin 2\omega_c t\right]}_{\text{SSB signal at } 2\omega_c}$

After low pass filter

$m(t)$

→ The product of $\varphi_{SSB}(t) \cdot 2\cos\omega_c t$, yields the base band signal and another SSB signal with a carrier $2\omega_c$. A low-pass filter will suppress the unwanted SSB term, giving the desired baseband signal $m(t)$.

Ex Tone Modulation : SSB

Find $\varphi_{SSB}(t)$, when the modulating signal is a sinusoid $m(t) = \cos\omega_m t$. Also determine the coherent demodulation.

→ Since Hilbert transformation delays the phase of each spectral component by $\pi/2$

Since, $m(t) = \cos\omega_m t$ (only one spectral component of frequency ω_m)

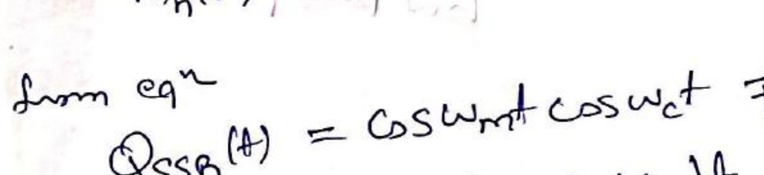
$$m_h(t) = \cos(\omega_m t - \tfrac{\pi}{2}) = \sin\omega_m t$$

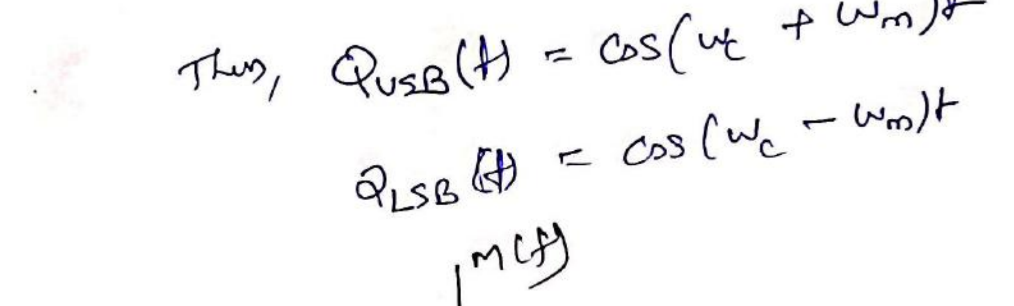
from eqn

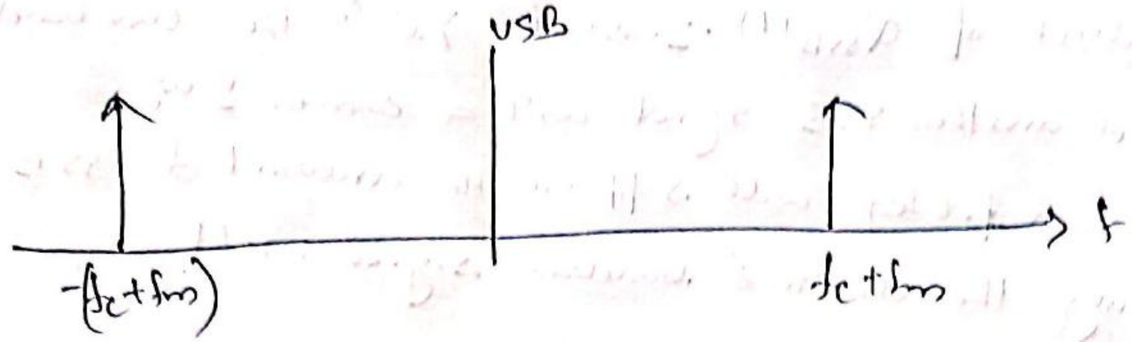
$$\varphi_{SSB}(t) = \cos\omega_m t \cos\omega_c t \mp \sin\omega_m t \sin\omega_c t$$
$$= \cos(\omega_c \pm \omega_m)t$$

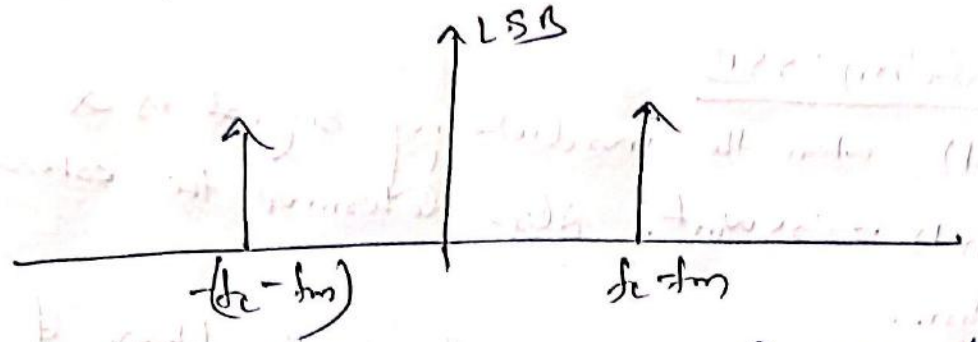
Thus, $\varphi_{USB}(t) = \cos(\omega_c + \omega_m)t$

$\varphi_{LSB}(t) = \cos(\omega_c - \omega_m)t$









for the coherent demodulation of SSB tone modulation

$$\varphi_{SSB}(t)\cdot 2\cos\omega_c t = 2\cos(\omega_c \pm \omega_m)t\cdot\cos\omega_c t$$
$$= \cos\omega_m t + \cos(\omega_c \pm \omega_m)t$$

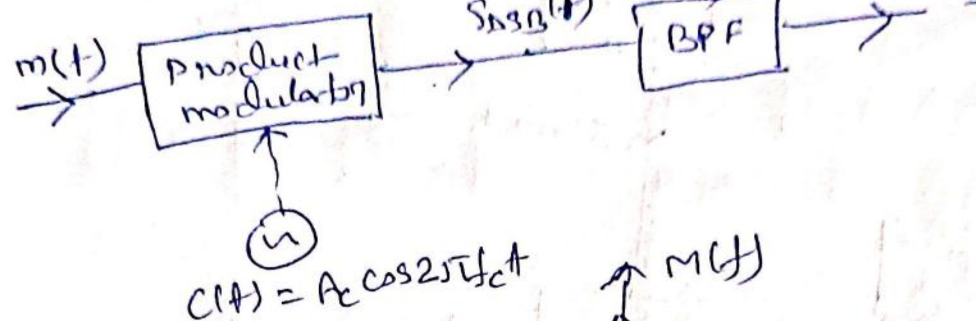
$\downarrow$ low pass filter $\rightarrow \cos\omega_m t$

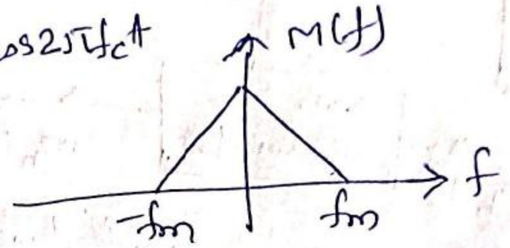
↳ Generation of SSB

The Generation methods of SSB are
(i) Frequency discrimination method
(ii) Phase discrimination method

* **Frequency Discrimination Method.**

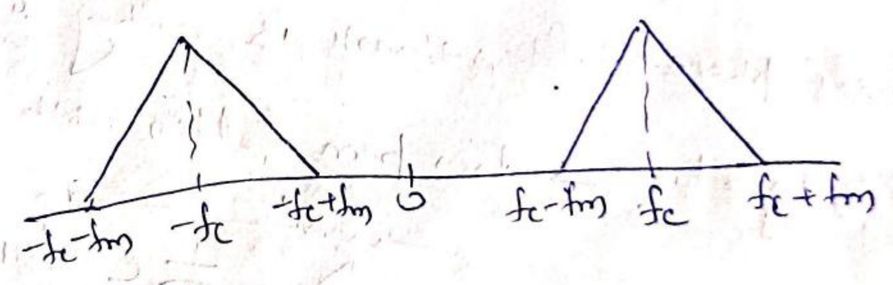
A DSB signal is passed through a sharp cut off filter to eliminate the undesired side-band to generate SSB.



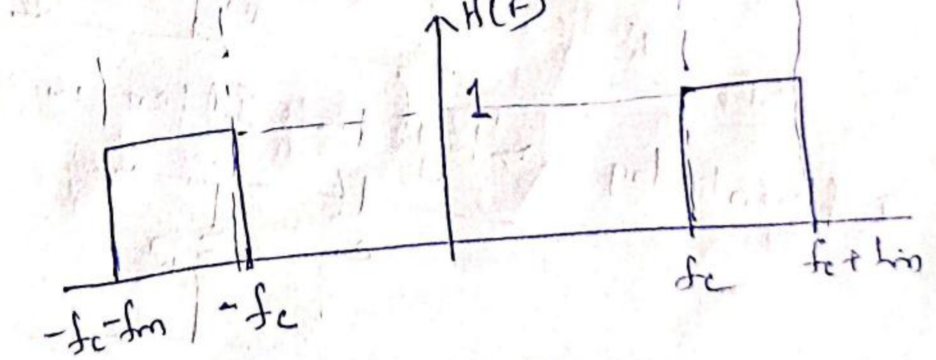


m(t) ⟷

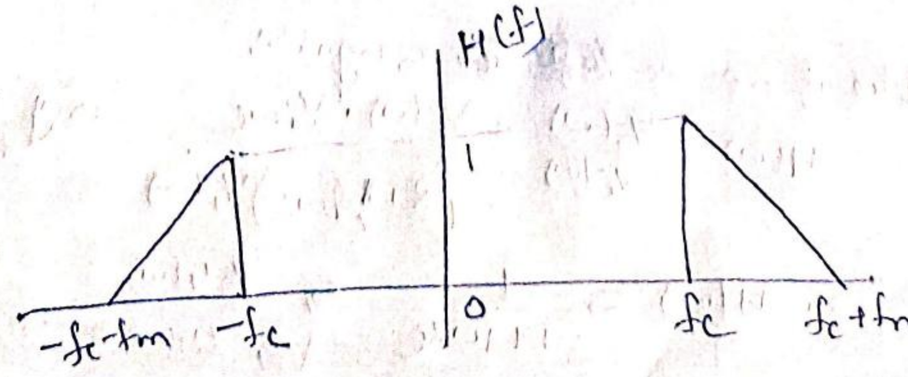
$S_{DSB}(t)$ ⟷



(BPF) ⟷



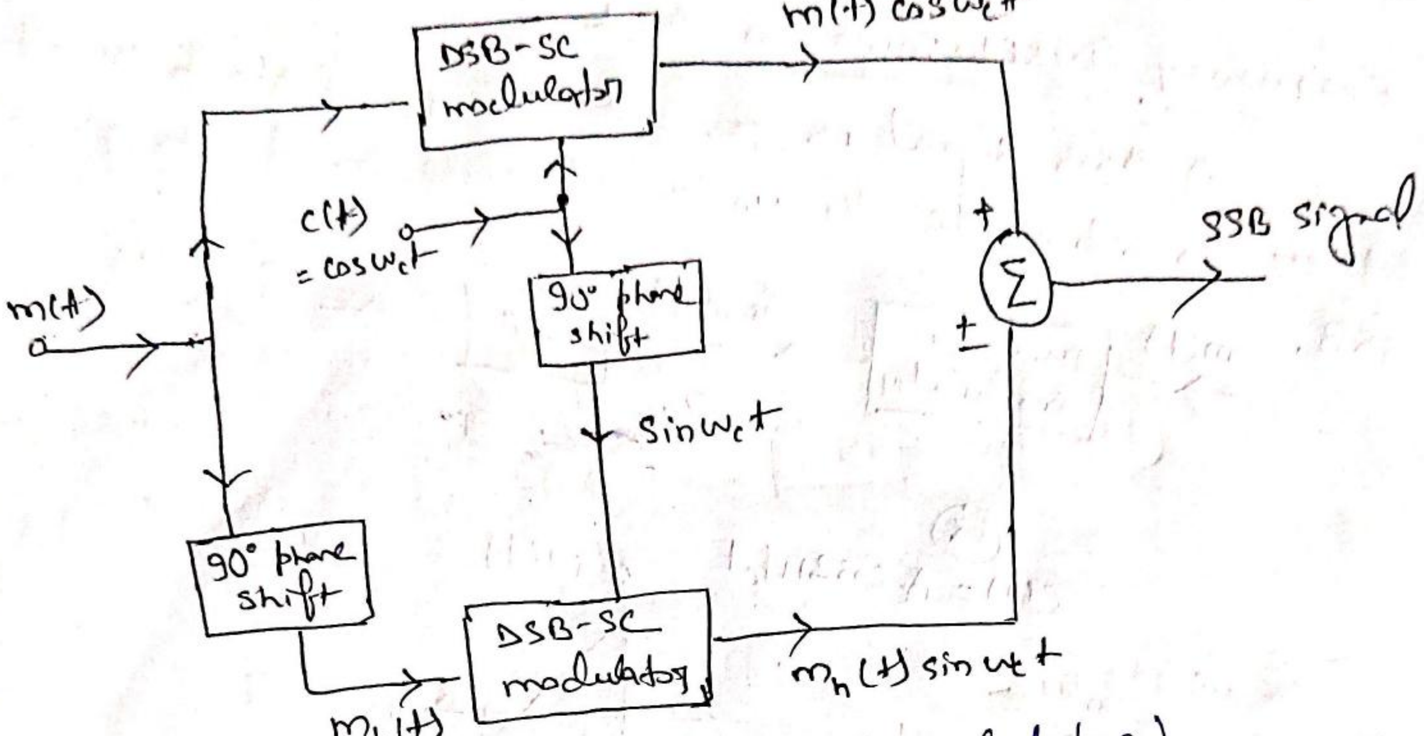
after the BPF
the o/p



→ **Phase Discrimination Method**.

