DARBHANGA COLLEGE OF ENGINEERING,DARBHANGA

MODERN CONTROL THEORY
(SEM-VIII:EEE)

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Module 1
Lecture-2

State Space Systems

General Form of a linear time invariant state space model

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$

- $t \in \mathbb{R}$ denotes time
- $\mathbf{x}(t) \in \mathbb{R}^n$ is the state (vector)
- $\mathbf{u}(t) \in \mathbb{R}^m$ is the input or control
- $\mathbf{y}(t) \in \mathbb{R}^p$ is the output

- $A(t) \in \mathbb{R}^{n\times n}$ is the dynamics matrix
- $B(t) \in \mathbb{R}^{n\times m}$ is the input matrix
- $C(t) \in \mathbb{R}^{p\times n}$ is the output or sensor matrix
- $D(t) \in \mathbb{R}^{p\times m}$ is the feedthrough matrix

Obtaining state space equations from differential equations

consider an n^{th} order linear plant model described by the differential equation

$$\frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1}y}{dt^{n-1}} + \cdots + a_1\frac{dy}{dt} + a_0 y = u(t)$$

We can define a useful set of state variables as

$$x_1 = y, \quad x_2 = \dot{y}, \quad x_3 = \ddot{y}, \ldots, x_n = \frac{d^{n-1}y}{dt^{n-1}}$$

Taking derivatives of the first $n-1$ state variables, we have

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = x_3, \quad \ldots, \quad \dot{x}_{n-1} = x_n$$

Finally:

$$\dot{x}_n = -a_0 x_1 - a_1 x_2 - \cdots - a_{n-1} x_n + u(t)$$

How does this look in matrix form?

Obtaining state space equations from differential equations

In matrix form, this looks like

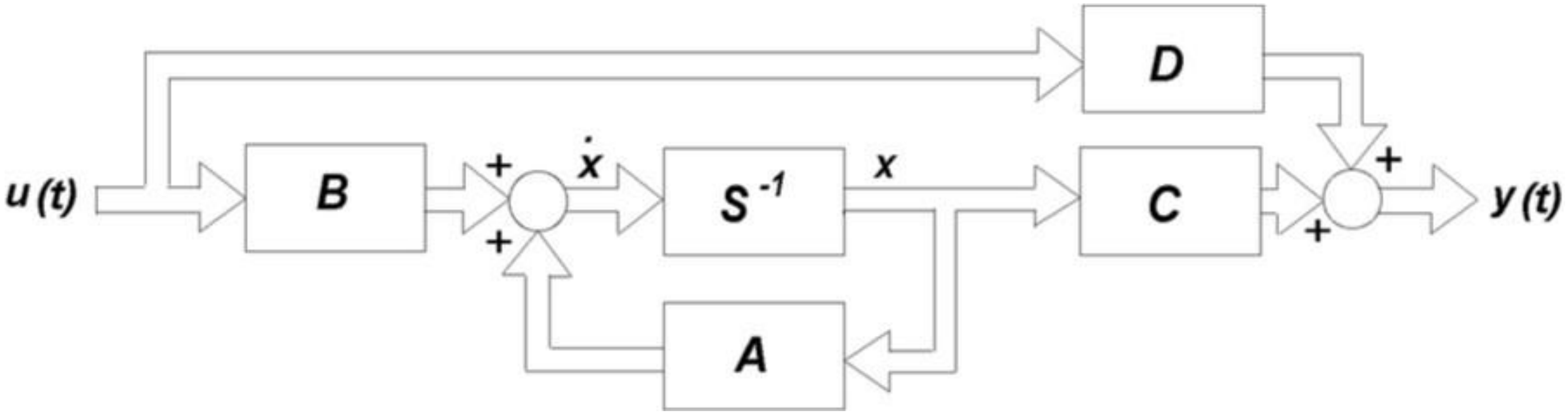
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t)$$

If we are measuring only one state, then we have

$$y = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \end{bmatrix} \vec{x}$$

General Form of a linear time invariant state space model

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t)$$
$$\mathbf{y}(t) = C\mathbf{x}(t) + D\mathbf{u}(t)$$



Example

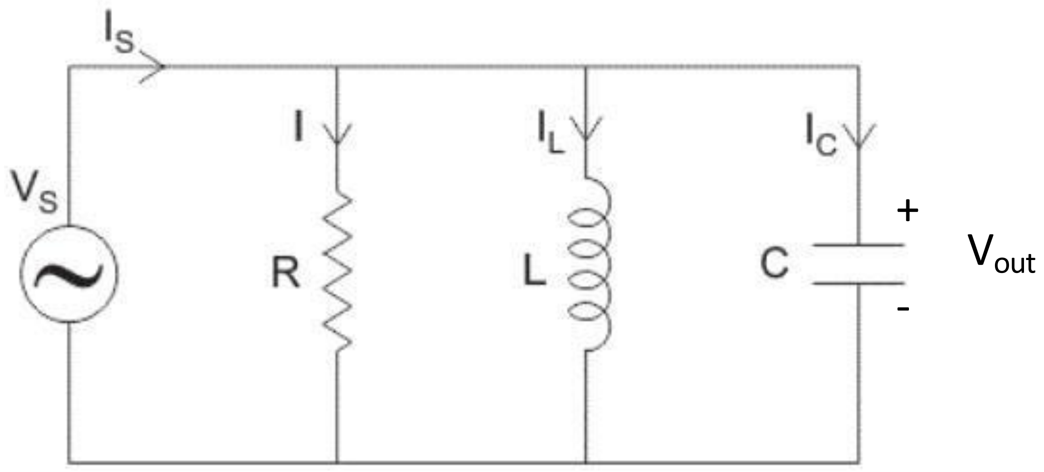
Obtain the state equation in phase variable form for the following differential equation:

$$2\frac{d^3y}{dt^3} + 4\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 10u(t)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Simple RLC example