The general expression of SSB signal is given as

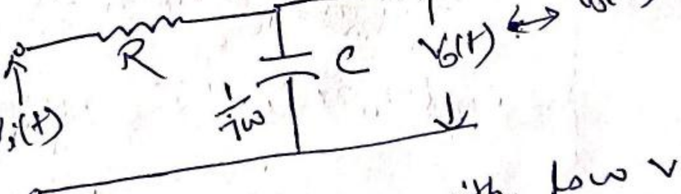
$$\varphi_{SSB}(t) = m(t)\cos w_c t \mp m_h(t)\sin w_c t$$



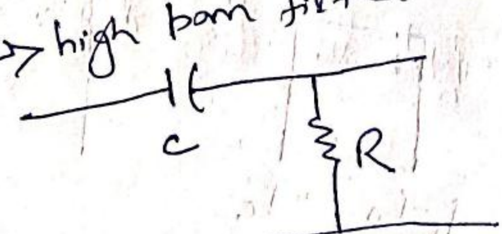
(Generally use for multitone modulation)

Here phase shift network is nothing but Integrator or Differentiator

Integrator → low pass filter with high RC



Differentiator → high pass filter with low value of RC



like if we see Integrator,

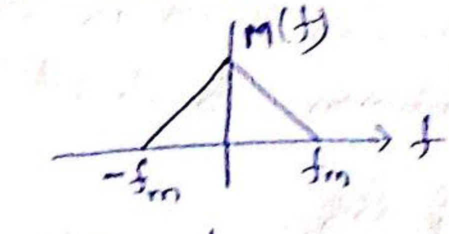
$$H(w) = \frac{V_o(w)}{V_i(w)} = \frac{V_i(w)\cdot 1/jwC}{(R + 1/jwC)V_i(w)}$$ using voltage division method,

So $H(w) = \frac{1}{1 + jwRC}$ so, $\angle H(w) = -\tan^{-1}(wRC)$

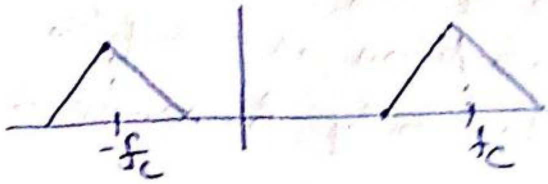
for, $\angle H(w) = -\pi/2$, $wRC \to \infty$ but practically it is also not possible.

→ Drawbacks of Frequency discrimination method:

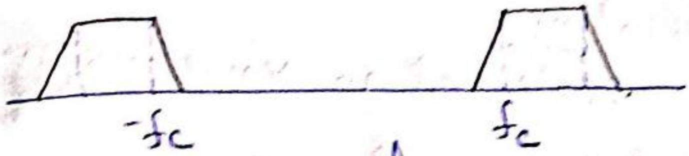
Let, $m(t) \xleftrightarrow{F.T}$



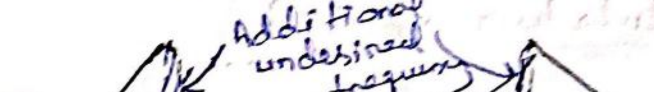
$\varphi_{DSB}(t) \xleftrightarrow{F.T}$



the practical BPF $\longleftrightarrow$



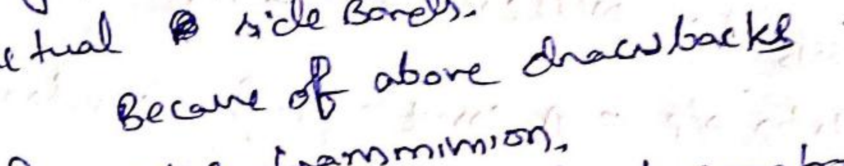
So, $(BPF)_{o/p} \rightarrow \varphi_{SSB}(t)$

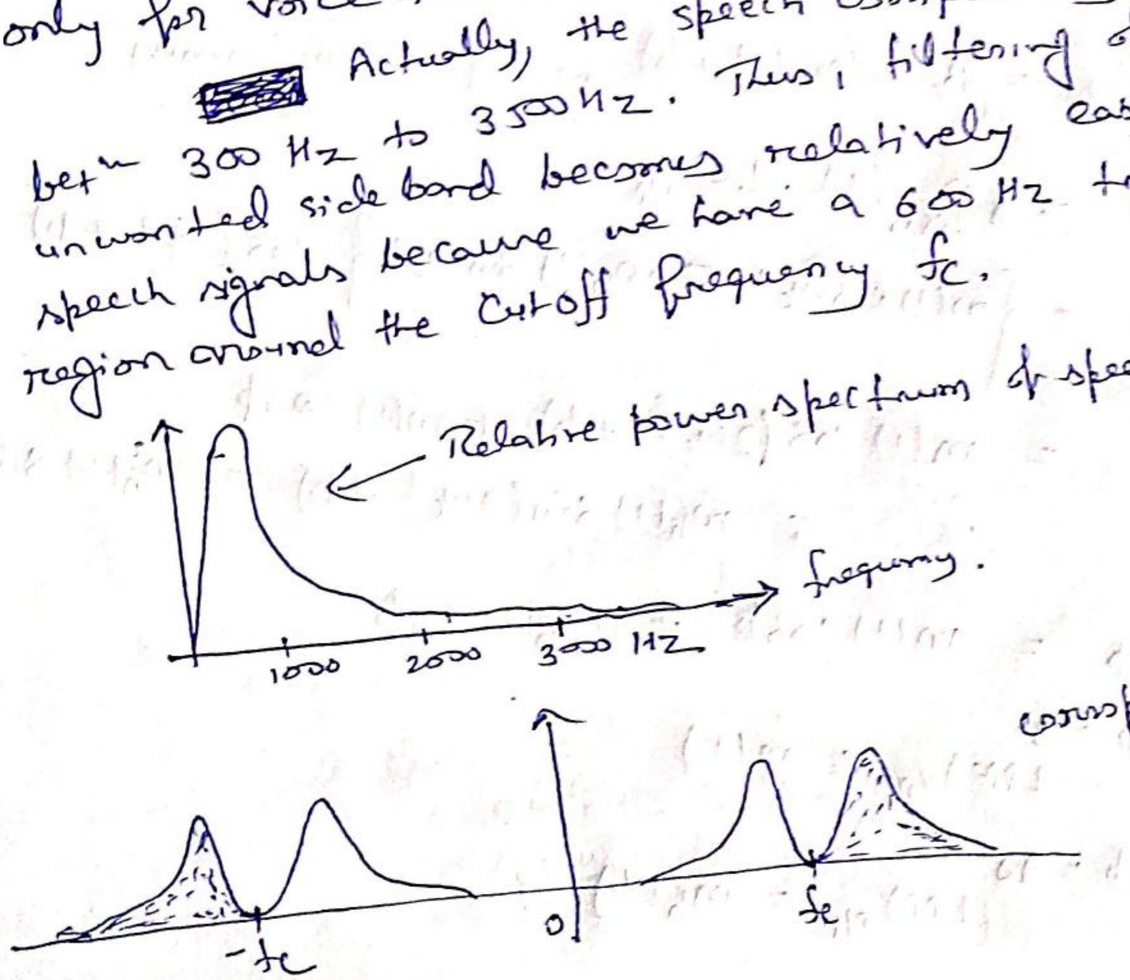


Since, Ideal BPF can not be constructed, the resulting SSB signal contains undesired frequency in addition to actual side bands.

Because of above drawbacks SSB is limited only for voice transmission.

Actually, the speech components lies betn 300 Hz to 3500 Hz. Thus, filtering of the unwanted side band becomes relatively easy for speech signals because we have a 600 Hz transition region around the cut off frequency f_c.





Advantage of SSB

i) Transmission power is saved

ii) Transmission Bandwidth is saved

Drawbacks of SSB

Demodulation is complex

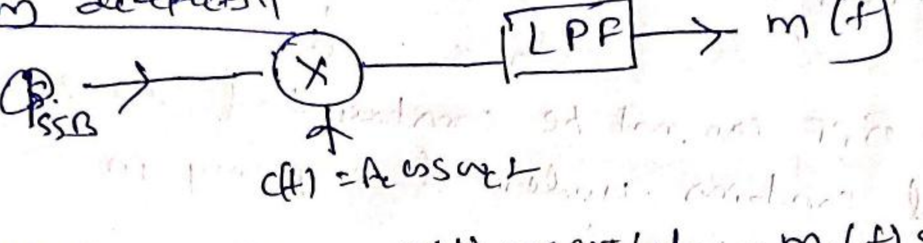
limited only for voice signal transmission.

Application

Preferred in voice - signal transmission.

→ Demodulation of SSB

synchronous detection



General expression, $\phi_{SSB}(t) = m(t)\cos 2\pi f_c t \mp m_h(t)\sin 2\pi f_c t$

Case-I Local oscillator o/p $= A_c \cos\omega_c t$ (perfect synchronous)

Case-II $(LO)_{o/p} = A_c\cos(2\pi f_c t + \phi)$ no perfect synchronous

$$(\text{Multiplier})_{o/p} = \phi_{SSB}(t) \cdot (LO)_{o/p}$$

$$= \{m(t)\cos\omega_c t \mp m_h(t)\sin\omega_c t\}\cos(\omega_c t + \phi)$$

$$= m(t)\cos(2\omega_c t + \phi) + m(t)\cos\phi \pm m_h(t)\sin\phi$$

$$\mp m_h(t)\sin(2\omega_c t + \phi)$$

$$(LPF)_{o/p} = m(t)\cos\phi \pm m_h(t)\sin\phi$$

when $\phi = 0$ $(LPF)_{o/p} = m(t)$

when $\phi = 90^\circ$ $(LPF)_{o/p} = m_h(t)$

↳ VESTIGAL SIDEBAND (VSB)

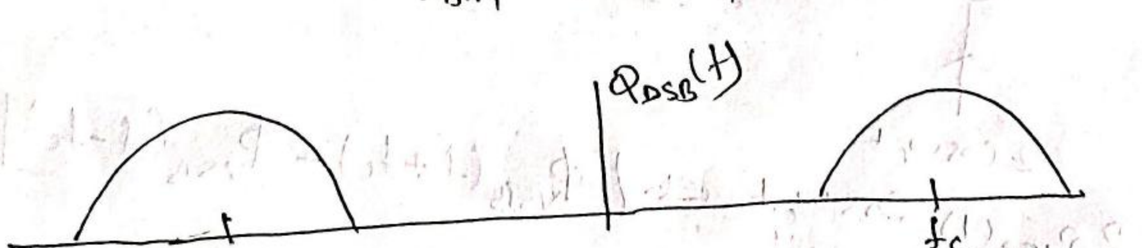
As discussed earlier, it is rather difficult to generate exact SSB signals. A phase shifter, required in the phase shift method, is unrealizable, or only approximately realizable.

The generation of DSB signal is much simpler, but it requires twice the signal bandwidth.

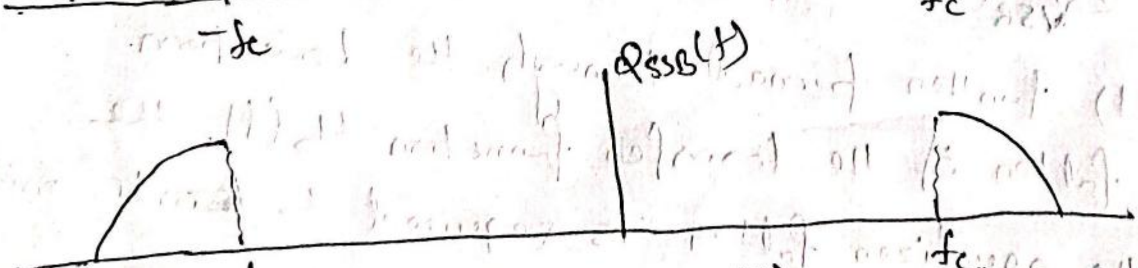
Vestigal Sideband (VSB) modulation, also called the ~~asymmetric~~ asymmetric sideband system, is a compromise betn DSB and SSB. It inherits the advantage of DSB and SSB but avoids their disadvantages at a small cost. VSB signals are relatively easy to generate, and, at the same time, their bandwidth is only a little (typically 25 %) greater than that of SSB signals.

↳ In VSB, instead of rejecting one sideband completely (as in SSB), a gradual cutoff of one sideband as shown in figure (d) is accepted

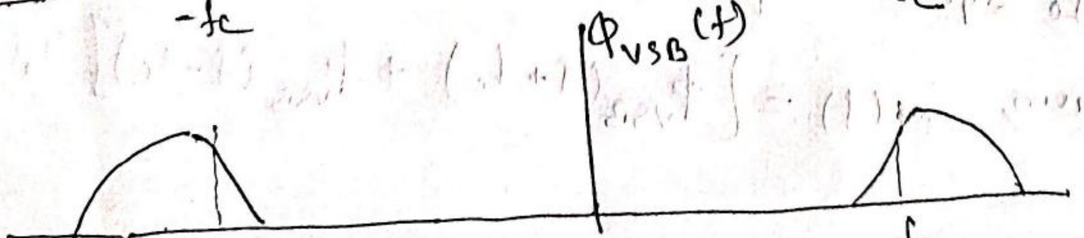
(a)



(b) DSB

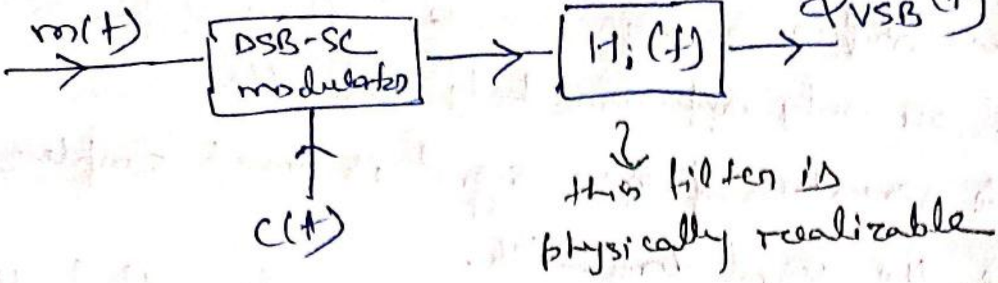


(C) SSB



(d) VSB

VSB modulator & Demodulator.

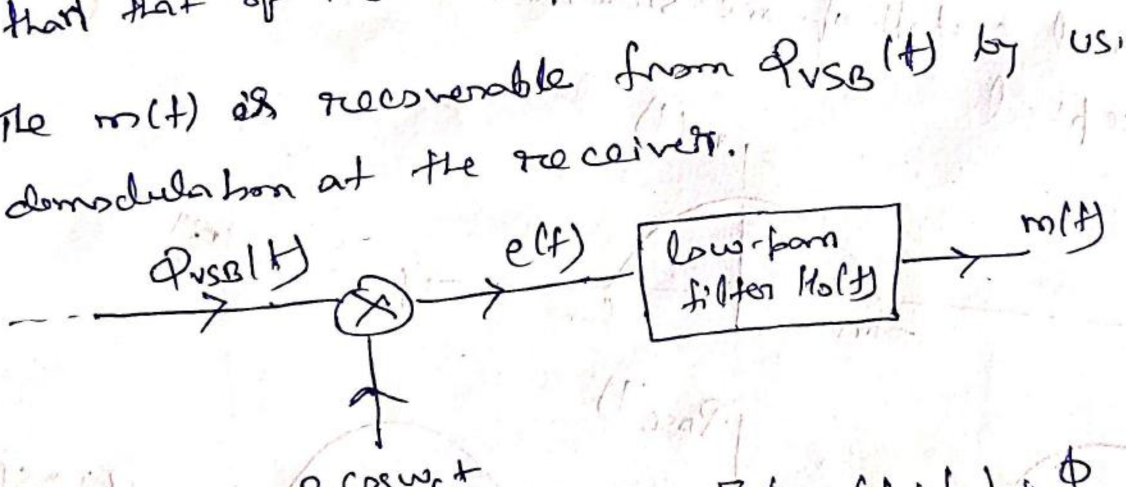


$$\varphi_{VSB}(f) = [M(f+f_c) + M(f-f_c)]\, H_i(f)$$

This VSB shaping filter $H_i(f)$ allows the transmission of one sideband but suppresses the other sideband, not completely, but gradually.

This makes it easy to realize such a filter, but the transmission bandwidth is now somewhat higher than that of the SSB.

↳ The m(t) is recoverable from $\varphi_{VSB}(t)$ by using synchronous demodulation at the receiver.



$$e(t) = 2\varphi_{VSB}(t)\cos\omega_c t \Leftrightarrow [\varphi_{VSB}(f+f_c) + \varphi_{VSB}(f-f_c)]$$

The signal e(t) further passed through the low-pass equalizer filter of the transfer function $H_0(f)$. The output of the equalizer filter is required to be m(t).

Hence spectrum, $M(f) = [\varphi_{VSB}(f+f_c) + \varphi_{VSB}(f-f_c)]\, H_0(f)$

So, Since, $\Phi_{VSB}(f) = [M(f+f_c) + M(f-f_c)]\, H_i(f)$

Substituting this, into eqn of $M(f)$.

$$M(f) = [\Phi_{VSB}(f+f_c) + \Phi_{VSB}(f-f_c)]\, H_o(f)$$

$$= [\cancel{M(f+2f_c)} + M(f)]\, H_i(f+f_c) \cdot H_o(f)$$

$$+ [M(f) + \cancel{M(f-2f_c)}]\, H_i(f-f_c) \cdot H_o(f)$$

replace, the term at $2f_c$

$$\cancel{M(f)} = H_o(f)\left[\cancel{M(f)} \cdot H_i(f+f_c) + \cancel{M(f)}\, H_i(f-f_c)\right]$$

Hence,
$$\left\{ H_o(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)} \right\} \quad |f| \le f_m$$

Since $H_i(f)$ is a Band pass filter so, the terms, $H_i(f \pm f_c)$ contains low-pass components.

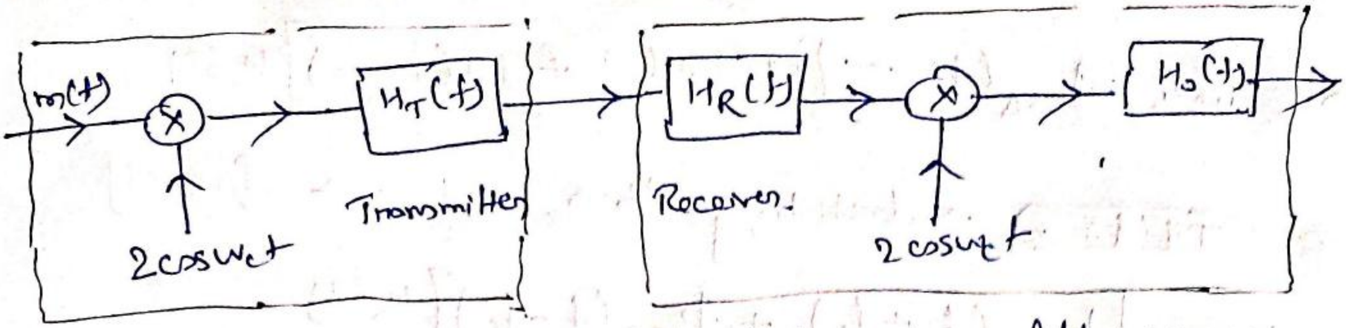
→ Use of VSB in Broadcast television

VSB is a clever compromise betn SSB and DSB, which makes it very attractive for television broadcast system.

→ The Baseband video signal of television occupies an enormous bandwidth of 4.5 MHz.

So, DSB signal needs → 9 MHz

needs of SSB → to save Bandwidth.

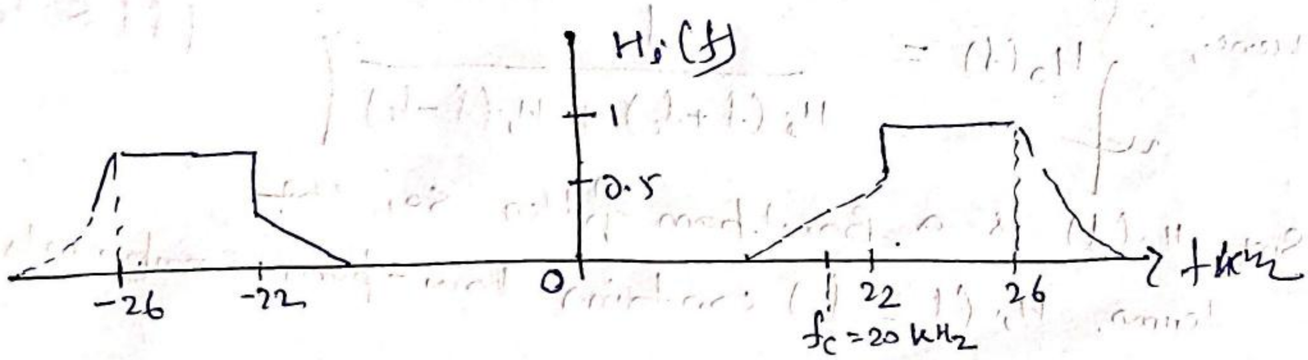


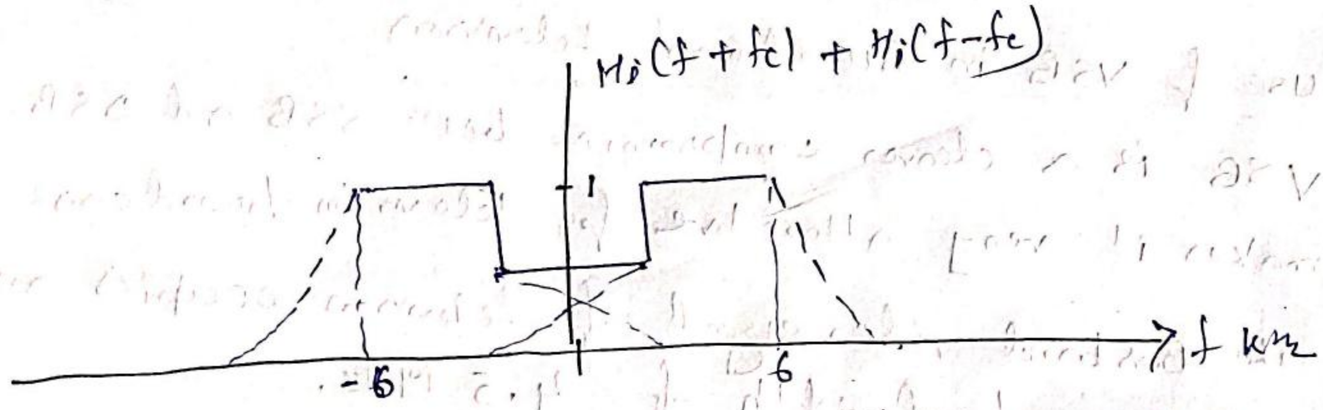
The vestigial spectrum is controlled by two filters: The transmitter filter $H_T(f)$ and the receiver filter $H_R(f)$. Jointly we have

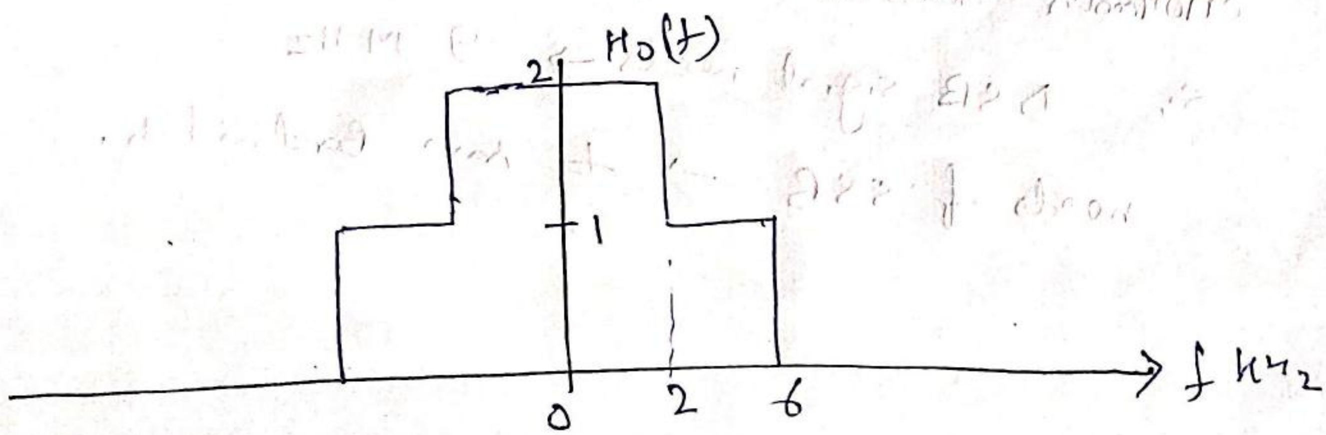
$$H_i(f) = H_T(f) \cdot H_R(f)$$

Hence the design of receiver output filter $H_o(f)$.

$$H_o(f) = \frac{1}{H_i(f+f_c) + H_i(f-f_c)}$$







ANGLE MODULATION AND DEMODULATION

Here we discuss the non linear modulation of frequency (FM) and phase (PM), collectively known as angle modulation

⇒ Non-linear Modulation

A general sinusoidal carrier signal

$$\varphi(t) = A\cos(\omega_c t + \theta_0)$$

ω_c → $2\pi f_c$ varies in proportional to the message signal as frequency - modulation (FM)

θ_0 (phase) varies in proportional to message signal as phase modulation (PM)

→ The effort were focused on finding a modulation scheme that would reduce the bandwidth. Initially, it feels that the FM can be soln for this. But the FM bandwidth is always greater that AM bandwith. But FM is usefull in other application.

→ The concept of Instantaneous Frequency

In FM, the instantaneous carrier frequency vary in proportion to the modulating signal $m(t)$. This means that the carrier frequency is changing continously every instant.

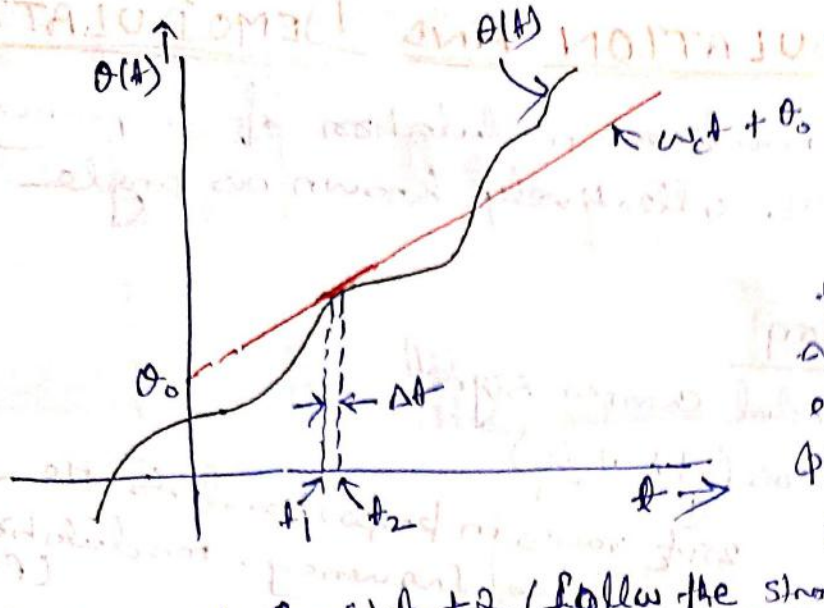
Let us consider a generalized sinusoidal signal $\varphi(t)$ given by — (1)

$$\varphi(t) = A\cos\theta(t) = A\cos(\omega_c t + \theta_0)$$

where $\theta(t)$ is the generalized angle and is a function of t.

so, $\theta(t) = \omega_c t + \theta_0$ → initial phase

if ω_c is const, the $\theta(t)$ is straigh line eqn and if w_c will change continously with message signal $m(t)$ then we talk about at any instantaneous t (for very small interval) and at that point we get the instantaneous frequency.



the important point key around t, over a small interval $\Delta t \to 0$, the $\varphi(t) = A\cos\theta(t)$ and, $A\cos(\omega_c t + \theta_0)$ is same,

i.e. $\theta(t) = \omega_c t + \theta_0$ (follow the straight line eqn) for $t_1 < t < t_2$

and, $(\omega_c t + \theta_0)$ is tangential to $\theta(t)$, the angular frequency of $\varphi(t)$ is the slope of angle $\theta(t)$ over this small interval.

instantaneous frequency ω_i is: —— (2)

$$\omega_i(t) = \frac{d\theta}{dt}$$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\, d\alpha$$

so the possibility of transmitting of information of $m(t)$ by varying the angle θ of a carrier are known as Angle modulation or Exponential modulation.

Two such possibilities are:
- Phase Modulation (PM)
- Frequency Modulation (FM)

In PM the angle $\theta(t)$ is varies linearly with $m(t)$:

$$\theta(t) = \omega_c t + \theta_0 + k_p m(t)$$

assume initial phase $\theta_0 = 0$

$$\theta(t) = \omega_c t + k_p m(t) \quad \text{—— 3(a)}$$

where $k_p = \text{const}$.

so, the resulting PM waves

$$\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)] \quad \text{—— 3(b)}$$

φ_{PM} phase modulated signal (PM)

So, the instantaneous angular frequency at that time

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t) \quad \text{where } \dot{m}(t) = \frac{dm(t)}{dt} \qquad \text{— 3(c)}$$

Hence in PM, the instantaneous angular frequency ω_i varies linearly with the derivative of modulating signal.

⇒ In FM, the instantaneous frequency ω_i is varied linearly with the modulating signal,

$$\omega_i(t) = \omega_c + k_f\, m(t) \qquad \text{— 4(a)}$$

where k_f is a constant.