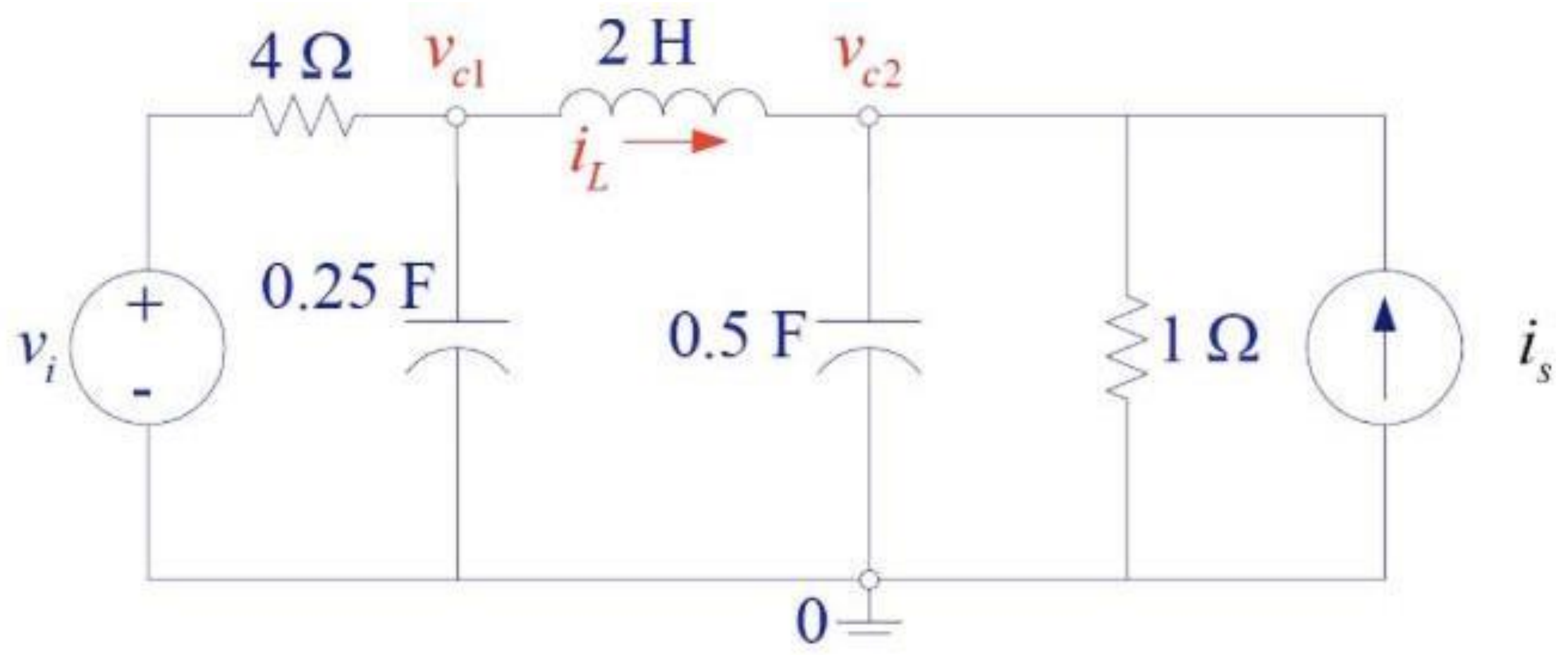


The final state space model will be as below

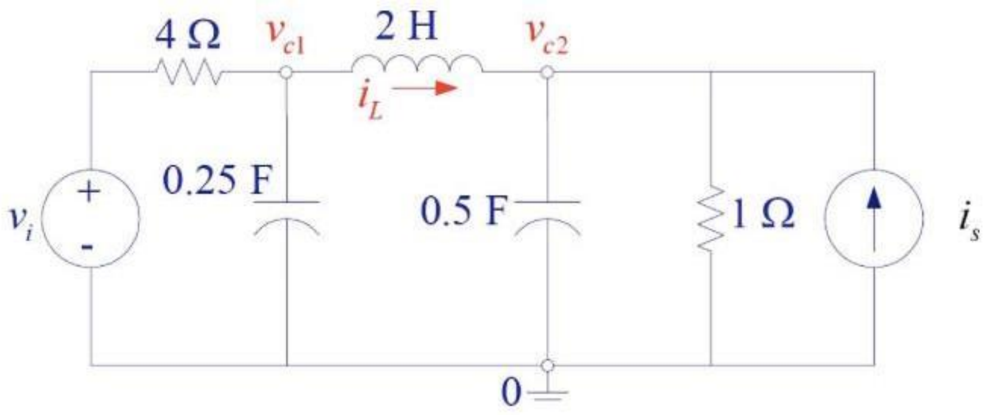
$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{L} \\ -\frac{1}{C} & -\frac{1}{RC} \end{bmatrix} \begin{bmatrix} I_L \\ V_c \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C} \end{bmatrix} I_s$$