So, the angle $\theta(t) = \int_{-\infty}^{t} [\omega_c + k_f\, m(\alpha)]\, d\alpha$

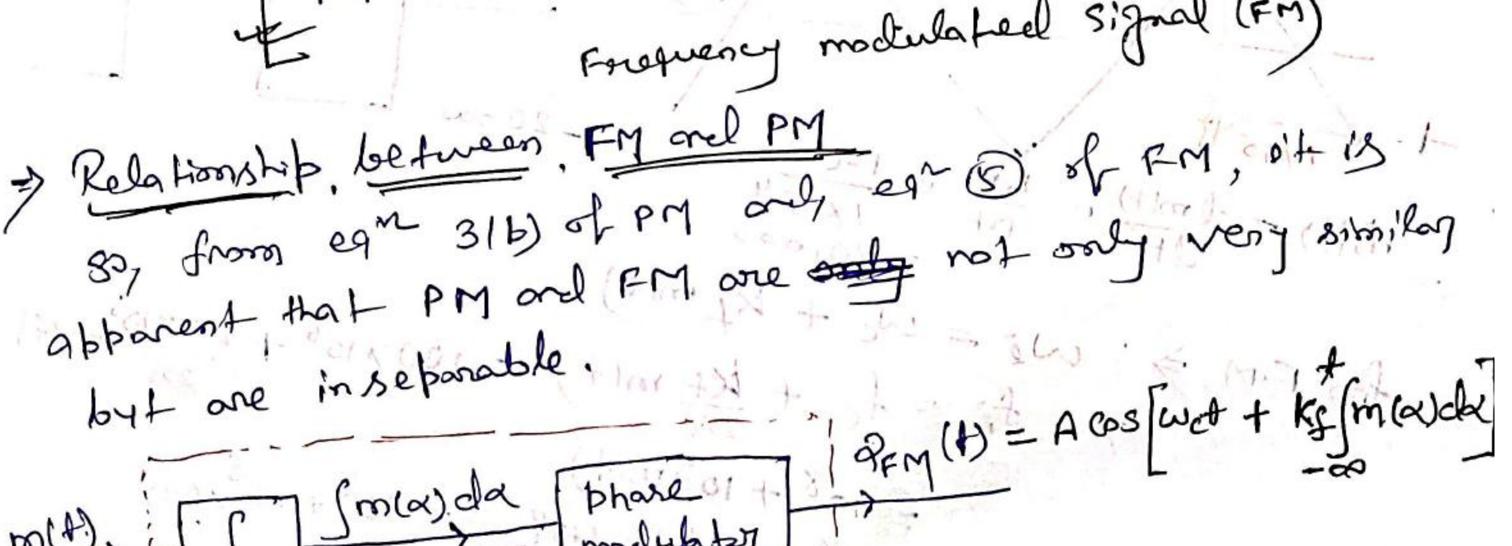
$$\theta(t) = \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha \qquad \text{— 4(b)}$$

So, $$\varphi_{FM}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right] \qquad (5)$$

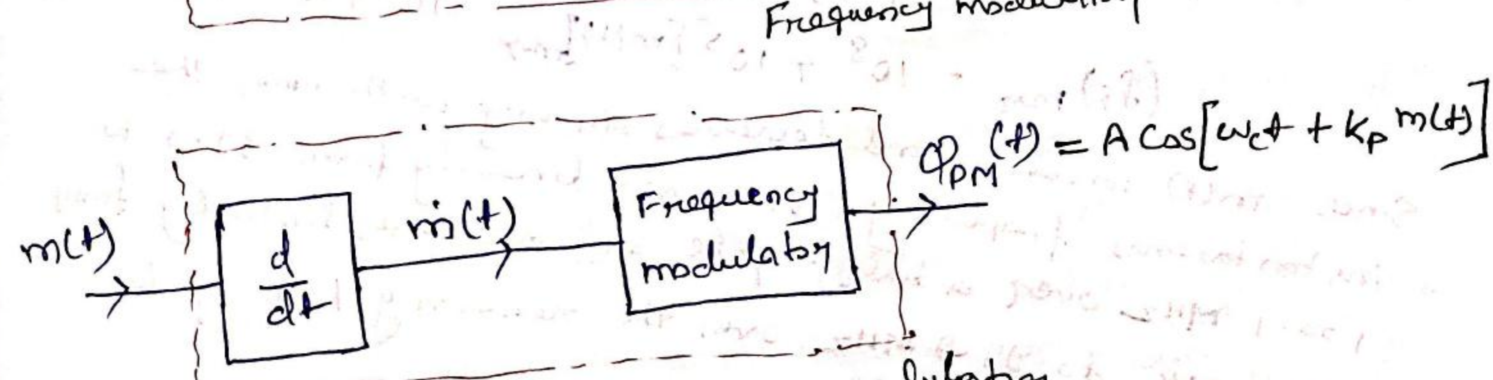
Frequency modulated signal (FM)

⇒ Relationship between FM and PM

So, from eqn 3(b) of PM and eqn (5) of FM, it is apparent that PM and FM are ~~only~~ not only very similar but are inseparable.



Frequency modulator



Phase modulator

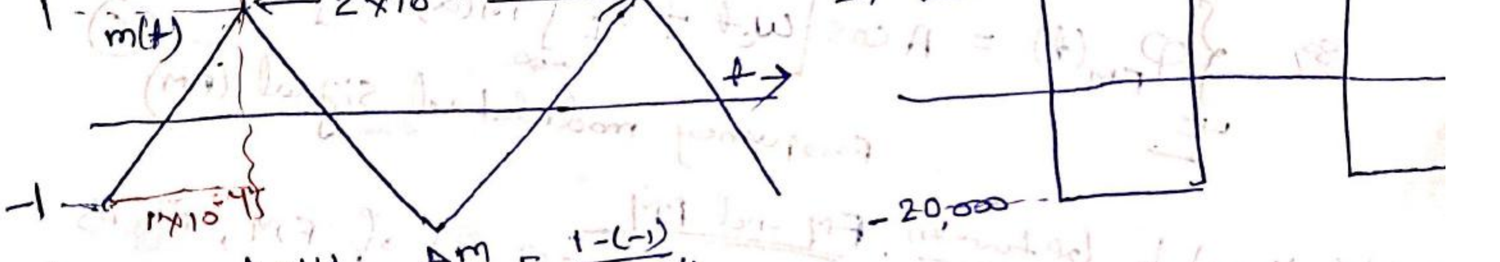
→ So, the PM and FM are inseparable in nature. In PM the angle of carrier directly proportional to m(t) whereas in FM the angle of carrier is proportional to integral of m(t). and the method of generation and demodulation of each type of modulation is same

→ Power of Angle - modulated wave.

in PM or FM, the instantaneous phase and frequency can vary with time, but Amplitude remains constant

so, $\text{Power}(P) = \frac{A^2}{2}$ always

Ex Sketch PM and FM wave for the modulating signal m(t) as shown in figure. The constant, k_f and k_p are $2\pi \times 10^5$ and 10π, respectively, and $f_c = 100$ MHz.



$\dot{m}(t) = \frac{d\,m(t)}{dt} = \frac{\Delta m}{\Delta t} = \frac{1-(-1)}{1\times10^{-4}}$
$= 20,000$

for F.M ⇒ $\omega_i = \omega_c + k_f\, m(t)$

or, $f_i = f_c + \frac{k_f}{2\pi} m(t) = 100\times10^6 + \frac{2\pi\times10^5}{2\pi} m(t)$

$= 10^8 + 10^5 m(t)$

so, $(f_i)_{min} = 10^8 + 10^5[m(t)]_{min} = 10^8 + 10^5 \times(-1)$

$= 99.9$ MHz

$(f_i)_{max} = 10^8 + 10^5[m(t)]_{max} = 10^8 + 10^5 = 100.1$ MHz

Since, m(t) increases and decreases linearly with time, the instantaneous frequency increases linearly from 99.9 to 100.1 MHz over a half-cycle and decreases linearly from 100.1 MHz to 99.9 MHz over the remaining half-cycle of the modulating signal for PM, then PM of $m(t)$ is same as FM for $\dot{m}(t)$

actually, in PM, $\theta_i(t) = \omega_c t + k_p m(t)$

so, instantaneous frequency, $\omega_i = \frac{d\theta_i(t)}{dt} = \omega_c + k_p \dot{m}(t)$

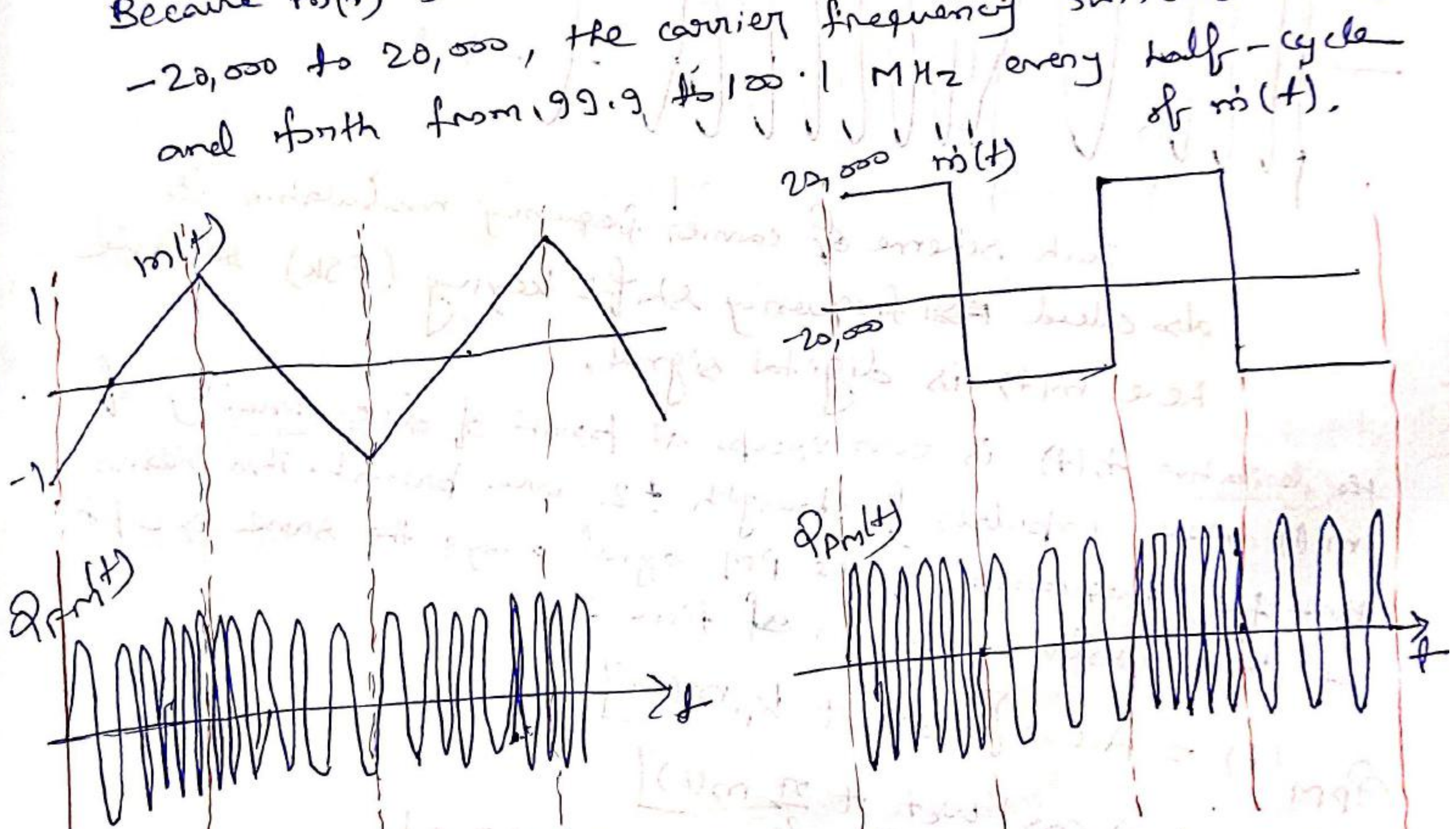
$$\text{or } f_i = f_c + \frac{k_p}{2\pi}\dot{m}(t)$$

$$= 10^8 + \frac{10\pi}{2\pi}\dot{m}(t) = 10^8 + 5\dot{m}(t)$$

so $(f_i)_{min} = 10^8 + 5|\dot{m}(t)|_{min} = 10^8 - 10^5 = 99.9$ MHz

$(f_i)_{max} = 10^8 + 5|\dot{m}(t)|_{max} = 10^8 + 10^5 = 100.1$ MHz

Because $\dot{m}(t)$ switches back and forth from a value of −20,000 to 20,000, the carrier frequency switches back and forth from 99.9 to 100.1 MHz every half-cycle of $\dot{m}(t)$.

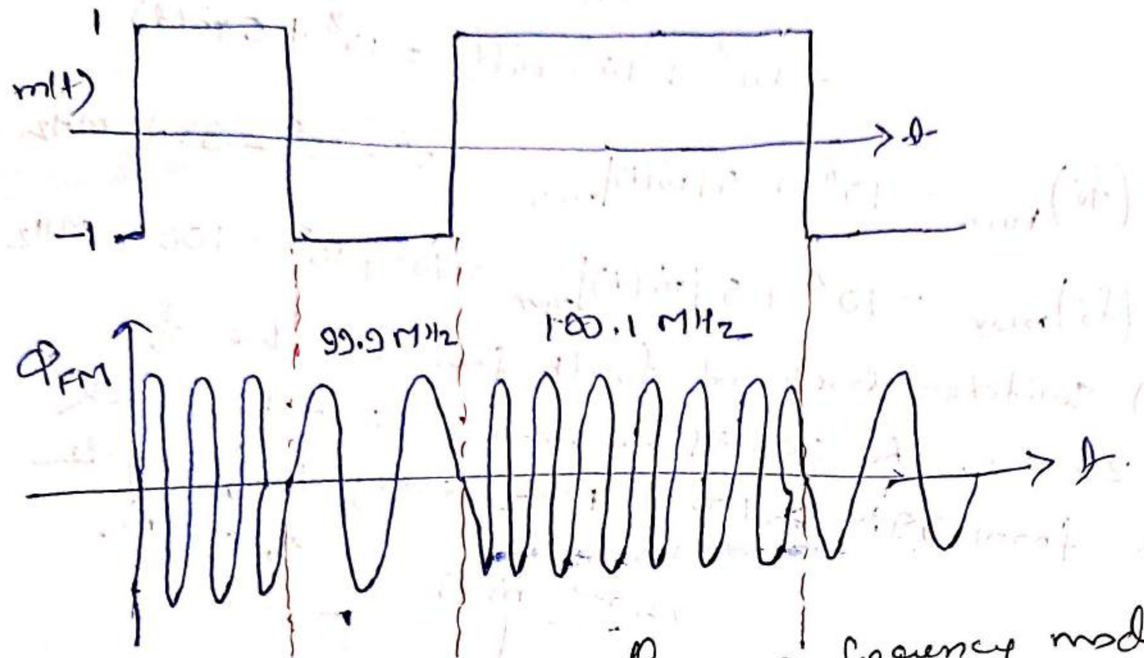


→ This indirect method of sketching PM [using $\dot{m}(t)$ to frequency-modulate a carrier] works as long as $m(t)$ is continuous signal. If $m(t)$ is discontinuous, it means that the PM signal has sudden phase changes and hence, $\dot{m}(t)$ contains impulses. This indirect method fails at points of the discontinuity. In such case, a direct approach should be used.

Ex $K_f = 2\pi \times 10^5$, $k_p = \pi/2$ and $f_c = 100$ MHz.

for FM, $f_i = f_c + \frac{k_f}{2\pi} m(t) = 10^8 + 10^5 m(t)$

Since, m(t) switches from 1 to −1 and vice versa, the FM wave frequency switches back and forth betn 99.9 to 100.1 MHz.



Such scheme of carrier frequency modulation is also called frequency shift keying (FSK) because here m(t) is digital signal.

the derivative $\dot{m}(t)$ is zero except at point of discontinuity of m(t) where impulses of strength ±2 are present. This means that the frequency of the PM signal stays the same except at these isolated points of time.

$$\varphi_{PM}(t) = A\cos[\omega_c t + k_p m(t)]$$

$$= A\cos[\omega_c t + \frac{\pi}{2} m(t)]$$

$$= \begin{cases} A\sin\omega_c t & \text{when } m(t) = -1 \\ -A\sin\omega_c t & \text{when } m(t) = 1 \end{cases}$$

This scheme of carrier PM by a digital signal is called phase shift keying (PSK) because information digits are transmitted by shifting the carrier phase.

Phase & Frequency Modulation

→ **Phase Modulation**

$$\phi_{PM}(t) = A_c \cos[\omega_c t + k_p m(t)] \quad \text{——— (6)}$$

→ **Frequency Modulation**

$$\phi_{FM}(t) = A \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right] \quad \text{——— (7)}$$

$f_i = f_c + k_f m(t)$

k_f = frequency sensitivity, (Hz/volt)

So, when, $m(t) = 0$, $f_i = f_c$

$m(t) = +ve$, $f_i > f_c$

$m(t) = -ve$, $f_i < f_c$

So, max^m frequency of FM signal ⇒ $f_{max} = f_c + k_f A_m$

min^m frequency of FM signal ⇒ $f_{min} = f_c - k_f A_m$

So, max^m frequency deviation = Max $\{k_f m(t)\}$

So, $\Delta f = k_f A_m$

→ Total frequency Swing of FM signal $= f_{max} - f_{min}$
$= 2\Delta f$

since, $f_{max} = f_c + \Delta f$

$f_{min} = f_c - \Delta f$

Q An unmodulated carrier frequency is given by 1MHz. After frequency modulation max^m frequency is given by 1.4 MHz. Find Δf and f_{min}?

$f_{max} = 1.4$ MHz
$= f_c + \Delta f$ ⇒ $\Delta f = 1.4 - 1 = 0.4$ MHz

and, $f_{min} = f_c - \Delta f$
$= 1 - 0.4 = 0.6$ MHz

Q For an FM signal f_{max} is given by 1.5 MHz, total frequency swing is given by 900 kHz. Find f_c, Δf & f_{min}

Given, $2\Delta f$ = Total frequency swing = 900 kHz

$\Delta f = 450$ kHz

Now, $f_{max} = 1.5$ MHz
$= f_c + \Delta f$

$f_c = f_{max} - \Delta f$
$= 1.5 \times 10^6 - 450 \times 10^3$
$= 1050$ k $= 1.05$ MHz

and, $f_{min} = f_c - \Delta f$
$= 1.05 \times 1000 - 450 = 600$ kHz

~~Single Tone FM~~

Q A sinusoidal carrier of 20V, 2 MHz is frequency modulated by a message signal of $10 \sin 4\pi \times 10^3 t$. K_f is given by 50 kHz/volt. Find Δf; f_{max} & f_{min}?

Given, $A_c \cos 2\pi f_c t = c(t)$

So, $A_c = 20V$; $f_c = 2$ MHz

message signal, $m(t) = A_m \sin 4\pi \times 10^3 t$

$A_m = 10$; $f_m = 2$ kHz

$K_f = 50$ kHz/volts

So, $f_{max} = f_c + K_f\, m(t)$
$= 2000 + 50 \times 10$
$f_{max} = 2500$ kHz

$f_{min} = f_c - K_f\, m(t)$
$= 2000 - 50 \times 10$
$f_{min} = 1500$ kHz

So, $\Delta f = \dfrac{\text{total frequency swing}}{2} = \dfrac{2500 - 1500}{2} = 500$ Hz

⇒ Single tone FM

The general expression for FM

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + 2\pi k_f \int m(t)\,dt\}$$

so for single tone,

$$m(t) = A_m \cos 2\pi f_m t$$

so $S_{FM}(t) = A_c \cos\{2\pi f_c t + 2\pi k_f \int A_m \cos 2\pi f_m t\, dt\}$

$$= A_c \cos\left\{2\pi f_c t + 2\pi k_f \cdot A_m \frac{\sin 2\pi f_m t}{2\pi f_m}\right\}$$

$$= A_c \cos\left\{2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right\} \quad \text{—(8)}$$

So, $\left[\frac{k_f A_m}{f_m} = \frac{\Delta f}{f_m} = \beta\right]$ modulation index of FM —(9)

$$\text{modulation Index } (\beta) = \frac{\text{max}^m \text{ Frequency deviation}}{\text{message signal frequency}}$$

So, $\{S_{FM}(t) = A \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$ —(10)

for single tone FM

↳ Depending up the value of β, the FM is classified as:

- → Narrow Band FM ($\beta \leq 1$)
- → Wide Band FM ($\beta \geq 1$)

~~[illegible]~~

↳ Narrow Band FM (NBFM)

The single tone FM is given as:

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$$S_{NBFM}(t) = A_c \left\{ \cos(2\pi f_c t) \cos(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) - \sin(2\pi f_c t) \sin(\underbrace{\beta \sin 2\pi f_m t}_{\theta}) \right\}$$

for, NBFM ; $\beta \leq 1$

so, $\theta = \beta \sin 2\pi f_m t \simeq 0$

so, for very small value of θ,

$$\cos\theta \cong 1 , \quad \sin\theta \cong \theta$$

$$\therefore S_{NBFM}(t) = A_c \cos 2\pi f_c t \cdot 1 - A_c \sin 2\pi f_c t \cdot \beta \sin 2\pi f_m t$$

$$\therefore S_{NBFM}(t) = A_c \cos 2\pi f_c t - \frac{A_c\beta}{2}\{\cos 2\pi(f_c - f_m)t\} + \frac{A_c\beta}{2}\{\cos 2\pi(f_c + f_m)t\} \quad \text{(ii)}$$

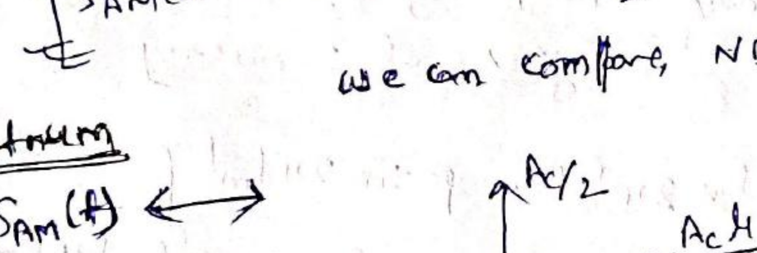
Now, the similarity bet^n, NBFM & AM

$$S_{AM}(t) = A_c \cos 2\pi f_c t + \frac{A_c \mu}{2}\cos 2\pi(f_c - f_m)t + \frac{A_c \mu}{2}\cos 2\pi(f_c + f_m)t$$

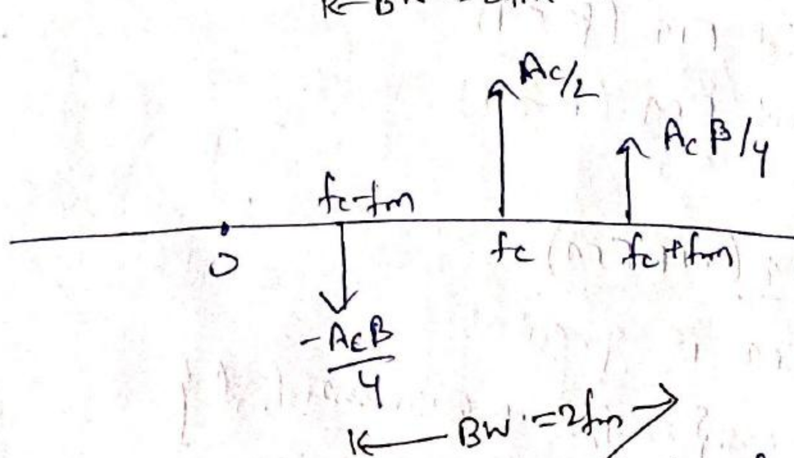
we can compare, NBFM & AM

→ **Spectrum**

$S_{AM}(f) \longleftrightarrow$ **AM**



NBFM



so, the general Expression of AM & NBFM are same except 180° phase shift at LSB frequency component.

→ Power of NBFM

As, $P_t = P_c + P_{USB} + P_{LSB}$

where $P_c = \frac{A_c^2}{2R}$; $P_{USB} = \left(\frac{A_c \beta}{2}\right)^2 / 2R = \frac{A_c^2 \beta^2}{8R} = P_{LSB}$

So, $P_t = \frac{A_c^2}{2R} + \frac{2 \times A_c^2 \beta^2}{8R} = \frac{A_c^2}{2R} + \frac{A_c^2 \beta}{4R}$

So, $P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\beta^2}{2}\right\} = P_c\left\{1 + \frac{\beta^2}{2}\right\}$ —— (12)

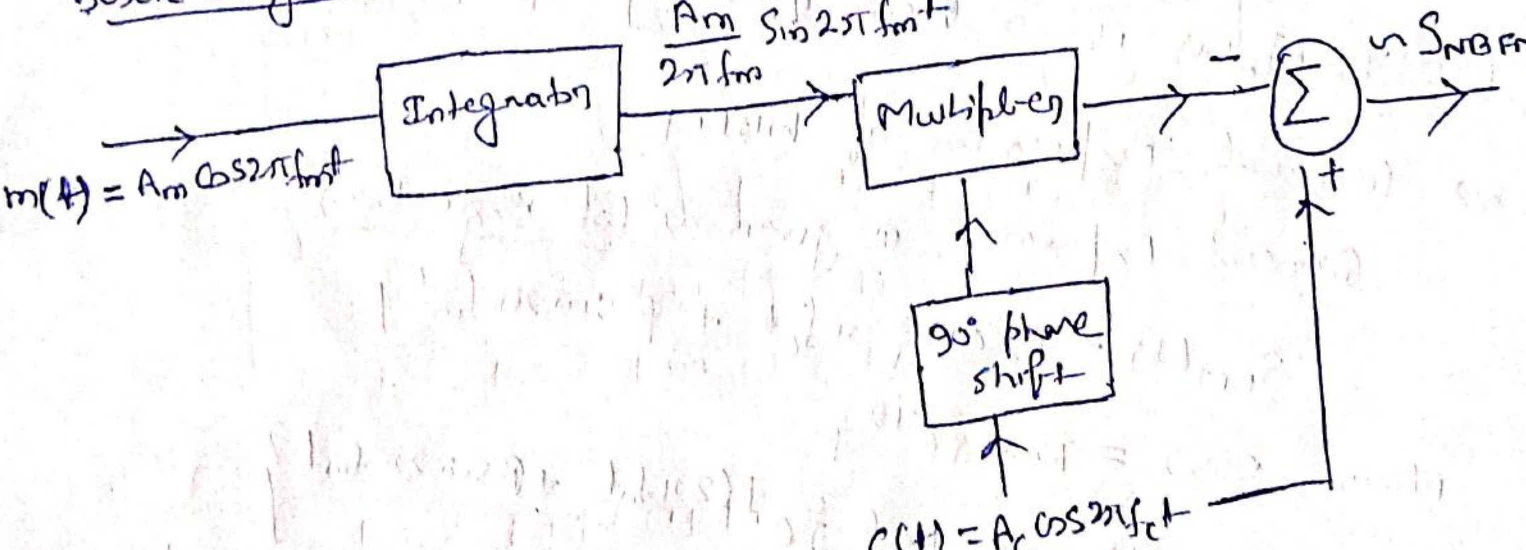
So, the NBFM has much similarity with AM, hence practical significance of NBFM is negligible.

→ Generation of NBFM

Since, $S_{NBFM}(t) \cong A_c \cos 2\pi f_c t - A_c \beta \sin 2\pi f_c t \cdot \sin 2\pi f_m t$

$\cong A_c \cos 2\pi f_c t - \frac{A_c k_f \cdot A_m}{f_m} \sin 2\pi f_c t \cdot \sin 2\pi f_m t$ —— (13)

Block diagram



⇒ WIDE BAND FM (WBFM)

↳ **Bessel function**

Standard definition is given as:-

$$J_n(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(x\sin\theta - n\theta)}\, d\theta \quad \text{—— (14)}$$

Property of Bessel function $J_n(x)$:-

(i) $J_n(x) \downarrow$ decreases as $n \uparrow$ increases

So, $J_0(x) > J_1(x) > J_2(x) > J_n(x)$ —— (15a)

(ii) $J_{-n}(x) = (-1)^n J_n(x)$

So, $J_{-n}(x) = -J_n(x)$; n = odd

$J_n(x)$; n = even —— (15b)

(iii) $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$ —— 15(c)

(iv) $J_n(x)$ is a real quantity

↳ General Expression of WBFM.