Example

Write the state equation for the network shown



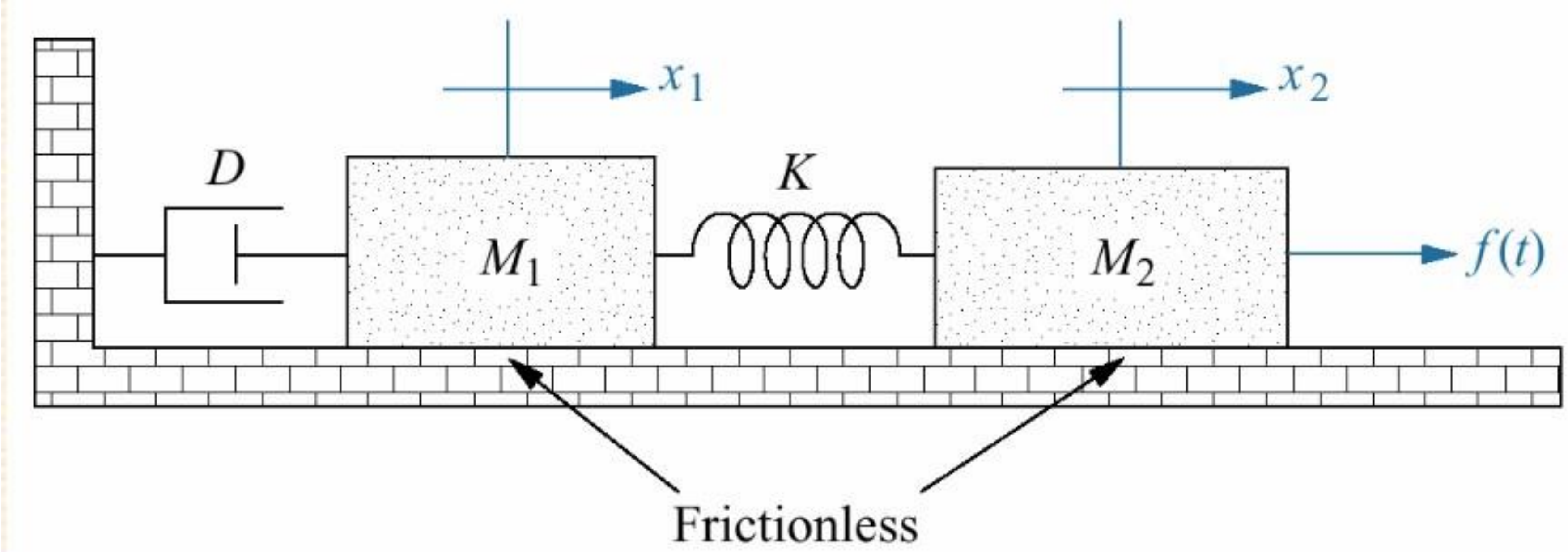
Example



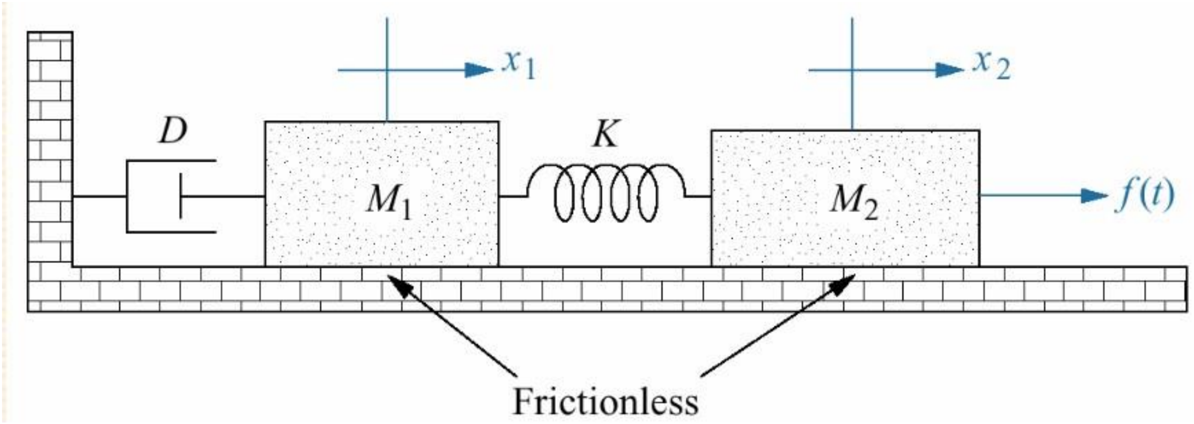
The state space model will be

$$\begin{bmatrix} \dot{v}_{c1} \\ \dot{v}_{c2} \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -1 & 0 & -4 \\ 0 & 2 & -2 \\ 0.5 & -0.5 & 0 \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_L \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & -2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_i \\ i_s \end{bmatrix}$$

Example – coupled mass spring damper



Example – coupled mass spring damper



$$M_1 \frac{d^2 x_1}{dt^2} + D \frac{dx_1}{dt} + K(x_1 - x_2) = 0$$

$$M_2 \frac{d^2 x_2}{dt^2} + K(x_2 - x_1) = f(t)$$

Define new variables: $$v_1 \triangleq \frac{dx_1}{dt}, \quad v_2 \triangleq \frac{dx_2}{dt}$$

Example – coupled mass spring damper

System equations

$$\frac{dv_1}{dt} = -\frac{K}{M_1}x_1 - \frac{D}{M_1}v_1 + \frac{K}{M_1}x_2$$

$$\frac{dv_2}{dt} = \frac{K}{M_2}x_1 - \frac{K}{M_2}x_2 + \frac{1}{M_2}f(t)$$

The state space eqns in standard form are then:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{M_1} & -\frac{D}{M_1} & \frac{K}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{K}{M_2} & 0 & -\frac{K}{M_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{M_2} \end{bmatrix} f(t)$$

States are position and velocity of each mass

Overview

- Summary: Lecture

- Introduction
- Simple example (mass-spring-
- damper)
- Definition of a state space system
- Three examples for state space modeling – mechanical and electrical
- Relation between state space and transfer function