General Expression is given as (of single tone):

$$S_{FM}(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

Now, $\cos\theta = \text{Real}\{e^{j\theta}\}$

So, $S_{FM}(t) = A_c \,\text{Real}\{e^{j(2\pi f_c t + \beta\sin 2\pi f_m t)}\}$

$$= A_c \,\text{Re}\{e^{j2\pi f_c t} \cdot \underbrace{e^{j\beta \sin 2\pi f_m t}}\} \quad \text{—— (16)}$$

it is a continuous periodic signal with $T = 1/f_m$

Since, $e^{j\beta \sin 2\pi f_m t}$ is periodic So, $x(t) = x(t+T)$ (13)

$\downarrow 1/f_m$

$$e^{j\beta \sin 2\pi f_m t} = e^{j\beta \sin 2\pi f_m (t + 1/f_m)}$$

$$\boxed{\text{So, to find the fourier series} \rightarrow f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \; ; \omega_0 = \frac{2\pi}{T} \quad \text{where, } C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t) e^{-jn\omega_0 t}\, dt}$$

So, $$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn 2\pi f_m t}$$

and, $$C_n = \frac{1}{(1/f_m)} \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin 2\pi f_m t} e^{-jn 2\pi f_m t}\, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)}\, dt \qquad (17)$$

So, Now, as, $J_n(x) = \frac{1}{2\pi} \int e^{j(x \sin\theta - n\theta)}\, d\theta$

So, we want to replace, C_n in the form of J_n

So, $\theta = 2\pi f_m t$

or, $d\theta = 2\pi f_m\, dt$

when, $t = -1/2f_m \Rightarrow \theta = -\pi$

$t = 1/2f_m \Rightarrow \theta = +\pi$

By replacing all the above

$$C_n = f_m \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \cdot \frac{d\theta}{2\pi f_m}$$

(14)

$$\left\{ C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin\theta - n\theta)} \, d\theta = J_n(\beta) \right\} \quad \text{——— (18)}$$

so
$$\boxed{C_n = J_n(\beta)} \quad \text{——— (19)}$$

putting this value we got the fourier series,

$$e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{jn2\pi f_m t}$$

substituting this value in the $S_{FM}(t)$

$$S_{FM}(t) = S_{WBFM}(t) = A_c \, Re \left\{ e^{j2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) e^{jn2\pi f_m t} \right\}$$

$$= A_c \, Re \left\{ \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c + nf_m)t} \right\}$$

So, the general the expression of WBFM is given as:-

$$\left\{ S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + nf_m) t \right. \quad \text{——— (20)}$$

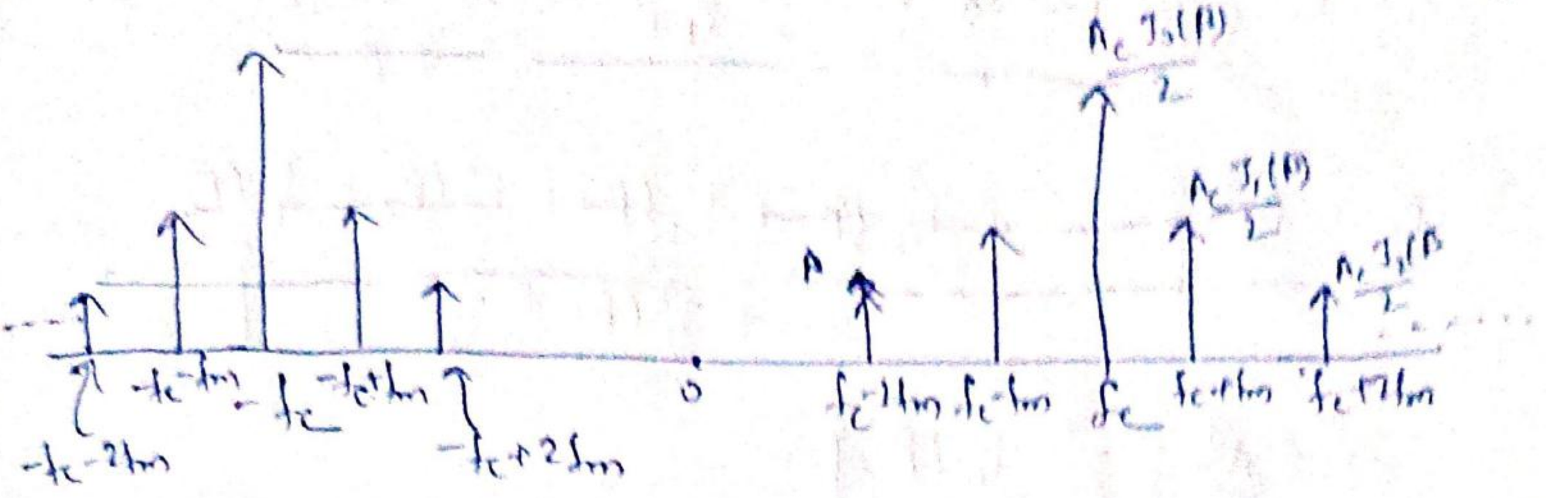
<u>Analysis</u>

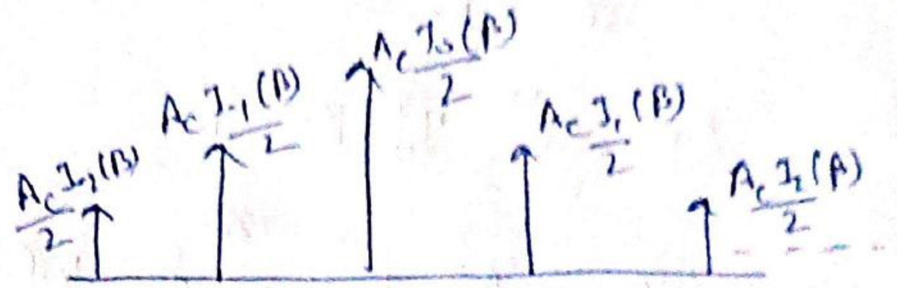
$$S_{WBFM}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c + nf_m) t$$

$$= A_c J_0(\beta) \cos 2\pi f_c t + A_c J_1(\beta) \cos 2\pi (f_c + f_m) t$$

$$+ A_c J_{-1}(\beta) \cos 2\pi (f_c - f_m) t + A_c J_2(\beta)$$
$$\cos 2\pi (f_c + 2f_m t) + A_c J_{-2}(\beta) \cos 2\pi (f_c - 2f_m)t$$

$$+ \text{——} \ldots \quad \text{——— (21)}$$





property
$$J_0(x) > J_1(x) > J_2(x) > J_n(x) \ldots$$

Conclusion.

(i) WBFM consists of carrier frequency component, ∞ no. of USBs and ∞ no. of LSBs

(ii) so, the actual BW of WBFM is ∞

(iii) For WBFM, strength of higher order side bands go on decreasing and finally becomes zero.

(iv) so, the lower order sidebands are said to be significant sidebands and higher order sidebands are insignificant.

→ The Power of WBFM

$$P_t = P_c + (P_{USB_1} + P_{USB_2} + \cdots) + (P_{LSB_1} + P_{LSB_2} + \cdots)$$

where, $P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$ and, $P_{USB_1} = \frac{A_c^2 J_1^2(\beta)}{2R}$; $P_{LSB_1} = \frac{A_c^2 J_{-1}^2(\beta)}{2R}$

(16)

$$P_{USB_2} = \frac{A_c^2 J_2^2(\beta)}{2R} \quad ; \quad P_{LSB_2} = \frac{A_c^2 J_{-2}^2(\beta)}{2R}$$

So, $$P_t = \frac{A_c^2}{2R}\left\{ \cdots + J_{-2}^2(\beta) + J_{-1}^2(\beta) + J_0^2(\beta) + J_1^2(\beta) + J_2^2(\beta) + \cdots \right\}$$

$$= \frac{A_c^2}{2R}\left\{ \sum_{n=-\infty}^{\infty} J_n^2(\beta) \right\}$$

Since, $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

So, $$\boxed{P_t = \frac{A_c^2}{2R}} \quad (22)$$

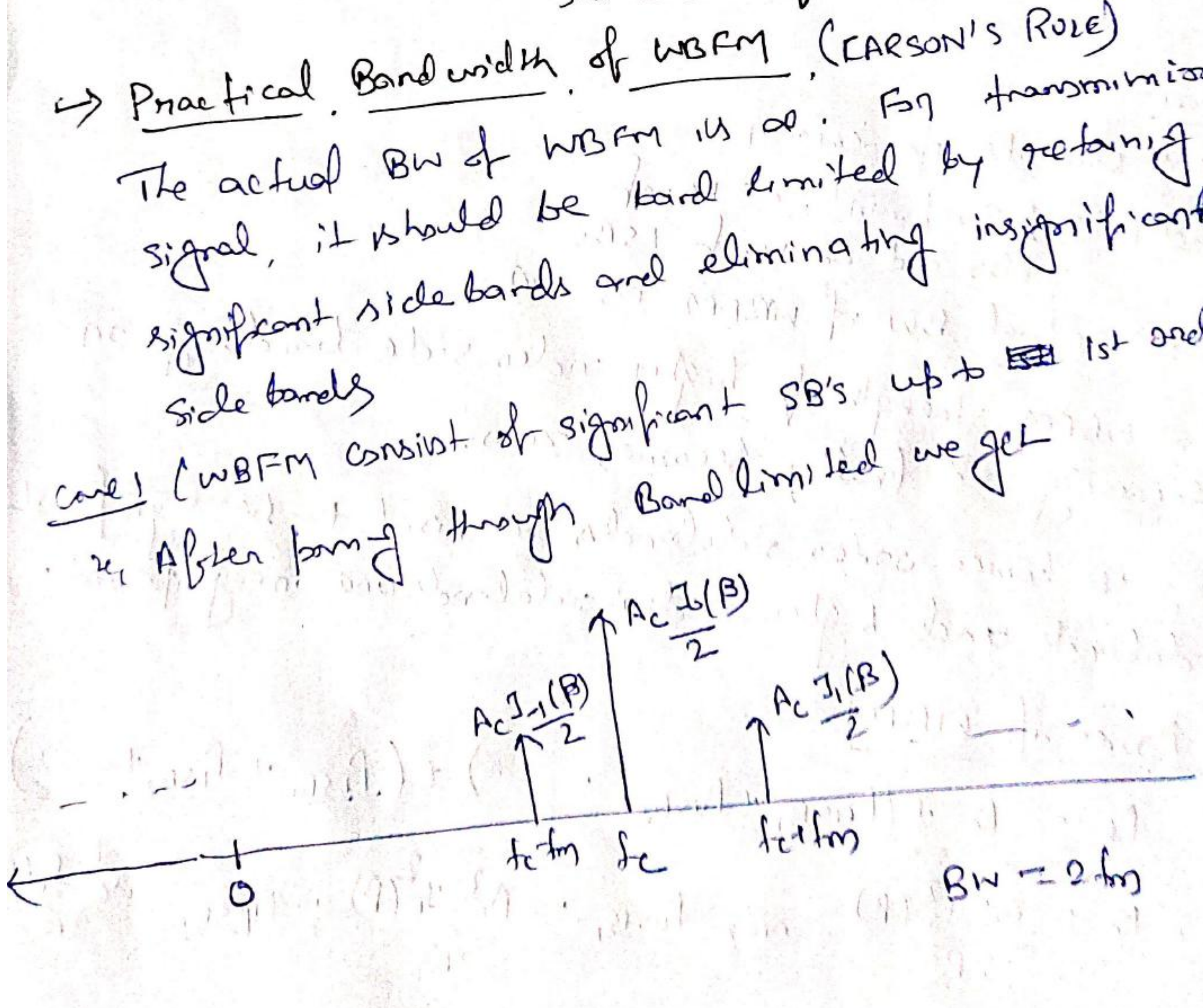
So for WBFM, the power of carrier before modulation is same as after modulation.

→ **Practical Bandwidth of WBFM** (CARSON'S RULE)

The actual BW of WBFM is ∞. For transmission of signal, it should be band limited by retaining on significant side bands and eliminating insignificant side bands

case 1 (WBFM consist of significant SB's up to 1st order)

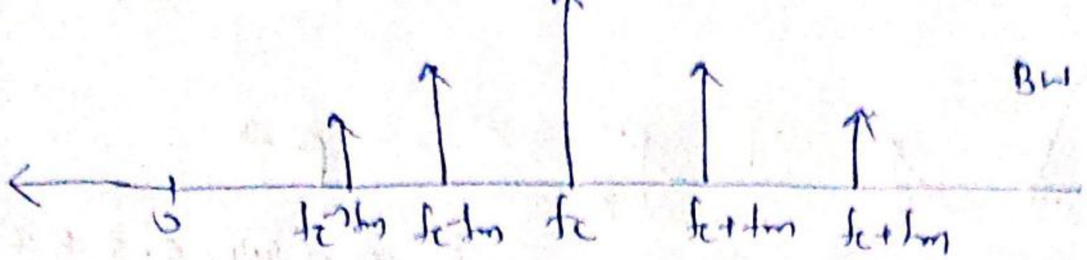
i.e. After passing through Band limited we get



BW = $2f_m$

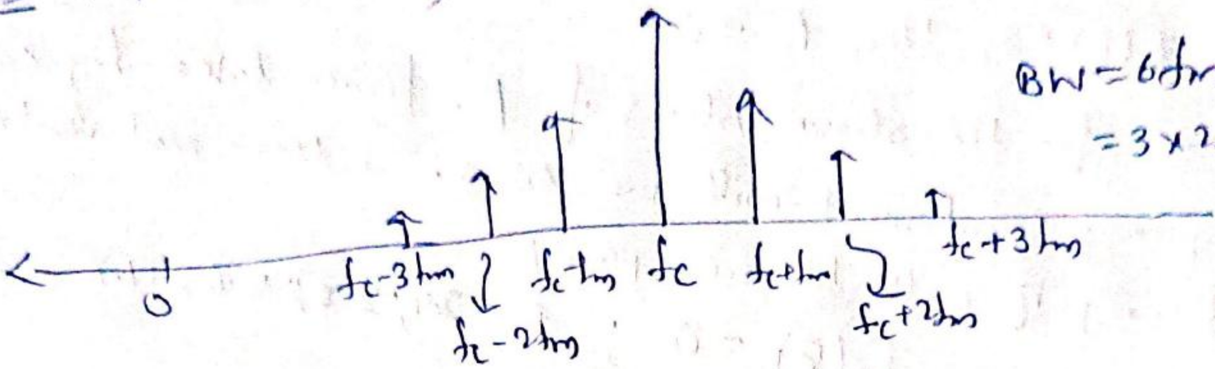
Case 2 (WBFM consists of significant SB's upto 2nd order)

After ~~significant~~ Band limited the signal we get:-



$BW = 4f_m$
$= 2 \times 2f_m$

Case 3 (up to 3rd order)



$BW = 6f_m$
$= 3 \times 2f_m$

CARSON'S RULE:

According to Carson, WBFM consists of the significant sideband upto "$\beta+1$", when the modulation index is β.

So, $BW = (\beta+1) \times 2f_m$

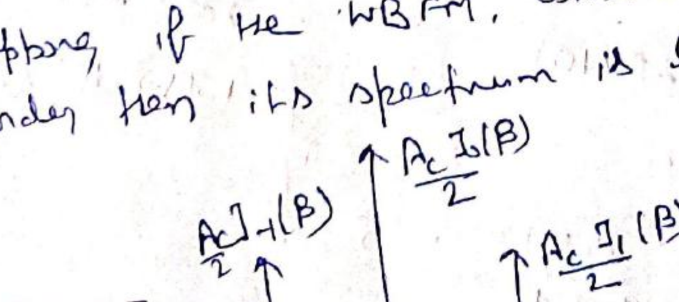
Since, $\beta = \frac{\Delta f}{f_m}$

So, $BW = \left(\frac{\Delta f}{f_m} + 1\right) 2f_m$

$$\boxed{BW = 2(\Delta f + f_m)} \quad \text{—— (23)}$$

Modulation Efficiency (η):

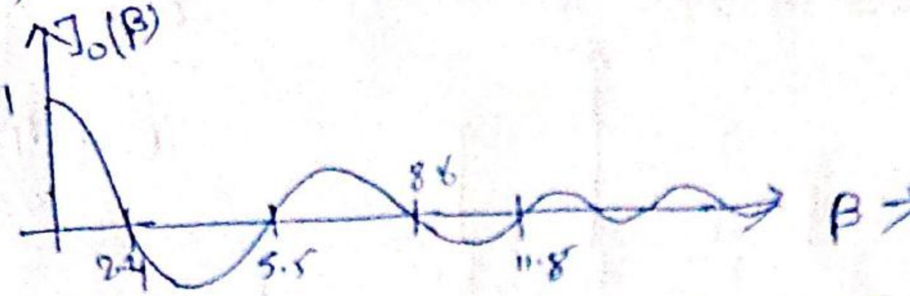
suppose, if the WBFM consists of significant SB's upto 1st order then its spectrum is like,



Here, the carrier power, at $f_c \Rightarrow P_c = \frac{A_c^2 J_0^2(\beta)}{2R}$

(18)

~~Hence~~ Since, the $J_0(\beta)$ with β is like:-



the standard values, $J_0(\beta) = 0$, for $\beta = 2.4, 5.5, 8.6, 11.8 \ldots$

If $J_0(\beta) = 0$, then $P_c = 0$

So for the above values of β, power taken by carrier frequency component will be zero, so that modulation efficiency η will become 100%

$$J_n(\beta) = 0 \; ; \; \beta = 2.4; 2.5; 8.6; 11.8$$

$$P_c = 0, \quad \eta = 100\%$$

Q. A sinusoidal carrier of 20V, 2MHz is frequency modulated by a sinusoidal message signal of 10V, 50 kHz

$K_f = 25 \text{ kHz/Volt}$

(a) find Δf; β; Bandwidth & power

(b) Repeat above if message signal amplitude is doubled.

Sol^n Given, $A_c = 20V$; $f_c = 2000$ kHz

$A_m = 10V$; $f_m = 50$ kHz

$K_f = 25 \dfrac{\text{kHz}}{\text{Volts}}$

So, $f_{max} = f_c + K_f\, m(t)$

$= (2000 + 25\times10)$

$= 2250$ kHz

$f_{min} = f_c - K_f\, m(t)$

$= 2000 - 25\times10$

$= 1750$ kHz

So, $\beta = \dfrac{\Delta f}{f_m} = \dfrac{K_f \cdot A_m}{f_m} = \dfrac{25\times10}{50\text{k}} = 5$

So, Bandwidth for WBFM

$BW = 2(\beta+1) f_m = 2(5+1) 50$

$= 600 kHz$

$P_t = \frac{A_c^2}{2R} = \frac{400}{2} = 200 W$

Q An FM signal is given by

$s(t) = 10 \cos\{2\pi \times 10^6 t + 8 \sin 4\pi \times 10^3 t\}$

(i) find β; Δf; BW & Power

~~(ii) Repeat above if~~

Given $s(t) = 10 \cos\{2\pi \times 10^6 t + 8 \sin 4\pi \times 10^3 t\}$

comparing with,

$s(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$

$A_c = 10 V$, $f_c = 1000k$; $\beta = 8$, $f_m = 2k$

So, $\Delta f = \beta f_m = 16K$

$BW = 2(\beta+1) f_m$

$= 2 \times 9 \times 2k = 36K$

$P_t = \frac{A_c^2}{2R} = \frac{100}{2} = 50 W$

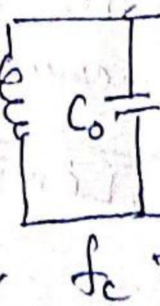
⇒ GENERATION OF FM

Generation of FM done by:

(i) Direct method

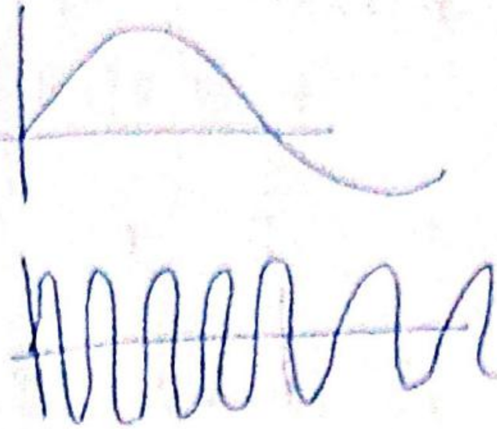
(ii) Indirect method or Armstrong Method

(i) Direct method

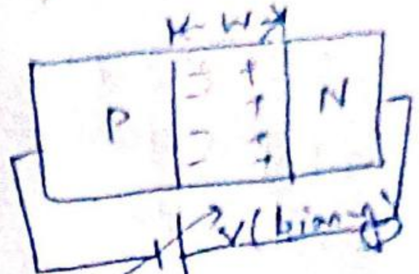


Here we use, the inductor and capacitor in parallel; it produces, $f_c = \frac{1}{2\pi\sqrt{LC_0}}$

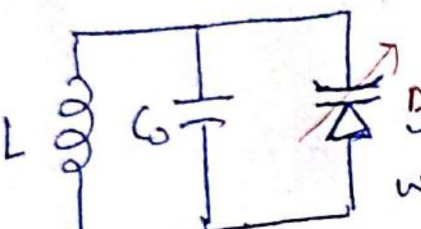
Now, for FM we want to varies the frequency of carrier signal according to the Amplitude of message signal



So, we use the **varactor Diode**
↓
it is a diode have variable internal capacitance in the Depletion region, which varies according to applied voltage.



Now, our ckt for FM



we assume here $x(t)$ is variable which is associated with message signal

$C = \frac{\epsilon A}{W}$, as reverse bias voltage ↑ then W ↑ so, C ↓

as, bias voltage ↓ then W ↓ so, C ↑

and that message signal is w applied as $V(t) \rightarrow$ biasing voltage of diode

→ So, as message signal increases, $x(t)$ increases
So W ↑ and ΔC ↓ and vice versa

So now, Capacitor
$$C\, x(t) = C_0 + \Delta C\, x(t)$$

and, $f_c = \frac{1}{2\pi\sqrt{LC_0}}$

Now, the instantaneous frequency

$$f_i = \frac{1}{2\pi\sqrt{LC\, x(t)}}$$

$$= \frac{1}{2\pi\sqrt{L(C_0 + \Delta C\, x(t))}}$$

$$f_i = \frac{1}{2\pi\sqrt{LC_0\left(1 + \frac{\Delta C}{C_0} x(t)\right)}}$$

$$f_i = \frac{1}{2\pi\sqrt{LC_0}\sqrt{1 + \frac{\Delta C}{C_0} x(t)}}$$

$$= f_c\left(1 + \frac{\Delta C}{C_0} x(t)\right)^{-1/2}$$

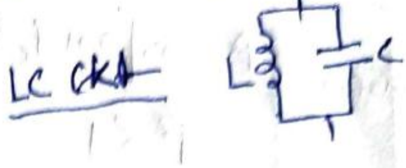
using binomial expansion

$$f_i = f_c\left[1 - \frac{\Delta C}{2C_0} x(t)\right]$$

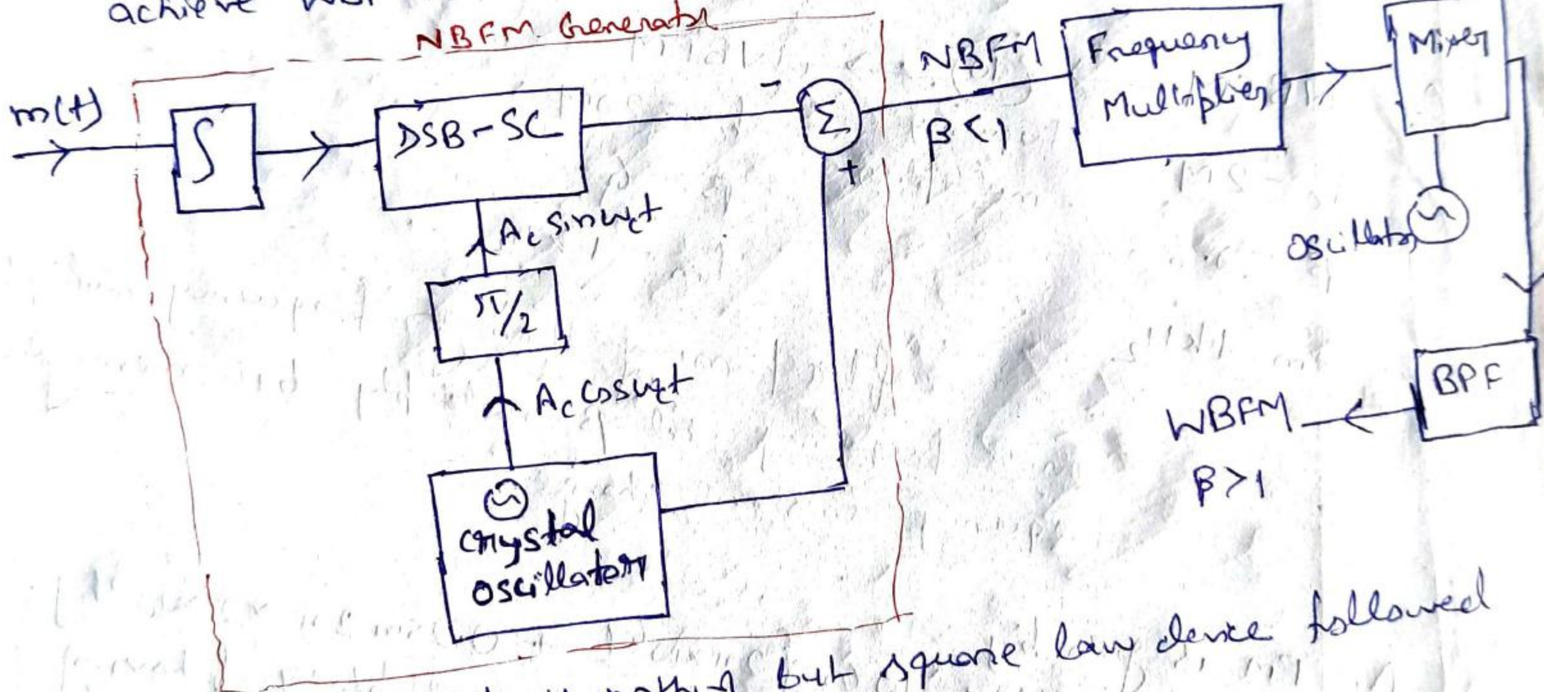
So, f_i varies according to $x(t)$

Indirect method of FM Generation.

In direct method, the frequency is generated with LC oscillator, but the frequency generated by LC ckt is not stable. So we didn't get the stable output frequency.



→ In indirect method, we generate the frequency from Crystal oscillator because it gives a stable output, but the problem is, it gives a stable frequency up to 1 MHz only. So, first we generate NBFM and using multiplier we achieve WBFM.



→ Frequency multiplier is nothing but square law device followed by proper pass band filter.

→ like $v_i \rightarrow$ squaring device $\rightarrow$ $v_0 = v_i^2$

$\cos 2\pi f_1 t$ → $\cos^2 2\pi f_1 t$

frequency multiplier, $(n=2)$

→ Frequency multiplier output $= S_{WBFM}$

$$= A_c \cos(2\pi n f_c t + n\beta \sin 2\pi f_m t)$$

n should be such that, $\boxed{n\beta > 1}$

Analysis of indirect method

NBFM ($\beta < 1$) $\rightarrow$ [Frequency Multiplier (n)] $\rightarrow$ WBFM ($\beta > 1$) the o/p get n times narrow band frequency

$S_{NBFM} = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$ $\rightarrow$ [Frequency multiplier (n)] $\rightarrow$ $A_c \cos[n(2\pi f_c t + \beta \sin 2\pi f_m t)]$

$\beta < 1$

$$S_{WBFM} = A_c \cos\{n 2\pi f_c t + n\beta \sin 2\pi f_m t\}$$

$$S_{WFM} = A_c \cos\{2\pi \underbrace{n f_c}_{\text{frequency increases n time}} t + \underbrace{n\beta}_{\beta \text{ increases n times}} \sin 2\pi f_m t\}$$

Ex NBFM $\rightarrow$ (10) frequency multiplier $\rightarrow$ WBFM

NBFM	WBFM
$f_c = 2M$	$f_c = 20M$
$\beta = 0.6$	$\beta = 6$
$f_m = 1kHz$	$f_m = 1kHz$

So after passing through frequency multiplier, carrier frequency and β multiply but message frequency does not change.

2. An FM is given by
$$s(t) = 10\cos\{2\pi \times 10^6 t + 0.2 \sin 2\pi \times 2 \times 10^3 t\}$$
It is passed through cascaded frequency multiplier of having multiplying const. of 4 and 5 respectively. Find all the parameters of FM signal at the o/p of each of the multiplier.

Sol^n Given $s(t) = 10\cos\{2\pi \times 10^6 t + 0.2 \sin 4\pi \times 10^3 t\}$

Comparing with standard eq^n
$$s(t) = A_c \cos\{2\pi f_c t + \beta \sin 2\pi f_m t\}$$

$A_c = 10V$

$\beta = 0.2$

$f_c = 1 MHz$

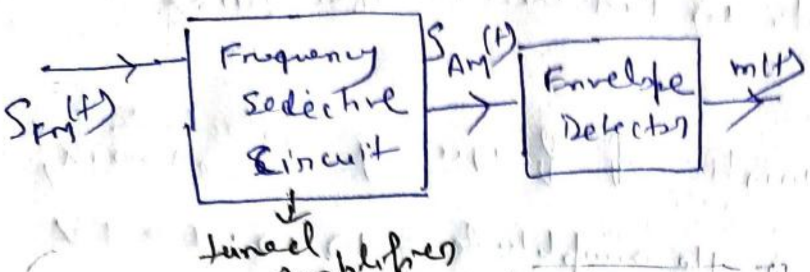
$f_m = 2 kHz$

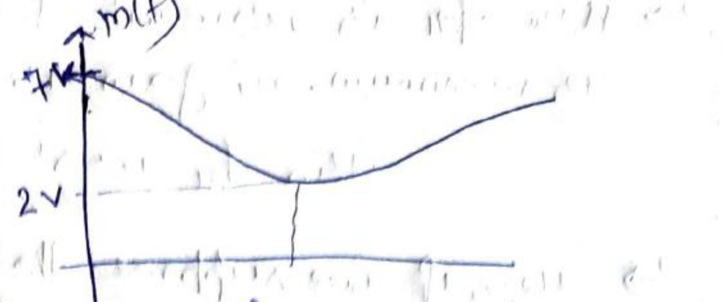
$\Delta f = \beta f_m = 0.4 kHz$

$s(t) \rightarrow$ [frequency multiplier (n=4)] $\xrightarrow{s'(t)}$ [freq^n Multiplier n=5] $\xrightarrow{s''(t)}$

↳ FM Demodulation

Slope Detection



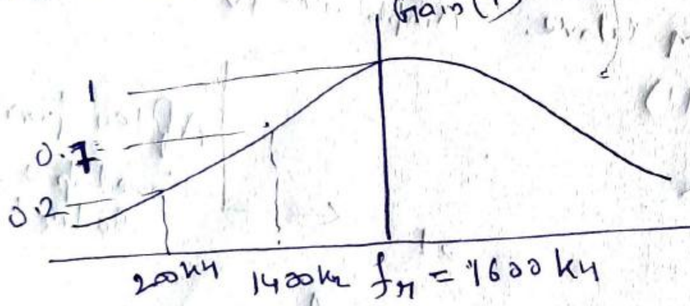


Let $F_{max} = 1400$ kHz
$F_{min} = 200$ kHz

frequency selective network

Envelope detector

the characteristic of frequency selective network



$f_c > f_{max}$

Due to the characteristic of frequency selective network, we convert FM to AM signal.

↳ Here, for different frequency different or variable Amplitude we get.

P.T.O

Remaining part of previous Question

After Passing ($n = 4$)

$A_c = 10$ V

$f_c = 4 \times 1 = 4$ MHz

$\beta = 4 \times 0.2 = 0.8$ (NBFM)

$f_m = 2$ kHz (no change)

$\Delta f = 4 \times 0.4$ (βf_m)
$= 1.6$ K

$BW = 2 f_m = 4$ k

$$P_t = \frac{A_c^2}{2R}\left\{1 + \frac{\beta^2}{2}\right\} = \frac{100}{2}\left\{1 + \frac{0.64}{2}\right\}$$
$$= 66 \text{ W}$$

After Passing ($n = 5$)

$A_c = 10$ V

$f_c = 5 \times 4$ MHz $= 20$ MHz

$\beta = 0.85 \times 5 = 4$ (WBFM)

$f_m = 2$ K

$\Delta f = 5 \times 1.6$ K $= 8$ K

$BW = 2(\beta + 1) f_m$
$= 2 \times 5 \times 2k = 20$K

$$P_t = \frac{A_c^2}{2R} = 50 \text{ W}$$

→ Here f_r is resonant frequency where the Amplitude is maximum. or, gain is max^m

this f_r must be greter than the input frequency.

→ Here, if we suppose that, in FM wave the, $f_{max} = 1400\,KHz$

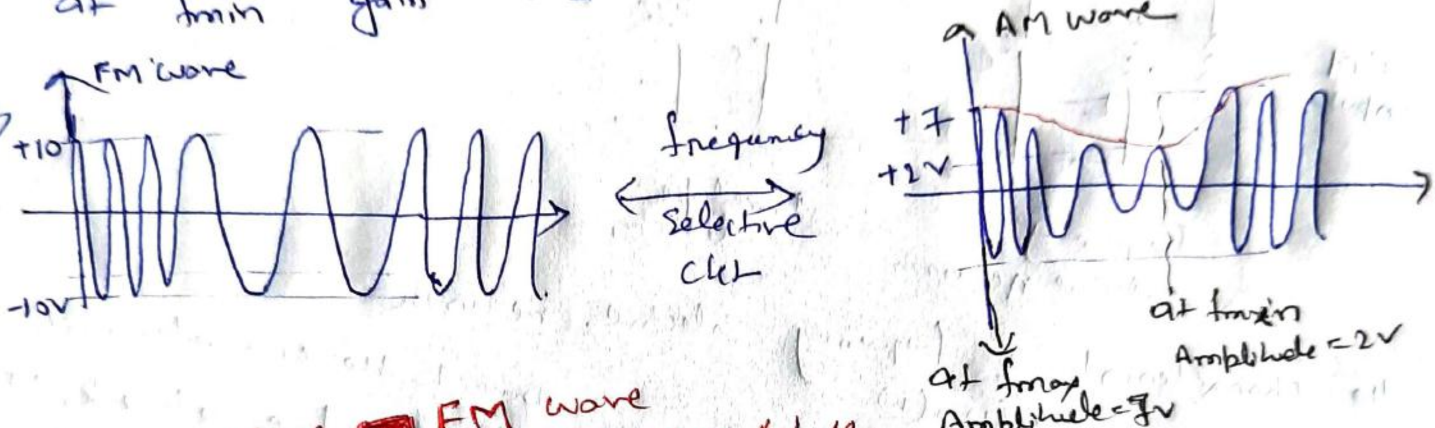
& $f_{min} = 200\,KHz$

by seeing the characteristic of frequency selective network,

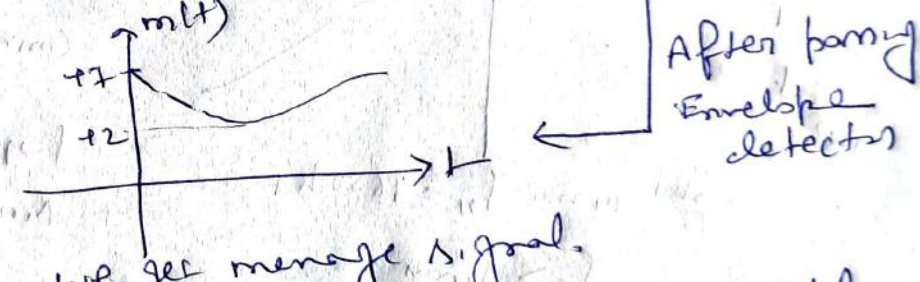
at f_{max}, gain $= 0.7$ So, The amplitude $= 10 \times 0.7 = 7V$

at f_{min} gain $= 0.2$ So, the amplitude $= 10 \times 0.2 = 2V$

So,



Here, FM wave converted into AM Wave



we get message signal.

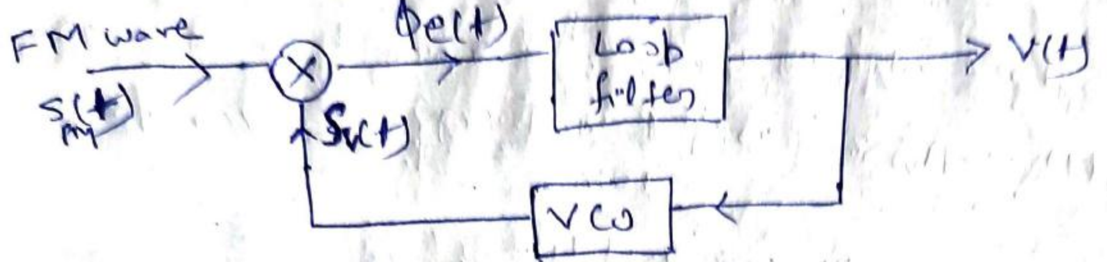
Note → Here, The gain frequency characteristic of Tuned Amplifier is non linear in nature so some amount of non-linearity will be introduce in frequency to voltage conversion.

So, the Reconstructed message signal is not perfectly corresponds to transmitted message signal and is called as SLOPE ERROR

PLL Method

Most widely used method (closed-loop feedback system)

→ It is used for indirect frequency demodulation and for carrier synchronisation.



Block diagram of PLL

Loop-filter → Remove high frequency component contained in multiplier output

VCO (voltage controlled oscillator) → Perform Frequency modulation on its own control signal

→ The frequency of FM signal changes continuously w.r.t to message signal voltage variations. So to maintain frequency synchronisation L.O of synchronous detector is replaced by VCO.

→ For the VCO, the message signal is taken as input, so that VCO o/p frequency changes continously w.r.t $m(t)$ voltage variations, and frequency synchronous can be achieved.

As → $S_{FM}(t) = A_c \cos\{2\pi f_1 t + \underbrace{2\pi k_f \int m(t)\,dt}_{\phi_1}\}$

and $(VCO)_{o/p} = A_v \cos\{2\pi f_2 t + \underbrace{2\pi k_v \int v(t)\,dt}_{\phi_2}\}$

for perfect reconstruction of message signal,

(i) f_1 should be made equal to f_2 i.e,

$f_1 = f_2 = f_c$

Then PLL is said to be working in the LOCK mode

(ii) $\phi_1(t)$ should be made equal to $\phi_2(t)$

$$\phi_1(t) = \phi_2(t)$$

then, PLL is said to be working in CAPTURE MODE

→ For Reconstruction of message signal VCO output should have 90° phase shift w.r.t. to transmitted carrier so,

$$S_v(t) = A_v \sin\{2\pi f_c t + \phi_2(t)\} \longrightarrow \text{VCO o/p}$$

$$\& \quad S_{FM}(t) = A_c \cos\{2\pi f_c t + \phi_1(t)\}.$$

So, the multiplier o/p is given as,

$$S_v(t) \times S_{FM}(t) = \frac{A_c A_v}{2}\left[\{\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)\} + \sin(\phi_2(t) - \phi_1(t))\}\right]$$

$$S_v(t) \cdot S_{FM}(t) = \frac{A_c A_v}{2} \sin\left(4\pi f_c t + \phi_1(t) + \phi_2(t)\right) - \frac{A_c A_v}{2} \sin\left(\phi_1(t) - \phi_2(t)\right) \quad \text{(A)}$$

Let $\phi_1(t) - \phi_2(t) \cong \phi_e(t)$
phase error

for PLL, $\phi_1(t)$ make very close to $\phi_2(t)$ so that
$\phi_e(t)$ will be very small,

So, eqn (A) is the multiplier output, and if it is pass through LPF.

$$S_v(t) \cdot S_{FM}(t) = \underbrace{\frac{A_c A_v}{2} \sin\left(4\pi f_c t + \phi_1(t) + \phi_2(t)\right\}}_{\text{blocked by LPF}} - \frac{A_c A_v}{2} \sin\{\phi_e(t)\}$$

Now, for $\phi_e(t)$ is very small,

then, $\sin\{\phi_e(t)\} \cong \phi_e(t)$

$$\text{so} \quad (\text{MUL})_{o/p} \cong -\frac{A_c A_v}{2} \phi_e(t)$$

$$\cong -\phi_e(t)$$

This is given to loop filter,

$$v(t) = \phi_e(t) \otimes h(t)$$

taking fourier transform on both side, we get,

$$V(f) = \phi_e(f) \cdot H(f)$$

since, $\phi_e(t) = \phi_1(t) - \phi_2(t)$

$$= \phi_1(t) - 2\pi k_v \int v(t)\, dt$$

where, $v(t) = \phi_e(t) \otimes h(t)$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v\, v(t)$$

$$\frac{d\phi_e(t)}{dt} = \frac{d}{dt}\phi_1(t) - 2\pi k_v \{\phi_e(t) \otimes h(t)\}$$

taking fourier transform on both side

$$j\cancel{2\pi} f\, \phi_e(f) = j\cancel{2\pi} f\, \phi_1(f) - 2\pi k_v \{\phi_e(f) \cdot H(f)\}$$

since, $\frac{d\phi(t)}{dt} \leftrightarrow j\omega\, \phi(f)$

So, $\phi_e(f)\{jf + k_v H(f)\} = jf\, \phi_1(f)$

So, $$\boxed{\phi_e(f) = \frac{jf\, \phi_1(f)}{jf + k_v H(f)}}$$

Also, $$\phi_e(f) = \frac{\phi_1(f)}{1 + \frac{k_v}{jf} H(f)}$$

Now when pass band gain, $H(f)$ of LPF $= \infty$

then $\phi_e(f) = 0 \Rightarrow \phi_e(t) = \phi_1(t) = \phi_2(t)$

But practically it is not possible, so the pass band gain of LPF is made high

Note:

① If pass band gain of LPF is ∞ i.e. $H(f) = \infty$

then, $\phi_e(t) = 0$, but for practical LPF; $H(f)$ will be finite.

(ii) By making $H(f)$ = very large; $\phi_e(t)$ will be made very small quantity.

⇒ Now, substituting value of $\phi_e(f)$ in the eqn $V(f) = \phi_e(f) \cdot H(f)$

$$\text{So, } V(f) = \frac{\phi_1(f)}{1 + \frac{K_v}{jf} H(f)} \cdot H(f)$$

Since $H(f)$ is very large, so $1 + \frac{K_v}{jf} H(f) \simeq \frac{K_v}{jf} H(f)$

$$\text{So, } V(f) = \frac{\phi_1(f)}{\frac{K_v}{jf} \cdot H(f)} \cdot H(f)$$

$$V(f) = \frac{jf}{K_v} \phi_1(f) \times \frac{2\pi}{2\pi}$$

By taking inverse F.T we get,

$$v(t) = \frac{1}{2\pi K_v} \cdot \frac{d}{dt} \phi_1(t)$$

$$= \frac{1}{2\pi K_v} \cdot \frac{d}{dt} \left\{ 2\pi K_f \int m(t)\, dt \right\}$$

$$\boxed{v(t) = \frac{K_f}{K_v} m(t)}$$

So we get the o/p as message signal

Note:

(i) if $S_v(t) = A_v \cos\{2\pi f_c t + \phi_2(t)\}$

$S_{FM}(t) = A_c \cos\{2\pi f_c t + \phi_1(t)\}$

So, $(Mul)_{o/p} = \frac{A_c A_v}{2} \cos\{4\pi f_c t + \phi_1(t) + \phi_2(t)\} + \frac{A_c A_v}{2} \cos\{\phi_1(t) - \phi_2(t)\}$

and, $\phi_e(t) = \phi_1(t) - \phi_2(t) \approx 0$

so, $\cos\{\phi_e(t)\} \approx 1$

So input of LPF = 1, hence message signal couldn't be obtained

So, we use $S_v(t) = A_v \sin\{2\pi f_c t + \phi_2(t)\}$

In PLL, (i) –ve feedback is Responsible for LOCK MODE

(ii) LPF is responsible for CAPTURE MODE of PLL.

(iii) The frequency produced by VCO corresponds to LOCK MODE of PLL

⇒ PHASE MODULATION

carrier before phase modulation $= A_c \cos(2\pi f_c t)$

carrier After phase modulation

$S_{PM}(t) = A_c \cos\{2\pi f_c t + \underbrace{\phi(t)}_{\text{phase deviation}}\}$

where $\phi(t) = k_p m(t)$

k_p = phase sensitivity of phase modulator (rad/volt)

$$\boxed{S_{PM}(t) = A_c \cos\{2\pi f_c t + \underbrace{k_p m(t)}_{\text{phase deviation}}\}}$$

Let, $m(t) = A_m \cos\{2\pi f_m t\}$

so, max^n phase deviation

$$\Delta\phi = \max\{k_p\, m(t)\}$$

$$\boxed{\Delta\phi = k_p A_m}$$

so $S_{PM}(t) = A_c \cos\{2\pi f_c t + k_p A_m \cos 2\pi f_m t\}$

let $k_p \cdot A_m = \beta$ = modulation index of PM

so (i) the generation expression for FM & PM are same expect 90° phase shift at message frequency component

(ii) The Magnitude spectrum of PM will be same as FM so that BW and power requirements of PM & FM will be same.

→ BW of PM (WBFM)

$$BW = 2(\beta + 1) f_m$$

$$= 2(\Delta\phi + 1) f_m$$

Q $S_{PM}(t) = 10 \cos\{2\pi \times 10^6 t + 6 \sin 6\pi \times 10^3 t\}$

so, find all parameters of PM.

→ The standard eq^n of PM

$$S_{PM}(t) = A_c \cos\{2\pi f_c t + \beta \cos 2\pi f_m t\}$$

so, $A_c = 10$, $f_c = 1\ MHz$, $f_m = 3\ kHz$

$\beta = k_p A_m = 6\ rad = \Delta\phi$

$BW = 2(\beta + 1) f_m = 2 \times 7 \times 3$

$= 42\ kHz$